INTERNATIONAL
ECONOMIC
REVIEW

May 2017
Vol. 58, No. 2

WHEN AND HOW THE PUNISHMENT MUST FIT THE CRIME*

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In repeated normal-form (simultaneous-move) games, *simple penal codes* (Abreu, *Journal of Economic Theory* 39(1) (1986), 191–225; and *Econometrica* 56(2) (1988), 383–96) permit an elegant characterization of the set of subgame-perfect outcomes. We show that in repeated *extensive*-form games such a characterization no longer obtains. By means of examples, we identify two types of settings in which a subgame-perfect outcome may be supported *only* by a profile with the property that the continuation play after a deviation is tailored not only to the identity of the deviator but also to the nature of the deviation.

My object all sublime
I shall achieve in time
To let the punishment fit the crime,
The punishment fit the crime;
And make each prisoner repent
Unwillingly represent
A source of innocent merriment,
Of innocent merriment!
W. S. Gilbert (1885), *The Mikado*

1. INTRODUCTION

Many popular applications of game theory are naturally modeled as simultaneous-move stage games, such as the Cournot oligopoly model of collusion. But other interesting applications have a dynamic structure whose stage game interactions are more naturally represented by nontrivial extensive-form stage games. Examples include the interaction between government and the private sector in the time-inconsistency literature, between upstream and downstream firms in the vertical relations literature, between bidders in open-outcry auctions, between firms that first

*Manuscript received February 2015; revised November 2015.
[1] Mailath thanks the National Science Foundation for research support (grants SES-0350969). Nocke is grateful for financial support from the European Research Council (grant no. 313623). This is a major revision of "When the Punishment Must Fit the Crime: Remarks on the Failure of Simple Penal Codes in Extensive-Form Games," originally circulated in 2004. We thank the editor Simon Board and three anonymous referees for helpful comments and Larry Samuelson for helpful discussions. Please address correspondence to: Volker Nocke, UCLA Department of Economics, Bunche Hall 8283, 315 Portola Plaza, Los Angeles, CA 90095, U.S.A. Phone: +1 (310) 794-6617. Fax: +1 (310) 825-9528. E-mail: *volker.nocke@gmail.com*.

invest or choose standards and then compete, between proposer and responders in bargaining games, and between principal and agent in contracting games. Such applied models often feature a socially desirable (or "cooperative") outcome that is inconsistent with equilibrium in a one-shot game because of opportunistic actions available to players, but that might be attainable in a repeated game. Economists are often interested in exploring how changes in institutions and/or repetition of the game might allow the more desirable outcomes to be supported as equilibria (see, e.g., the seminal work of Friedman, 1971). But with a few exceptions,[2] such analysis has been largely confined to repeated *normal-form* games, where all players are modeled as moving simultaneously in the stage game.

When players are sufficiently patient, one may ignore the detailed dynamic structure within the stage-game interaction and apply standard folk theorem arguments (Wen, 2002). The reason is that even a small difference in continuation values will dominate any short-term gain for patient players, so deviations are relatively easy to deter. However, applied theory is often concerned with the impact of a change in institutions or the environment on the set of equilibrium outcomes, and such an analysis is meaningful only for *impatient* players. In this article, we show that for impatient players, the dynamic structure of the stage game can be important and should not be neglected.

The literature has so far directed surprisingly little attention toward the study of repeated extensive-form games with impatient players. For applications of repeated simultaneous-move games, the techniques developed by Abreu (1986, 1988) are central. Abreu (1988) shows that any pure-strategy subgame-perfect equilibrium outcome can be supported by a set of simple punishment strategies called *simple penal codes*: If a player, i say, deviates from the proposed equilibrium play in a given period, in the next period, players switch to player i's worst equilibrium play (called i's *optimal penal code*). A similar rule applies to any player who deviates from play during an optimal penal code. In other words, the continuation play after a deviation by a player is *independent* of the nature of the deviation, depending only on the identity of the deviator. This result vastly simplifies the task of finding the set of equilibria that can be supported in repeated simultaneous-move games: One needs only to characterize a worst equilibrium for each player, and from there one can proceed to fill in all other equilibria that can be supported by using these punishment strategies. Abreu's result has been correspondingly important for applications employing normal-form stage games.

But what about applications that involve extensive-form stage games? In this article, we show that a similar simplification is not available for characterizing the set of subgame-perfect equilibrium outcomes when the stage game has a nontrivial dynamic structure. We begin by discussing the appropriate restrictions that simple penal codes should satisfy in this case. We then present two settings in which simple penal codes can fail to support behavior as an equilibrium that is supportable with more complicated strategies (for a given discount factor). The forces driving the equilibrium incentives in these settings are intuitive and natural.

Consider a deviation by some player in a repeated simultaneous-move game. Since the stage game is a simultaneous-move game, the other players can respond only in the next period. Abreu's (1988) results rely on the observation that, in repeated simultaneous-move games, every subgame is strategically equivalent to the original repeated game.[3] Hence, the worst punishment is simply the worst subgame-perfect equilibrium of the original game. Consider now a deviation in a repeated extensive-form game. In contrast to normal form games, the other players may be able to respond not only in the next period but also within the same period. Moreover, the deviation may lead to a subgame that is not strategically equivalent to the original game. Consequently, the appropriate notion of simple penal code for a repeated extensive-form game is not obvious. Nonetheless, for any history that ends at the end of a

[2] Repeated extensive-form games have been analyzed in the relational contracting literature (Levin, 2003), in the policy games literature (Athey et al., 2005), and in the vertical relations literature (Nocke and White, 2007).

[3] This observation is a direct implication of the property that every subgame's initial node is at the beginning of some period.

period, the associated subgame is strategically equivalent to the original repeated game. This suggests that any definition of a simple penal code should have the feature that after a player deviates from the candidate equilibrium, subsequent play *beginning in the next period* should be independent of the precise nature of the deviation.

Our two sets of examples show that there are equilibria in repeated extensive-form games that cannot be supported by continuation play satisfying this property: Even though it is feasible to play the same outcome path after two different deviations starting in the next period, sustaining a candidate equilibrium outcome may require that different outcome paths be played.

Say that an action for a player is *myopically suboptimal* if, given the specified behavior for the other players, that action is not optimal for that player when payoffs from future periods are ignored. Simple penal codes are not sufficient to support all equilibria in repeated extensive-form games because some equilibria require the use of within-period myopically suboptimal "punishments" to ensure that deviations are not profitable. In contrast, in repeated normal-form games, the sequential rationality of "within-period" punishments is not an issue. In each of our examples, the within-period punishment is myopically suboptimal for the potential punisher(s) but is required to sustain the desired equilibrium.

In our first set of examples, the need for different continuations arises because the interests of the deviator and the potential punisher are aligned, though imperfectly. Many important settings have this structure—for example, policy games, models of relational contracting, and models where players engage in repeated investment or production. In such settings, using continuation play to reward a player for carrying out myopically suboptimal within-period punishment of the earlier deviator may necessarily also reward the deviator. Consequently, the rewards for the punisher (and hence the outcome path following the deviation) may have to be fine-tuned to the particular deviation chosen by the deviator. Within-period punishment is valuable after some, but not all, deviations, depending on how effective and costly is within-period punishment after a particular deviation. We first analyze a simple example to illustrate the difficulty and then show how the same logic affects a repeated game of bilateral investment with hold-up in the spirit of Klein et al. (1978) and Grossman and Hart (1986).

In our second set of examples, we highlight a contrasting problem, which arises because players moving after a deviator are required to coordinate to inflict effective within-period punishment. In contrast to the first set, where complications arise when interests are aligned, here, difficulties occur when the interests of the players required to inflict punishment are in conflict. The simplest case has three players, and sustaining the desired equilibrium requires that the two later movers both inflict within-period punishment on the first mover in the event of a deviation. Think, for instance, of a situation where a deviation is profitable if and only if at least one other player 'accepts' it. Such a structure is common in many important settings. Examples include colluding upstream firms selling through downstream firms (Nocke and White, 2007), attempted expropriation by a sovereign power (where citizen groups can cooperate to successfully resist expropriation, as in Weingast, 1995, 1997), bargaining over a series of proposals where a majority vote is sufficient for acceptance (Baron and Ferejohn, 1989), or attempted entry when an entrant requires more than one customer to break even (as in Rasmusen et al., 1991; Segal and Whinston, 2000). The conflict of interest between the two potentially enforcing players in these cases means that it is not possible to provide the maximum "reward" to both the punishers in any given equilibrium. Intuitively, the continuation equilibrium chosen (and hence the "reward" that each punisher receives) must depend on the degree of sacrifice which the punisher makes in inflicting the myopically suboptimal punishment. Which of the two later moving players receives more of the "carrot" in future play optimally depends on for which of them it was more costly to apply the "stick." Again, we first analyze a simple example and then turn to the classic "naked exclusion game" of Rasmusen et al. (1991) and Segal and Whinston (2000) to illustrate the operation of optimal punishments in that context.

Apart from Rubinstein and Wolinsky (1995), Sorin (1995), Wen (2002), and Mailath and Samuelson (2006), the repeated-game literature has focused on repeated normal-form games,

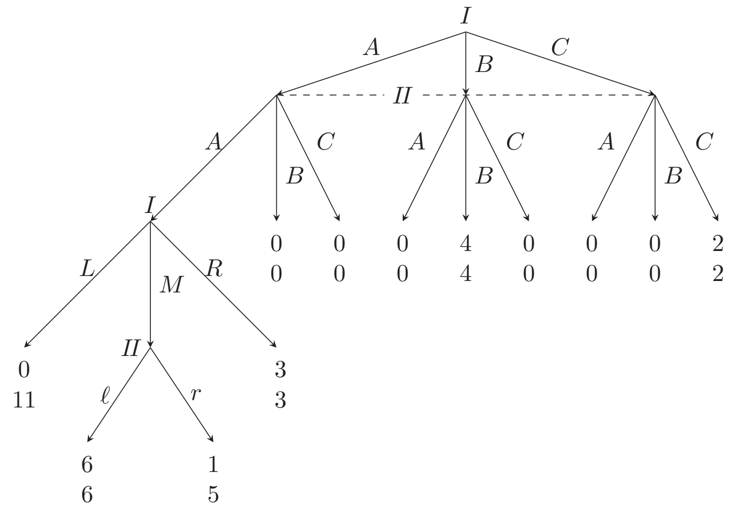


FIGURE 1

THE GAME Γ_1

ignoring the dynamic structure within the stage game. Rubinstein and Wolinsky (1995) present some examples illustrating the difference between the set of subgame-perfect equilibrium payoffs of repeated extensive and normal-form games for patient players when the standard full dimensionality condition of Fudenberg and Maskin (1986) does not hold. Sorin (1995) discusses the implications of the different information that players have available across periods in repeated extensive, instead of normal, form games. Wen (2002) extends the arguments of Abreu (1988) to prove a folk theorem for repeated sequential-move games under a weaker condition than Fudenberg and Maskin's (1986) full dimensionality condition. Finally, Mailath and Samuelson (2006, section 9.6) prove a folk theorem for repeated extensive form games via an extension of the tools of Abreu et al. (1990). None of these papers is concerned with penal codes or with characterizing the set of subgame-perfect equilibrium payoffs with impatient players.

2. THE PUNISHMENT SHOULD FIT THE CRIME

"Is it her fault or mine?
The tempter or the tempted—who sins the most?"
William Shakespeare, *Measure for Measure*, Act 2, Scene 2.

In this section, we highlight the value of tailoring the punishment to fit the deviation in games where the potential deviator and punisher have a commonality of interest. Thus our games have a strong coordination flavor. The most effective punishments can be complicated in such settings, because it is difficult both to punish the deviator and reward the punishing player for applying the costly punishment.

We first present a simple stylized example to illustrate the issues that arise. For ease of exposition, we focus on equilibria in pure strategies. In the following subsection, we will present a more interesting application to a game of repeated investment and hold-up.

2.1. *A Simple Example.* We begin with the extensive-form game Γ_1, presented in Figure 1. Consider the perfect information subgame reached after both players choose A. The unique subgame-perfect equilibrium of this subgame has player I playing M and player II playing ℓ (after M). In the subgame, player II, of course, prefers that player I plays L.[4] Thus, in any

[4] The subgame of Γ_1 following both players choosing A has an interpretation as an entry game between a potential entrant (player I) and an incumbent (player II). The entrant can decide to stay out (play L), enter with product

subgame-perfect equilibrium of Γ_1, *AA* yields payoffs of (6, 6). As the payoffs of the two players are perfectly correlated across the three subgame-perfect equilibria (involving the play of *AA*, *BB*, or *CC* at the first information set), Γ_1 has a coordination game structure.

The game Γ_1 is repeated once, and payoffs are summed over the two periods. The aim of the example is to show the need to tailor the punishment to fit the deviation in situations where players' interests are closely aligned. To this end, note that in the second period, the payoffs of the two players in any subgame-perfect equilibrium are, by construction, identical. It is therefore impossible to punish (or reward) one player without simultaneously doing the same to the other.

We will show that the choice of *L* by player *I* in the first period can be supported only if the responses to each of the two attractive deviations for player *I*, *M* and *R*, differ. Note first that the payoffs in this example have been chosen so that the variation in second-period payoffs alone is insufficient to deter player *I* from playing *M* when player *II* chooses ℓ, her myopic best reply: Player *I*'s first-period incentive to deviate by playing *M* is then 6, whereas the largest punishment the second period can impose is the profile *CC*, with associated loss of payoff of 4.[5]

However, if player *II* can be induced to play *r* following a deviation to *M*, the benefit to player *I* of *M* will be drastically reduced (from 6 to 1). The trick is to provide appropriate incentives to prevent *II* from myopically optimizing, that is, in specifying a higher continuation payoff after *Mr* than after $M\ell$. At the same time, the continuation play after *Mr* must not ignore *I*'s original deviation. This motivates the following specification of second-period play: Play *AA* after *L*, *BB* after *Mr*, and *CC* after $M\ell$ and *R*. It is straightforward to check that this specification supports *Lr* as equilibrium first-period choices.

In the profile described in the previous paragraph, *different* continuation equilibria are specified after *I*'s deviation to *M* and to *R*. We have already seen that the second-period play of *BB* after *Mr* is needed to make *II*'s choice of *r* optimal. At the same time, a play of *BB* after *R* does not provide a sufficient disincentive for *I*, so *R* must be followed by *CC*.

The play of *L* in the first period cannot therefore be sustained by a "simple" penal code in which continuation play is independent of the particular deviation chosen by player *I*. As we have seen, the play of *L* is sustained by a more complex strategy profile that employs different punishments after different deviations.

The discussion above shows that there is a subgame-perfect equilibrium of the repeated game in which, in the first period, both players choose *A*, and then player *I* chooses *L*. (The only issue we have not addressed is a deviation by *I* or *II* to *B* or *C* in the first period. Player *II* clearly cannot benefit from such a deviation. For player *I*, the period-1 payoff from this deviation is 0, the same as from *Mr* in Γ_1, and the second-period play is the same as well, and so the deviation is not profitable.)

To conclude our discussion of this example, it is useful to compare our analysis with that of an analysis of the repeated normal form of Γ_1. The normal form is given in Figure 2. Treating the simultaneous-move normal form of Figure 2 as the stage game, the profile in which $(AL,\ Ar)$ is played in the first period and $(AR,\ A\ell)$ in the second, with *any* deviation by player 1 resulting in *CC* in the second period, is a subgame-perfect equilibrium of the repeated game. However, this profile is a subgame-perfect equilibrium only because the simultaneity of moves means that there is no subgame beginning with *II*'s choice between ℓ and *r*, and so subgame perfection does not require that choice to be optimal.

line *M* (play *M*), or enter with product line *R* (play *R*). (One can also interpret *M* as small-scale entry and *R* as large-scale entry.) Following entry with product *M*, the incumbent can choose to acquiesce (play ℓ) or fight (play *r*). We assume here for simplicity that fighting is possible only when product *M* is chosen, but one can also allow fighting after choice of *R* with, for example, payoffs (−1,1) without changing any of the conclusions reached in the analysis below.

[5] This conclusion would not change if we allowed for mixed strategies. The most severe punishment that can be inflicted involves playing the mixed-strategy equilibrium with probabilities 2/11, 3/11, and 6/11 on actions *A*, *B*, and *C*, respectively, resulting in an expected payoff of 12/11 for each player.

	$A\ell$	Ar	B	C
AL	$1, 11$	$1, 11$	$0, 0$	$0, 0$
AM	$6, 6$	$1, 5$	$0, 0$	$0, 0$
AR	$3, 3$	$3, 3$	$0, 0$	$0, 0$
B	$0, 0$	$0, 0$	$4, 4$	$0, 0$
C	$0, 0$	$0, 0$	$0, 0$	$2, 2$

FIGURE 2

THE (REDUCED) NORMAL FORM OF Γ_1

2.2. *Application: Bilateral Investment and Hold-Up.* We now consider a simplified version of the classic hold-up model with bilateral investment in the spirit of Klein et al. (1978) and Grossman and Hart (1986). Here, the standard Nash bargaining stage is replaced by a simple noncooperative bargaining model where one player makes a take-it-or-leave-it offer to the other player. The stage game has three stages:

Stage 1 (Investment) Both players simultaneously decide whether or not to make a relation-specific investment. The cost of the investment is $c > 0$. Unless both players have made the investment, the stage game ends, with player i's payoff being $-c$ if $i \in \{1, 2\}$ had invested and zero otherwise. If both players have made the investment, the game proceeds to the next stage.

Stage 2 (Offer) Player 1 makes a take-it-or-leave-it offer $x \geq 0$ to player 2.

Stage 3 (Acceptance) Player 2 decides whether to accept or reject the offer. If the offer is accepted, payoffs are $(B - x - c, x - c)$; otherwise payoffs are $(-c, -c)$, where B is the investment revenue.

The interesting case arises when investment is efficient: $B > 2c$. Despite the efficiency of investment, it is well known that ex post bargaining over terms leads to inefficiently low investment—here, the one-shot game has a unique subgame-perfect equilibrium with no investment by either player, resulting in payoffs $(0, 0)$.

2.2.1. *The infinitely repeated game.* Now consider the infinite repetition of the stage game just described, with δ denoting the common discount factor. We characterize the best stationary equilibrium for player 2 and show that this equilibrium depends on whether behavior is restricted to simple penal codes. An implication is that, for some parameter values there are equilibrium behaviors that cannot be supported using simple penal codes.

For simplicity, we focus on player 2's best stationary equilibrium.[6] This equilibrium (with and without simple penal codes) involves in each period either (i) no investment by either player (as in the static equilibrium), resulting in payoffs $(0, 0)$, or (ii) investment by both parties, with player 1 making a subsequent offer of $\hat{x}$, which is accepted by player 2, resulting in payoffs $(B - c - \hat{x}, \hat{x} - c)$. As each player can ensure himself a payoff of 0 by not investing, if an investment equilibrium exists, player 1's offer $\hat{x}$ must satisfy $B - c \geq \hat{x} \geq c$.

2.2.2. *The stationary equilibrium maximizing player 2's payoff.* We first analyze the features of stationary equilibria without the restriction to simple penal codes. Since infinite repetition of the static equilibrium (i.e., the subgame-perfect equilibrium of the stage game), resulting in no investment, is trivially a subgame-perfect equilibrium of the repeated game, we turn to the

[6] For ease of exposition (and without loss of generality), we assume that the play is stationary along the equilibrium path, and also that—following a deviation—continuation play in all subsequent periods is stationary as well. We do not impose any form of stationarity when showing the inadequacy of simple penal codes.

incentive constraints for any equilibrium involving investment by both parties and an accepted offer of $\hat{x}$ in every period.

There are two possible ways in which a deviation by player 1 to an offer less generous than $\hat{x}$ can be punished. First, and standardly, deviations could be deterred by the threat of reversion to the static equilibrium in all future periods. Such reversion may well be sufficient to deter a small deviation by player 1 (to an offer x not much smaller than $\hat{x}$) because his short-run benefit from such a deviation is small. But such a punishment scheme may not deter a large deviation (to a small offer x) since the short-term benefit of having a very low offer accepted may be too large. So second, it may be necessary to have player 2 reject player 1's offer (leaving both players with a payoff of $-c$ this period). This second strategy will be useful to deter large deviations by player 1 to low offers that are particularly attractive to player 1 and relatively unattractive to player 2.

But with this second strategy, we need to consider player 2's incentives, since it is myopically suboptimal for player 2 to reject any offer from player 1 once investment costs are sunk. Therefore, for player 2 to be willing to punish player 1 by rejecting his offer, she must be subsequently rewarded by a sufficiently attractive continuation play. This may work for low offers from player 1 but may be insufficient to induce player 2 to reject offers x close to $\hat{x}$, which must therefore be followed by reversion to the no-investment equilibrium. Thus, intuitively, there may be a need to fine-tune the continuation play to the size of the deviating offer player 1 makes.

These considerations suggest that the strategy profile supporting the largest possible $\hat{x}$ as a stationary equilibrium offer requires the following continuation play after a deviant offer $x < \hat{x}$ by player 1:[7]

- If the deviant offer satisfies $x \in [\tilde{x}, \hat{x})$, where $\tilde{x}$ is a (still-to-be-determined) cutoff, then player 2 accepts it (which is the myopically optimal decision). This acceptance is followed by the infinite reversion to the static (no-investment) equilibrium from next period onward.
- If the deviant offer satisfies $x \in [0, \tilde{x})$, then player 2 rejects it. This rejection is followed by reversion to an accepted offer of $\hat{x}$ in every future period. If player 2 were to deviate by accepting the deviant offer, play would revert to the static (no-investment) equilibrium in all future periods.

Player 1 will find it unprofitable to make a deviant offer $x \in [\tilde{x}, \hat{x})$ only if the smallest deviant offer $\tilde{x}$ (inducing the same continuation play) is unprofitable, that is,

$$\frac{B - c - \hat{x}}{1 - \delta} \geq B - c - \tilde{x}.$$

This relation implies the following lower bound on $\tilde{x}$, the lowest deviant offer that player 2 can accept:

$$\tilde{x} \geq \frac{\hat{x} - \delta(B - c)}{1 - \delta}. \tag{1}$$

Player 2 will be willing to reject a deviant offer $x \in [0, \tilde{x})$ only if

$$-(1 - \delta)c + \delta(\hat{x} - c) \geq (1 - \delta)(\tilde{x} - c).$$

[7] A deviation by player 1 to an offer $x > \hat{x}$ is myopically suboptimal and therefore does not need to be punished: Continuation play is as if player 1 had not deviated. Similarly, continuation play in future periods is not affected in case one player deviates by not investing as such a deviation is myopically suboptimal.

This equation implies an upper bound on $\tilde{x}$, the largest deviant offer that player 2 is willing to reject:

$$\frac{\delta(\hat{x}-c)}{1-\delta} \geq \tilde{x}. \tag{2}$$

Combining inequalities (1) and (2), we obtain, as a necessary condition, an upper bound on $\hat{x}$, namely,

$$\hat{x} \leq \frac{\delta}{1-\delta}(B-2c). \tag{3}$$

For players to be willing to invest, their individual rationality constraints for investment must also be satisfied, which requires that each player's payoff be nonnegative. So we must also have

$$\hat{x} \geq c \quad \text{and} \quad B-\hat{x} \geq c. \tag{4}$$

Thus, if $(\tilde{x}, \hat{x})$ describes the stationary equilibrium maximizing player 2's payoff, $\hat{x}$ is the largest offer satisfying (3) and (4). Since

$$\frac{\delta}{1-\delta}(B-2c) \geq c \iff \delta \geq \frac{c}{B-c} \equiv \delta',$$

if $\delta < \delta'$, there is no such $\hat{x}$.[8] If $\delta \geq \delta'$, then

$$\hat{x} = \min\left\{\frac{\delta}{1-\delta}(B-2c), B-c\right\}.$$

Let δ'' be the smallest value of δ at which $\hat{x} = B-c$, that is,

$$\delta'' \equiv \frac{B-c}{2B-3c}.$$

For $\delta \in (\delta', \delta'')$, (3) holds with equality and so both (1) and (2) hold with equality, pinning down $\tilde{x}$. It is straightforward to verify that $\tilde{x} \in (0, \hat{x})$ in this case.

We are now in a position to describe the best equilibrium for player 2 (the verification is straightforward).

PROPOSITION 1. *Suppose $\delta \in (\delta', \delta'')$. The stationary equilibrium that maximizes player 2's payoff has the following structure: The equilibrium offer $\hat{x}$ is given by*

$$\hat{x} := \frac{\delta}{1-\delta}(B-2c).$$

The cutoff $\tilde{x}$ is given by

$$\tilde{x} := \frac{\delta(\hat{x}-c)}{1-\delta} \in (0, \hat{x}).$$

The equilibrium has two phases, "invest" and "don't invest," and begins in the "invest" phase. In the "invest" phase,

[8] Indeed, if $\delta < \delta'$, then there is no equilibrium with investment, since even an offer $\hat{x} = \delta(B-c)$ (the largest offer consistent with $\tilde{x} \leq 0$) violates player 2's individual rationality constraint in (4) above.

(1) on the path of play, both players invest and player 1 offers $\hat{x}$ to player 2,
(2) player 2 accepts all offers $x \geq \tilde{x}$, and
(3) player 2 rejects all offers $x < \tilde{x}$.

Play stays in the invest phase as long as both players invest, and the last offer satisfies $x \notin [\tilde{x}, \hat{x})$, which player 2 accepts if $x \geq \hat{x}$ and rejects if $x < \tilde{x}$. Otherwise, play switches to the don't invest phase, in which the static no-investment equilibrium is played in every period.

The equilibrium described in Proposition 1 does not have a simple penal code structure, since deviations by player 1 to relatively generous offers $x \geq \tilde{x}$ lead to different continuation play than deviations to less generous offers $x < \tilde{x}$.

However, for $\delta \geq \delta''$, we can take $\tilde{x} = \hat{x} = B - c$, since for large δ,

$$B - c \leq \frac{\delta(\hat{x} - c)}{1 - \delta}.$$

(It is straightforward to verify that the relevant incentive constraints hold.) In this case, the simple penal code in which player 2 rejects all offers less than $B - c$ (and accepts all larger offers), followed by the play of the stationary equilibrium maximizing player 2's payoff from next period onward, supports the same equilibrium outcome.

2.2.3. *Simple penal code for $\delta \in (\delta', \delta'')$.* We have just seen that in this game, a simple penal code can support the stationary equilibrium that maximizes player 2's payoff as long as $\delta \geq \delta''$, and the only equilibrium involves no investment for $\delta \leq \delta'$. We now investigate the efficacy of simple penal codes for intermediate values of $\delta \in (\delta', \delta'')$. We both prove that $\hat{x}$ cannot be supported using simple penal codes and characterize x^{SPC}, the largest offer that player 2 can receive in any stationary equilibrium supported by a simple penal code.

Fix a candidate stationary equilibrium offer $\bar{x}$. There are two candidates for a simple penal code in this setting:

(1) Any deviating offer $x \neq \bar{x}$ triggers reversion to the static no-investment equilibrium from next period onward. Since this static equilibrium is the worst not only for player 1 but also for player 2, player 2 will accept any offer $x > 0$.
(2) Any deviating offer $x \neq \bar{x}$ is followed by the continued play of the equilibrium maximizing 2's payoff from next period onward (providing player 2 with maximal incentives not to deviate from the prescribed response). If $x > \bar{x}$, equilibrium may prescribe that player 2 accepts the offer, as such a deviation is myopically suboptimal for player 1. If $x < \bar{x}$, however, equilibrium must prescribe that player 2 reject the offer (as otherwise the deviation would be profitable for player 1). Failure by player 2 to do so leads to no investment in the future, the worst possible punishment.

Consider the first simple penal code. Player 1 will have no incentive to make a deviant offer $x < \bar{x}$ only if he has no incentive to make an arbitrarily small deviant offer, that is,

$$\frac{B - c - \bar{x}}{1 - \delta} \geq B - c,$$

which implies the following upper bound on $\bar{x}$:

$$\bar{x} \leq \delta(B - c). \tag{5}$$

It is straightforward to check that the profile in which both players invest, player 1 always makes the offer $\delta(B - c)$, which is accepted (as is any lower off-the-equilibrium path offer), and any deviation by player 1 to an inferior offer results in future no investment is an equilibrium for

$\delta \in (\delta', \delta'')$. Since $\delta > \delta', \bar{x} = \hat{x}$ violates (5) and the offer $\hat{x}$ cannot be supported as an equilibrium using the first simple penal code.

Now consider the second simple penal code, and let v_2 be the maximum of player 2's discounted sum of payoffs over all subgame-perfect equilibria.[9] Let $x^\dagger$ be the maximum equilibrium offer accepted by player 2 in an equilibrium giving payoffs v_2. Note that $\bar{x} \leq x^\dagger$, for any stationary equilibrium offer $\bar{x}$, and $x^\dagger - c \geq (1-\delta)v_2$. In the period in which $x^\dagger$ is offered, player 2 will be prepared to reject any offer $x \in [0, x^\dagger)$ only if

$$-c + \delta v_2 \geq x^\dagger - c,$$

which implies

$$-c + \delta v_2 \geq (1-\delta)v_2,$$

that is,

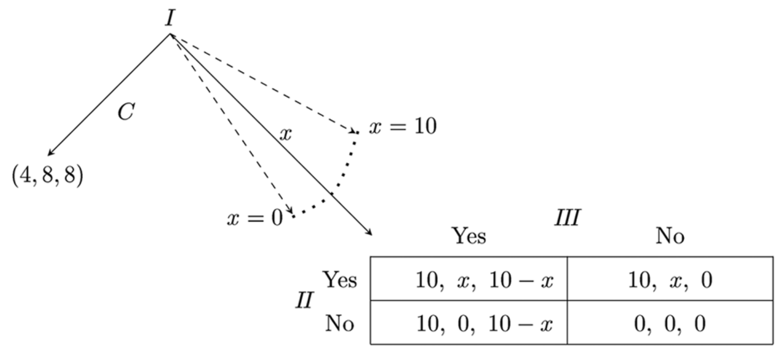
$$(2\delta - 1)v_2 \geq c.$$

If $2\delta \leq 1$, this inequality is clearly impossible. What if $2\delta > 1$? Since $\delta < \delta''$ (and $2\delta - 1$ is increasing in δ), we evaluate the above inequality at δ'' to obtain the condition $v_2 \geq 2B - 3c$, which is also impossible since $v_2 \leq (B-2c)/(1-\delta) < (B-2c)/(1-\delta'') = 2B - 3c$.

Hence, if $\delta \in (\delta', \delta'')$, the best simple penal code involves the play of the no-investment equilibrium in all periods following a deviant offer by player 1. The largest offer that player 2 can receive in any equilibrium supported by a simple penal code is $\delta(B-c) =: x^{SPC}$, which is strictly smaller than $\hat{x}$.

2.2.4. *Discussion.* When stationary equilibria cannot be supported by simple penal codes, they have an interesting feature. If the offerer deviates from the expected price by shading just slightly, the responder will accept (shrug it off) but will no longer invest in the relationship (one can also think about the parties as walking away from the relationship and then receiving outside options of zero on the spot market). By contrast, if the offerer deviates from the expected price by making a much lower offer, then instead the—perhaps insulting—offer is rejected, but in the expectation that next period the investment relationship will be reestablished on better terms. The structure of this equilibrium might at first glance be considered counterintuitive, since one might think that the relationship would be more likely to continue if the shading of the price is only slight and not large. But if the shading of price is only slight, it is expensive to induce the responder to reject the offer to inflict punishment; punishment is instead inflicted by a return to the no-investment equilibrium next period. If the shading is larger, the deviation is particularly profitable for the responder, and, at the same time, it is relatively cheap to inflict within-period punishment by refusing the offer (which is rather unattractive). But the offer will nevertheless be refused only in the expectation that this will lead to the relationship being resumed, on better terms, the following period. Moreover, the offerer's credible incentive to inflict large costs by failing to make a mutually beneficial trade in a period of deviation ("cutting off one's nose to spite one's face") can help to sustain a price above that which would otherwise be possible. So the equilibrium yields a prediction about the pattern of punishment strategies in relationships with hold-up: Other things being equal, small deviations result in walking away whereas large deviations result in a costly standoff and then resumption of trade. By contrast, a simple penal code would involve the responder taking the same action—in particular, walk away—independently of how low the offer he receives is and never rejecting a myopically beneficial offer—no matter how paltry—in a period when costs are sunk.

[9] We can take the maximum, instead of the supremum, because the set of subgame-perfect equilibrium payoffs is compact.



NOTES: The choice x for player I ranges over the nonnegative integers, $\{0, 1, \dots, 10\}$.

FIGURE 3

THE EXTENSIVE-FORM GAME Γ_2

3. THE REWARD SHOULD FIT THE TEMPTATION

"You oughtn't to yield to temptation."
"Well somebody must, or the thing becomes absurd."
Anthony Hope, *The Dolly Dialogues*

We have just highlighted how the value of deviation-dependent punishments can arise from the *commonality* of interest between the punisher and the punished. We now examine how the value of deviation-dependent punishments arises for a very different reason—because there is a *conflict* of interest between two players who are not supposed to acquiesce to a deviation by a third player. As in the first set of examples, we begin with a simple stylized example to illustrate the main force at work, which is that continuation play must be specified such that the reward to each player is adapted to the sacrifice that they made in inflicting within-period punishment. We then provide an application to a more complex game of greater applied interest—in this case, a repeated version of the "Naked Exclusion" game analyzed by Rasmusen et al. (1991) and Segal and Whinston (2000).

3.1. *A Simple Example.* The stage game for our second simple example is the extensive form Γ_2, presented in Figure 3. We interpret the choice of $x \in \{0, 1, \dots, 10\}$ by player I as a bribe to player II (with $10 - x$ the bribe to player III).[10] If player I chooses the "cooperative" action C, then the stage game ends. If player I chooses instead to offer a bribe of 10 to players II and III (with x representing the split), players II and III then simultaneously decide whether to accept or reject the bribe. (Notice that for player I to receive his maximum payoff of 10, it suffices that only one of the other two players accepts the bribe; hence, he needs to bribe only one of them to accept and their interests are conflicting.) The stage game then ends, and all actions become common knowledge. The game has many subgame-perfect equilibria, but they all share some common features: Player I attempts to bribe the other players instead of behaving cooperatively, and both players II and III accept any positive bribe offered. Moreover, the set of subgame-perfect equilibrium payoffs is given by

$$\{(10, x, 10 - x) : x \in \{0, 1, \dots, 10\}\}.$$

The conflictual nature of the game is reflected in the fact that the payoffs of players II and III are negatively correlated across the equilibria of the stage game.

[10] The extensive form Γ_2 has a natural interpretation as a bargaining game where player I has a pie of 20 to be split between himself and two others, with decisions being taken by majority voting.

We are interested in the possibility of using these multiple equilibria to construct an equilibrium of the once-repeated game where player *I* behaves cooperatively in the first period by playing *C*.

We begin by arguing that it is impossible to support this cooperative play by *I* in the first period using continuation play in the second period that is independent of the nature of *I*'s bribe. If player *I* deviates by attempting the bribe x, then equilibrium requires that players *II* and *III* both reject player *I*'s bribe. For *II* to reject, her continuation payoff from rejection must be at least x. If the continuation play is independent of *I*'s bribe, then *II*'s continuation payoff after rejection must be at least 10 (otherwise *II* would accept a bribe of 10, undermining *I*'s incentive to play *C*). But at the same time, for *III* to reject, his payoff must be at least $10 - x$, again for all x. And this requires that *III*'s continuation payoff after rejection is also 10, which is impossible.

On the other hand, it is easily verified that the following profile is a subgame-perfect equilibrium: In the first period, player *I* plays cooperatively; if he were to deviate all bribes would be rejected by both players *II* and *III*. In the second period, after cooperative play in the first period, player *I* bribes at some level x in the second period (any level works). If player *I* deviated in the first period, and his bribe x was rejected by both *II* and *III*, then player *I* offers the *same* bribe in the second period, which is accepted by both *II* and *III*. (If *I* offers a deviant bribe in the second period, both players *II* and *III* accept.) If only one player $k \in \{II, III\}$ accepts a deviant first-period bribe, then in the second period, player *I* offers the bribe that leaves player k with 0. (It is irrelevant whether player k accepts the bribe in the second period; the other player of course accepts.) Finally, if both players *II* and *III* accept the first-period bribe, an arbitrary continuation equilibrium is played (since both players accepting is a simultaneous deviation by *II* and *III*, these payoffs are irrelevant for the purposes of checking for subgame perfection).

3.2. *Application: Naked Exclusion.* In this section, we analyze a repeated version of the classic "naked exclusion" model of Rasmusen et al. (1991) and Segal and Whinston (2000). The game has three long-lived players, an incumbent monopolist (I) and two buyers (B_1 and B_2). Each period, the same incumbent faces a challenge from a different short-lived entrant who is more efficient than the incumbent. For simplicity, we do not model the potential entrants as players. In each period, the incumbent can choose whether or not to offer exclusive dealing contracts to the two buyers. Exclusive dealing contracts last one period, and the incumbent can make a transfer payment to the buyers to compensate them for signing an exclusive dealing contract. The incumbent's exclusive dealing offers are publicly observable, and buyers make their acceptance/rejection decisions simultaneously. If at least one buyer signs an exclusive dealing contract for the current period, no entry occurs, and the incumbent subsequently charges the monopoly price to each buyer, earning a profit π^m on each. If no buyer has signed an exclusive dealing contract for the current period, the entrant enters the market, resulting in zero profit for the incumbent and an increase in rents of S for each buyer.

The stage game proceeds as follows:

Stage 1 I chooses between offering no exclusive dealing contracts, N, and a pair $(x_1, x_2) \in [0, \infty)^2$, where $x_i \geq 0$ is the offered transfer payment to B_i in return for signing an exclusive dealing contract. If I chooses N, then the period ends, and payoffs for I and the two buyers are $(0, S, S)$; otherwise the game proceeds to Stage 2. (Here, S denotes the increase in buyer surplus due to entry.)

Stage 2 Facing public offers (x_1, x_2), B_1 and B_2 simultaneously choose whether to accept the offer ($a_i = 1$) or not ($a_i = 0$). If at least one buyer accepts the offer, payoffs are $(2\pi^m - a_1x_1 - a_2x_2, a_1x_1, a_2x_2)$, where π^m is the monopoly profit that the incumbent can extract from each buyer; if both buyers reject the offer, payoffs are $(0, S, S)$.

We assume that payoffs satisfy $2\pi^m > S > \pi^m > 0$. The first inequality ensures that the incumbent's monopoly profit is sufficiently large so that it is worthwhile for him to offer a large enough "bribe" to one buyer to make it a dominant strategy for that buyer to accept the exclusive dealing offer. The second inequality implies that entry is efficient. Although the stage game has multiple subgame-perfect equilibria, there is no entry in any of these equilibria, and aggregate payoffs are $2\pi^m$. The best equilibrium for buyer B_1 (buyer B_2) involves I offering $(S, 0)$ (respectively, $(0, S)$), making it a dominant strategy for that buyer to accept the offer and resulting in incumbent profit of $2\pi^m - S$. These are also the worst equilibria for I. The best equilibrium for I is the one in which I offers $(0, 0)$ and both buyers accept the offer, resulting in incumbent profit of $2\pi^m$.

3.2.1. *The infinitely repeated game.* We now investigate the effects of the infinite repetition of this game. We are interested in determining the conditions under which it is possible to sustain the play of N (no exclusive dealing, and thus entry) in each period as an equilibrium of the game. In this equilibrium, the incumbent's payoff is zero, and each buyer receives S in every period. The common discount factor is denoted $\delta \in (0, 1)$.

We first consider sustaining the perpetual play of N by a simple penal code. The simple penal code prescribes the following behavior when I deviates by making offers (x'_1, x'_2):

- If $x'_1 + x'_2 > 2\pi^m$, both buyers accept. (Since one buyer is accepting, it is myopically optimal for the other to do so as well, so no dynamic incentives need be provided. Moreover, such deviations are unprofitable for the monopolist, so no further punishment is required.)
- If $x'_1 + x'_2 \le 2\pi^m$, both buyers reject. (If one buyer deviates and accepts the offer, then, in all future periods, a static equilibrium is played in which that deviant buyer receives a zero payoff.)
- In either case, this is followed by the play of N in all future periods.

Note that this simple penal code maximally punishes the deviating I and maximally rewards the two buyers for rejecting offers, given that the rewards cannot be made dependent on the deviant offers.

Facing deviant offers (x'_1, x'_2) such that $x'_1 + x'_2 \le 2\pi^m$, buyer B_i is willing to reject his offer x'_i if and only if

$$x'_i \le \frac{S}{1-\delta}.$$

As long as $x'_i + x'_{-i} \le 2\pi^m$, this incentive constraint has to hold for any $x'_i \le 2\pi^m$, and the perpetual play of N can be sustained by a simple penal code if and only if

$$\delta \ge \frac{2\pi^m - S}{2\pi^m} \equiv \hat{\delta}^{SPC}.$$

We now show that, using more general continuations, the perpetual play of N can be sustained for even lower discount factors, namely, if and only if

$$\delta \ge \frac{2\pi^m - S}{4\pi^m - S} \equiv \hat{\delta}^* < \hat{\delta}^{SPC}.$$

The strategy profile differs from the simple penal code above only in the event in which I's deviant offers (x'_1, x'_2) are such that $\max(x'_1, x'_2) \in (S, 2\pi^m]$ and $x'_1 + x'_2 \le 2\pi^m$. Such deviant offers are rejected by both buyers and are followed, from the next period onward, by play of the equilibrium that maximizes the payoff of the buyer, B_j, who received the larger deviant offer. As we show below, in this continuation equilibrium, buyer B_j gets a per-period payoff of $2\pi^m$, whereas buyer B_{-j} and the incumbent I both get zero. So, I is maximally punished.

Turning to the buyers' incentive constraints following such deviant offers, note first that buyer B_{-j} receives S this period (and zero in all future periods) from rejecting his offer but only $x'_{-j} \le 2\pi^m - x'_j < S$ in this period (and zero in all future periods) from accepting. So his incentive constraint is satisfied for all discount factors. Buyer B_j is willing to reject his offer if and only if

$$x'_j \leq S + \frac{\delta}{1-\delta} 2\pi^m.$$

This inequality has to hold for all x'_j not exceeding $2\pi^m$, so B_j's incentive constraint is satisfied if and only if $\delta \geq \hat{\delta}^*$.

It remains to show that, for any $\delta \geq \hat{\delta}^*$, there is an equilibrium that gives buyer B_j a per-period payoff of $2\pi^m$.[11] Along the equilibrium path, I offers $x_j = 2\pi^m$ to B_j and $x_{-j} = 0$ to B_{-j}, both buyers accept, and no entry takes place. (Note that neither buyer has a myopic incentive to deviate, so equilibrium may as well prescribe play of the same equilibrium in all future periods following a deviation by a buyer.) Consider now a deviation by I to offers (x'_1, x'_2). If $x'_1 + x'_2 > 2\pi^m$, both offers are accepted, and the same equilibrium (giving I a zero payoff) is played forever after. (Accepting the offer is myopically optimal for each buyer, provided the other buyer does so as well, so no dynamic incentives need to be provided.) If instead $x'_1 + x'_2 \leq 2\pi^m$, both offers are rejected (so that entry occurs, yielding a payoff of S to each buyer in the current period); equilibrium play from the next period onward is exactly the same as was prescribed following the same vector of deviant offers in the nonsimple punishment scheme for sustaining the perpetual play of N above. As our analysis above shows, both buyers have an incentive to reject I's deviant offers if $\delta \geq \hat{\delta}^*$. Hence, I does not have a profitable deviation.

3.2.2. *Discussion.* In this extended example, "no exclusive dealing" can be supported as an equilibrium outcome in a repeated version of the naked exclusion game. On the equilibrium path, the incumbent is not supposed to offer exclusive dealing contracts to his buyers—but if he does, both of the buyers must reject these offers in order to inflict within-period punishment on the incumbent and reduce his temptation to deviate. Inducing such rejection does not require dynamic incentives if the incumbent offers a payment for exclusive dealing of less than S, since (when rejection by the other buyer is expected), rejection is myopically optimal. But inducing rejection of an exclusive dealing offer with a payment of more than S does require dynamic incentives since such rejection would decrease the buyer's current profit. Buyers must therefore be rewarded in the continuation game for rejecting such a tempting exclusive dealing contract.

In contrast to the first set of examples, in this setting, there is no trade-off between rewarding the punisher and punishing the deviator, since equilibria that reward the punishers (exclusive dealing or equilibria with entry) can be constructed that yield zero profits to the incumbent. There is, however, a trade-off between rewarding the two buyers who must both reject the deviant offers. The most that both buyers can receive at the same time is S per buyer (when entry occurs in every period), so the best simple penal code involves the play of this equilibrium after any deviation by the incumbent. But there exist equilibria with exclusive dealing that can provide more than S to one buyer (and less than S to the other one and zero to the incumbent). Thus, if the incumbent makes a deviation involving a bribe of more than S to one buyer (and less than S to the other, otherwise the deviation is unprofitable), the optimal punishment scheme provides a larger "carrot" to the buyer most tempted. In this setting, as in Section 2, the continuation play again optimally depends on the particular deviation chosen, but this time because the reward provided must fit the temptation resisted (or the sacrifice made) when rejecting the deviation.

The principle that the reward provided should be tailored to the sacrifice is natural. This is particularly so in the dynamic variant of the repeated naked exclusion game set out above, where, instead of entrants being short-lived, the entrant replaces the incumbent in the period

[11] There does not exist an equilibrium in which B_j earns more than $2\pi^m$ per period, because both I and B_{-j} can ensure themselves a payoff of at least zero, and when there is no exclusive dealing each buyer receives only $S < 2\pi^m$.

following rejection of all exclusive dealing contracts and hence becomes the new incumbent, offering exclusive dealing contracts against a new entrant in the following period.[12] In this dynamic game, if an incumbent deviates by offering an asymmetric set of exclusive dealing contracts (more than S to one buyer, less than S to another), then it is natural that the entrant, when he becomes the incumbent next period, plays a continuation equilibrium that provides larger rewards to the buyer that made a greater sacrifice in enabling the entrant to replace the old incumbent.[13]

4. CONCLUSION

A major concern in any long-run interaction is the provision of incentives to discipline the behavior of agents. In simple interactions, such as the repeated prisoners' dilemma, an opportunistic deviation is immediately profitable (since the other players cannot immediately react) and can only be deterred by appropriate specifications of continuation play (or future punishments). In many other applications, however, interactions are intrinsically dynamic, and opportunistic deviations are only profitable if they are validated by the complicit behavior of at least one other player. Moreover, this complicit behavior is often myopically optimal (since, for example, it may involve accepting a "bribe").

When agents are impatient, deterring deviations may therefore require preventing the complicit behavior, which requires specifying deviation-dependent continuations. In this article, we have indicated two different reasons why deviation-dependent continuations can be necessary: (1) Settings where rewarding a potentially complicit player for not being complicit also rewards the original deviator and (2) settings where there are multiple potentially complicit players and there is a trade-off in dividing rewards for not being complicit among these players.

We have illustrated these causes by providing applications to a repeated bilateral investment game with hold-up in the spirit of Klein et al. (1978) and Grossman and Hart (1986), and to a repeated naked exclusion game modeled on Rasmusen et al. (1991) and Segal and Whinston (2000). But we expect that the same phenomena will arise in many other interesting applications. The first cause of the need for deviation-dependent continuations—the commonality of interest between potential punisher and punishee—arises in many environments, such as the interaction between parent and child, between legislators of the same political party, between a monetary authority and the public; and between principal and agent in relational contracting. The second cause—the trade-offs between rewarding different punishing parties—arises in settings such as multiplayer bargaining or lobbying; collusion in vertically related markets, a sovereign dealing with multiple constituencies, and a defendant dealing with multiple claimants. As we hope our examples show, examining the particular structure of optimal punishment in an applied setting can yield new insights as to ways to make existing institutions work better.

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[12] Since we focus on equilibria in which the incumbent makes zero profit in all future periods, independently of whether or not he deviates, our analysis of the optimal punishment extends to this dynamic version of the game. In the dynamic version, it is the entrant who rewards the buyer rejecting the exclusive dealing contract.

[13] Similar forces are at work in Nocke and White (2007). That work studies collusion between upstream firms in a vertically related industry. An upstream firm can profitably deviate from the collusive equilibrium only if his deviant contract offer is accepted by at least one downstream retailer. Weingast (1995, 1997) has also analyzed a game with a structure similar to the naked exclusion game set out above. In Weingast's game, an incumbent sovereign can expropriate one or both of his subject groups. The subject groups can successfully resist expropriation only if they both do so, however, and resistance is costly.

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