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Hyperbolic discounting and secondary markets

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Abstract

Does the existence of secondary markets for durable goods affect price and allocation on primary markets? We study competitive equilibria for durable goods where the possibility of future trade on secondary markets does not affect consumer behaviour in the primary market, provided consumers are exponential discounters. If consumers are hyperbolic discounters, however, secondary markets are no longer neutral as they allow consumers to postpone their purchasing decisions. In this case, the equilibrium price in the primary market is decreasing in the number of periods in which the good can be traded. Hence, primary producers have an incentive to close down secondary markets. If secondary markets never close, hyperbolic discounters may use collusive intrapersonal strategies, which lead to a Pareto improvement for all incarnations of the same consumer. We characterise the set of all stationary equilibrium prices and show that the competitive equilibrium allocation may be inefficient.

JEL classification: L11; D40; D91

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1. Introduction

In this paper, we ask whether (and to what extent) the existence of secondary markets for durable goods may affect price and allocation on primary markets. To this end, we study competitive equilibria in a simple market environment and focus on the role of consumers'

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discounting. In the first period, the market clears for a fixed primary supply of the durable good. In subsequent periods, the durable good is traded in competitive secondary markets. There is no depreciation and no additional supply of the durable good over time. The only intertemporal aspect of our model is the durability of the traded goods. Durable goods we have in mind include, for example, collector's items such as coins, stamps, Meissner porcelain figures, etc.

In our model, perfectly competitive secondary markets have no effect on the primary market if consumers are time-consistent exponential discounters. Specifically, the initial price and the incentive to provide the primary supply are not affected by the possibility of future trade in secondary markets. Furthermore, the allocation of the durable good is efficient: consumers above a certain threshold type buy the good in each period, whereas consumers below that threshold type never buy the durable good.

Hyperbolic discounting applies a higher discount rate to the near future than to the distant future. Such discounting implies a conflict between today's preferences and future preferences. Time inconsistency potentially matters in our durable goods environment, since consumers incur a cost at the date of purchase and receive a continuing stream of benefits from consumption over time.

As we will explain, competitive secondary markets are no longer neutral if consumers are time-inconsistent hyperbolic discounters: the equilibrium price, primary supply as well as the set of consumers who obtain the good may change through the introduction of secondary markets.

With hyperbolic discounting, current and future incarnations play an intrapersonal game. A competitive equilibrium in our model satisfies two conditions:

(i) for a given price path, the strategy profile of each consumer forms a subgame perfect equilibrium in the intrapersonal game, and
(ii) the price path is such that the market clears in each period.

In the absence of secondary markets, future incarnations are given no choice: the initial period incarnation decides whether to consume the durable good in each period or never. Certain consumer types have first period incarnations which ideally would like to commit to future consumption and prefer not to buy the good at present. These types have an incentive to delay consumption. This incentive remains for later incarnations, conflicting with the ideal consumption path of the first period incarnation. In the presence of secondary markets, certain types may therefore end up never consuming, although they would be better off if all incarnations decided to buy and consume in each period.

As far as we are aware, this is the first paper to consider hyperbolic discounting in a durable good environment. Motivated by overwhelming evidence in the psychology literature on time-inconsistency, hyperbolic discounting has received a lot of attention in the economics literature recently.[1] The seminal paper in economics is

[1] For a discussion and references to the psychology literature, see for instance Ainslee (1992) or Loewenstein and Prelec (1992). See also the experiments documented in Thaler (1981). Although hyperbolic discounting is the most popular approach to the time inconsistency problem in the economics literature, there exist alternative explanations of time inconsistency; see Gul and Pesendorfer (2000, 2001), and Rubinstein (2000).

Strotz (1956).[2] Recent work on hyperbolic discounting has focused on task performance in decision problems (O'Donoghue and Rabin, 1999a, 1999b, 2001; Carrillo and Mariotti, 2000; Brocas and Carrillo, 1999) and on intertemporal consumption and savings decisions (Laibson, 1997; Harris and Laibson, 2001; Luttmer and Mariotti, 2000).[3] In the light of the work on task performance, we want to emphasise some characteristics of our framework. We assume that consumers are *sophisticated* in that they are aware of the intrapersonal game they play.[4] Another feature of our model is that the durable good can be purchased in each period. Hence, the "tasks" to buy the durable good are not mutually exclusive.[5] In contrast to the papers on task performance, the decision problem is embedded in a market environment so that the cost of performing the task is endogenous.

The industrial organisation literature on durable goods has mainly been concerned with the Coasian commitment problem of a monopoly supplier (see, for instance, Bulow, 1982; Stokey, 1981; Gul et al., 1986). In this paper, we want to abstract from the commitment problem of the supplier and focus instead on the commitment problem of consumers that arises in the context of hyperbolic discounting and secondary markets. This allows us to consider a stationary environment. As is well known for the case of exponential discounting, under certain circumstances it is indeed optimal for a monopolist to supply in the initial period only, provided she can commit to do so.

The outline of the paper is as follows. In Section 2, we present the model. In Section 3, we first characterise competitive equilibrium in the benchmark case of exponential discounting. For the case of hyperbolic discounting, we then analyse the (unique) competitive equilibrium when secondary markets close in finite time, and the (unique) competitive Markov equilibrium when secondary markets never close. We obtain the following nonneutrality results: the initial price decreases with the number of periods in which the good can be traded in secondary markets. Hence, a monopolist (or Cournot oligopolists) who provides the primary supply has an incentive to close down the secondary market. Moreover, a profit-maximising monopolist supplies less the larger is the number of periods in which secondary markets remain open. In Section 4, we analyse non-Markovian strategies in secondary markets which never close. In particular, we focus on collusive strategy profiles in which a deviation triggers a change in the future consumption path. A switch to the collusive strategy profile induces a Pareto improvement for all incarnations of the same consumer. If all consumers use collusive strategies, however, then all consumers are worse off for a given primary supply as the resulting competitive equilibrium restores the (high) equilibrium price in the absence of secondary markets. We then characterise the set of all stationary equilibrium prices. Moreover, we show that there

[2] The early paper on growth and intergenerational savings by Phelps and Pollak (1968) introduces the now-familiar β-δ-representation of preferences in the economics literature. However, in their paper, each generation lives for one period only.

[3] Other work includes Akerlof (1991), Caillaud et al. (1996), Benabou and Tirole (1999).

[4] Note, however, that in our model the consumer's degree of sophistication matters only for non-Markovian intrapersonal strategies. An incarnation's Markov strategy is independent of the consumer's degree of sophistication.

[5] This also holds, e.g., in Carrillo and Mariotti (2000), but not in O'Donoghue and Rabin (1999a, 1999b, 2001).

exist inefficient competitive equilibria. That is, an increase in the number of markets can lead to allocative inefficiency when consumers are time-inconsistent. Section 5 concludes.

2. The model

We consider a discrete time, infinite horizon model of a durable good market. Time is labelled by $t = 0, 1, 2, \ldots$. In each period, consumers may spend their disposable income on the durable good and a Hicksian composite commodity.

There is a unit mass of heterogeneous consumers with unit demand for the durable good. Consumers differ only in their valuations for this good, which are parameterised by v. Consumer type v is time-independent and uniformly distributed on $[0, 1]$. The durable good provides a constant utility stream. That is, there is no depreciation in the quality of the good and consumers' tastes do not change over time.[6]

Let us first consider a particular period t. Direct instantaneous utility is of the form $u(x_t, y_t; v) = vx_t + y_t$, where $x_t \in \{0, 1\}$ and $y_t \geqslant 0$ denote the period t consumption of the durable good and the Hicksian composite commodity, respectively. Type v can be interpreted as the utility derived from consuming the durable good in one period, which is measured in the units of the Hicksian composite commodity.

The Hicksian composite commodity is perishable, and its price normalised to one in each period. This normalisation is justified since we do not allow for income transfers over time. That is, we rule out saving and borrowing as well as forward markets for the durable good.[7] In each period, consumers have current income m (which, for simplicity, is independent of v) and inherit an endowment of $x_{t-1} \in \{0, 1\}$ units of the durable good. Hence, a consumer, who buys $x_t \in \{0, 1\}$ units of the durable good at price p_t and y_t units of the Hicksian composite commodity, faces the budget constraint $m + p_t x_{t-1} - p_t x_t - y_t \geqslant 0$. We assume that income m is sufficiently large such that a consumer can always afford to buy one unit of the durable good.

Following Strotz (1956) and Phelps and Pollak (1968), each consumer is composed of a sequence of incarnations indexed by their period of control over consumption. Consumer type v's period t incarnation chooses his consumption in period t so as to maximise the discounted sum of present and future instantaneous utilities. Discounting is of the exponential or hyperbolic form. In the latter case, the rate of substitution between periods

[6] The uniformity assumption on the unit interval is made for convenience; we could work with any continuous distribution function.

[7] If the interest rate on savings r is greater than $(1 - \beta\delta)/(\beta\delta)$, consumers would like to postpone consumption of the nondurable good indefinitely (for prices of the nondurable that are fixed as above) because instantaneous utility is linear in the consumption of the nondurable good. We could analyse a model with savings in which consumers' instantaneous utility functions are strictly concave in y_t to the effect that consumers maintain a positive consumption stream of the nondurable good. We conjecture that the nonneutrality of secondary markets for the durable good also holds in such a model. We do not pursue this avenue because, in a partial equilibrium environment, our model appears to be the natural model to start with: by abstracting from the issue of time inconsistency in saving behaviour, we can focus on the durable good market in isolation. Moreover, in our model, we are able to perform a simple welfare analysis.

t and $t+1$ is larger than the one between $t+\tau$ and $t+\tau+1$ for $\tau \geqslant 1$. Direct utility is of the form

$$U_t\big(\{x_s\}_{s=t}^{\infty}, \{y_s\}_{s=t}^{\infty}; v\big) = u(x_t, y_t; v) + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} u(x_{t+\tau}, y_{t+\tau}; v),$$

where $\beta \in (0, 1]$ and $\delta \in (0, 1)$. If $\beta = 1$, then consumers are exponential discounters and hence "time consistent," otherwise they are hyperbolic discounters and "time inconsistent."

Utility maximisation implies that the budget constraint in each period is satisfied with equality. Hence, y_t can be replaced by $m + p_t x_{t-1} - p_t x_t$. It is more convenient to work with incarnation t's indirect utility function conditional on $\{x_s\}_{s \geqslant t}$. In case the consumer never buys the durable good from t onward, i.e., $x_s = 0$ for all $s \geqslant t$, we normalise incarnation t's conditional indirect utility to 0. Consequently, incarnation t's conditional indirect utility does not reflect utility gains due to initial endowment effects in period t. If type v consumes the durable good in all periods from t onward, his incarnation t's conditional indirect utility is

$$V_t\big(\{p_s\}_{s \geqslant t}; v \,\big|\, \{x_s = 1\}_{s \geqslant t}\big) = (v - p_t) + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} v = \frac{1 - (1-\beta)\delta}{1-\delta} v - p_t.$$

More generally, incarnation t's indirect utility conditional on $\{x_s\}_{s \geqslant t}$ is

$$V_t\big(\{p_s\}_{s \geqslant t}; v \,\big|\, \{x_s\}_{s \geqslant t}\big) = x_t(v - p_t + \beta\delta p_{t+1}) + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} x_{t+\tau}(v - p_{t+\tau} + \delta p_{t+\tau+1}).$$

At $t = 0$, there is a fixed primary supply of q units of the durable good. This can be thought of as the aggregate supply of an industry, which we do not model explicitly at this point. For simplicity, no consumer has an initial endowment at $t = 0$. Below, we analyse the case of a monopolist who chooses q so as to maximise her profits. In each period, consumers are price takers. Given initial supply q, the equilibrium price p_0 is such that the durable good market clears in $t = 0$. From $t = 1$ onward, there is a perfectly competitive secondary market for the durable good. The good remains in constant supply of q, i.e., there is no additional production. Again, equilibrium price p_t is such that the market clears in period t.

A consumer's sequence of incarnations are assumed to play an *intrapersonal game*: period t incarnation makes his consumption choice, taking as given the strategies of all other incarnations (of the same consumer) and aggregate market conditions, as summarised by the price sequence $\{p_t\}_{t=0}^{\infty}$ in the durable goods market. Given $\{p_t\}_{t=0}^{\infty}$, a *pure strategy* of incarnation t in the intrapersonal game is a mapping from the private history into the action space $\{0, 1\}$. A consumer's private history in period t is summarised by the sequence of own past consumption of the durable good $\{x_s\}_{s \leqslant t-1}$. Note that $y_s = (m + p_s x_{s-1}) - p_s x_s$. Hence, we can suppress the sequence $\{y_s\}_{s \leqslant t-1}$ as part of the private history. A *mixed strategy* of incarnation t is a probability distribution over pure strategies. To generate a probability distribution over actions after a particular history, an incarnation uses a randomisation device. Let z_t be the realisation of the random variable with support Z

in period t and let μ be a probability measure on Z. (These random variables are assumed to be independent across individual consumers at any given time and across incarnations of the same consumer.) A consumer's private history in period t includes the sequence of realised outcomes of the randomisation devices used by his past incarnations.[8] That is, the consumer's private history in period t is now summarised by $\{x_s, z_s\}_{s\leqslant t-1}$. Since an incarnation's conditional indirect utility is independent of his private history and of past prices, a *pure Markov strategy* of incarnation t, given the price sequence $\{p_t\}_{t=0}^{\infty}$, is an element of $\{0, 1\}$.

Given $\{p_t\}_{t=0}^{\infty}$, a consumer's *intrapersonal equilibrium* is a subgame perfect equilibrium (SPE) in the intrapersonal game played by the consumer's different incarnations. (Sometimes, we will restrict attention to pure strategies or Markov strategies.) A *competitive equilibrium* in the durable goods market consists of the set of strategy profiles in the intrapersonal game and a sequence of prices $\{p_t\}_{t=0}^{\infty}$ in the durable goods market such that (i) each strategy profile forms an intrapersonal equilibrium at prices $\{p_t\}_{t=0}^{\infty}$, and (ii) the durable goods market clears in each period $t = 0, 1, 2, \ldots$. With the restriction to pure strategies in the intrapersonal game we refer to a *competitive equilibrium in pure strategies*. With the restriction to Markov strategies in the intrapersonal game we refer to a *competitive Markov equilibrium*.

3. (Non)neutrality of secondary markets

In this section, we investigate whether the existence of secondary markets from period 1 onward has any impact on the equilibrium price in the primary market (in period 0). The related question of interest is whether a monopolist offering the durable good in period 0 has an incentive to close down the secondary markets. Throughout, we assume that all incarnations use Markov strategies in the intrapersonal game. Before analysing the case of hyperbolic discounting, we consider the benchmark case of exponential discounting.

3.1. Exponential discounting: the neutrality of secondary markets

Under the assumption that consumers are exponential discounters ($\beta = 1$), we analyse the competitive equilibrium when secondary markets never close. If consumers are exponential discounters, the interests of the different incarnations of a given consumer coincide in the following sense. Suppose the consumer's period t incarnation could control consumption not only in period t, but also in all subsequent periods. Given prices $\{p_s\}_{s=0}^{\infty}$, period t incarnation's optimal sequence of consumption is denoted by $\{x_s^t\}_{s\geqslant t}$. This consumption sequence is consistent with the optimal consumption sequence of future incarnations: for any period $\tau \geqslant t$ incarnation, $\{x_s^t\}_{s\geqslant\tau} = \{x_s^\tau\}_{s\geqslant\tau}$. This shows that the solution to the period 0 incarnation's intertemporal decision problem under commitment, $\{x_s^0\}_{s\geqslant 0}$, can be sustained in an intrapersonal equilibrium without commitment.

[8] That is, we consider a randomisation device which is public among incarnations of a consumer in the intrapersonal game. This is similar to the public randomisation device used in the repeated game literature; see Fudenberg and Maskin (1986).

Lemma 1. *Under exponential discounting, the solution to the period* 0 *incarnation's decision problem with commitment is the unique equilibrium of the intrapersonal game without commitment.*

Proof. This follows from the fact that the problem is recursive. The complete proof is given in the discussion paper version of this paper, Nocke and Peitz (2001). □

Hence, a consumer's intrapersonal game can be solved as if it were a decision problem of the consumer's period 0 incarnation.

We start by analysing consumer v's intrapersonal game, given an arbitrary sequence of prices $\{p_s\}_{s=0}^{\infty}$. The equilibrium consumption choice in period t is given by

$$x_t = \begin{cases} 1 & \text{if } v - p_t + \delta p_{t+1} \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, the marginal consumer type in period t, who is just indifferent between buying and not buying the good, is $\hat{v}_t = p_t - \delta p_{t+1}$. In the intrapersonal equilibrium, all consumer types below $\hat{v}_t$ do not consume the durable good, while all other types do. In a competitive equilibrium, markets clear in each period. For a given aggregate supply q, the marginal type is given by $\hat{v}_t = 1 - q$ for all t. Hence, the equilibrium price in period t can be written as

$$p_t = \frac{1-q}{1-\delta} + \lim_{s\to\infty} \delta^s p_{t+s}.$$

If the price path is not allowed to explode exponentially, we obtain

$$p_t = \frac{1-q}{1-\delta} \quad \text{for all } t \geqslant 0. \tag{1}$$

That is, the equilibrium price is constant over time.

Suppose now that secondary markets are closed down *after* period $T \geqslant 0$. Consumers who buy (or keep) the durable good in the last period of trade will be able to consume the good forever. In period T, the willingness to pay of a type-v consumer is thus equal to $v/(1-\delta)$. Given the aggregate supply of q units, the competitive equilibrium price in period T is then given by $p_T = (1-q)/(1-\delta)$, which coincides with the equilibrium price when secondary markets never close. In period $T-1$, the willingness to pay for a type-v consumer is given by $v + \delta p_T = v + \delta(1-q)/(1-\delta)$. Since supply is equal to q, the equilibrium price in period $T-1$ is again $(1-q)/(1-\delta)$. More generally, the competitive equilibrium price in any period $t \in \{0, 1, \ldots, T\}$ is given by Eq. (1) and is independent of the time at which secondary markets close. We thus have the following result.

Proposition 1. *Under exponential discounting* ($\beta = 1$)*, the existence of a secondary market for the durable good does not affect consumer choice in the primary market.*

Consequently, secondary markets cannot affect a monopolist's optimal choice of primary supply q (nor can they affect the equilibrium price in the initial period, p_0). The neutrality of secondary markets under exponential discounting serves as a useful benchmark. Of course, in a richer model, secondary markets may play a role if consumers' evaluations change over time or goods can become faulty.

3.2. Hyperbolic discounting: the nonneutrality of secondary markets

Let us now turn to the equilibrium analysis when consumers are hyperbolic discounters, i.e., $\beta < 1$. In the intrapersonal game, we confine attention to Markov strategies.

Consider consumer type v's intrapersonal game for a given price path $\{p_s\}_{s=0}^{\infty}$. Since we assume that all incarnations use Markov strategies, future incarnations' consumption decisions will be independent of the action taken by the current incarnation. Hence, the consumer's period t incarnation optimally decides to consume in period t if and only if $v - p_t + \beta\delta p_{t+1} \geqslant 0$. Consequently, in period t, all consumer types above the marginal type $\hat{v}_t \equiv p_t - \beta\delta p_{t+1}$ choose $x_t = 1$; all types below $\hat{v}_t$ select $x_t = 0$. Market clearing implies that $\hat{v}_t = 1 - q$. The equilibrium price in period t is then given by

$$p_t = \frac{1-q}{1-\beta\delta} + \lim_{s\to\infty} (\beta\delta)^s \, p_{t+s}.$$

Assuming again that the price path does not explode exponentially, we obtain

$$p^M \equiv p_t = \frac{1-q}{1-\beta\delta} \quad \text{for all } t \geqslant 0. \tag{2}$$

In the limit as $\beta \to 1$, we are back in the case of exponential discounting, and the equilibrium price is again given by (1).

Let us now compare this equilibrium with the one that obtains when the secondary markets are closed down after period $T \geqslant 0$. In the special case when $T = 0$, trade is only possible in the primary market. Intrapersonal strategies are allowed to depend in an arbitrary way on the history of the game. For a given price path $\{p_s\}_{s=0}^{T}$, we can solve for the intrapersonal equilibrium by backward induction. Consider consumer v's period T incarnation. His conditional indirect utility from buying the durable good in period T is equal to $v - p + \beta\delta \sum_{s=0}^{\infty} \delta^s v$. Hence, independently of the history of the game, he optimally chooses

$$x_T = \begin{cases} 1 & \text{if } \left(\dfrac{1-\delta+\beta\delta}{1-\delta}\right) v - p_T \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Using backward induction, we can now solve for period $T-1$ incarnation's equilibrium strategy. Since period T incarnation's strategy does not condition on the action taken in $T-1$, period $T-1$ incarnation optimally chooses to consume if and only if $v - p_{T-1} + \beta\delta p_T \geqslant 0$. Using the same argument for all previous incarnations, the intrapersonal equilibrium strategies in periods 0 to $T-1$ are as before. That is, in the (unique) intrapersonal equilibrium all incarnations use Markov strategies. Market clearing implies that $\hat{v}_t = 1 - q$ for all $t = 0, \ldots, T$, where $\hat{v}_t$ is again the marginal consumer type in period t. From market clearing in period T, we obtain[9]

$$p_T = \frac{1-\delta+\beta\delta}{1-\delta}(1-q). \tag{3}$$

[9] As will become clear later, this price is equal to the "collusive price" p^C.

Market clearing in all previous periods implies the following equilibrium price in period $t \in \{0, \dots, T\}$:

$$p_t = (1-q) \sum_{s=0}^{T-t-1} (\beta\delta)^s + (\beta\delta)^{T-t} p_T$$
$$= \frac{1-(\beta\delta)^{T-t}}{1-\beta\delta}(1-q) + (\beta\delta)^{T-t} p_T.$$

Abusing notation, the equilibrium price may be rewritten as

$$p_t(T, q) = \frac{1-\delta+\delta(\beta\delta)^{T-t+1}(1-\beta)}{(1-\beta\delta)(1-\delta)}(1-q). \tag{4}$$

For a given final trading period T and an initial supply q, the equilibrium price is increasing over time: $p_t(T, q) < p_{t+1}(T, q)$ for all $t = 0, \dots, T-1$. The equilibrium price in a given period is lower if secondary markets close down later: $p_t(T+1, q) < p_t(T, q)$ for $t = 0, \dots, T$. In particular, as $T \to \infty$, the equilibrium price in any given period converges to the equilibrium price when the secondary markets never close and consumers use Markov strategies; this price p^M is given by (2). For a given price path, there exists a unique subgame perfect equilibrium (SPE) in each consumer's intrapersonal game, provided secondary markets close in finite time ($T < \infty$). In the limit as the final trading period T goes to infinity, this equilibrium converges to the unique Markov perfect equilibrium (MPE) of the infinite intrapersonal game ($T = \infty$). The uniqueness of equilibrium in each consumer's intrapersonal game for a given price path translates into the uniqueness of the competitive equilibrium. We summarise our findings by the following lemma.

Lemma 2. *Suppose consumers are hyperbolic discounters* ($\beta \in (0, 1)$). *If secondary markets close in finite time $T < \infty$, there exists a unique competitive equilibrium with a price path $\{p_t\}_{t=0}^{T}$ characterised by Eq.* (4). *If secondary markets never close, there exists a unique competitive Markov equilibrium with a price path $\{p_t\}_{t=0}^{\infty}$ characterised by Eq.* (2).

Under hyperbolic discounting, i.e., $\beta \in (0, 1)$, secondary markets are no longer neutral. For a given initial supply of q units, the equilibrium price in the primary market is the larger, the earlier secondary markets close down. This nonneutrality obtains although the set of consumers who buy the good along the equilibrium path is independent of the final trading period T. That is, all trade in the secondary markets is "trivial" (or "degenerate") in that the same set of consumers re-sell and re-purchase the good in each period.

Proposition 2. *For a given initial supply q, the competitive equilibrium price in the primary market, p_0, is the larger, the earlier secondary markets close down. However, the equilibrium allocation is independent of the final trading period.*

In order to understand the features of the competitive equilibrium consider the case where secondary markets never close down. Suppose the equilibrium price is p in all periods. If the consumer's period t incarnation could commit on the whole future consumption path, which path would he optimally choose?

Consider consumer type v's period t incarnation. Given a stationary price p, his indirect utility, conditional on the consumption path $\{x_{t+s}\}_{s=0}^{\infty}$, is given by

$$V_t\big(p; v \mid \{x_{t+s}\}_{s=0}^{\infty}\big) = x_t\big(v - (1-\beta\delta)p\big) + \beta \sum_{s=1}^{\infty} \delta^s x_{t+s}\big(v - (1-\delta)p\big).$$

If the consumer's period t incarnation could commit to the whole future consumption path, he would optimally choose

$$x_t = \begin{cases} 1 & \text{if } v \geqslant (1-\beta\delta)p, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad x_{t+s} = \begin{cases} 1 & \text{if } v \geqslant (1-\delta)p, \\ 0 & \text{otherwise,} \end{cases} \quad \text{for } s \geqslant 1.$$

Hence, any incarnation of type $v \in [(1-\beta\delta)p, 1]$ would like to commit to consume in the current period and in all periods thereafter. Any incarnation of type $v \in [(1-\delta)p, (1-\beta\delta)p)$ would like to commit to postpone consumption for a single period, whereas any incarnation of type $v \in [0, (1-\delta)p)$ would prefer never to consume.

However, commitments are not possible. Hence, in the intrapersonal Markov equilibrium for a given constant price p, all incarnations of consumer types $v \in ((1-\delta)p, (1-\beta\delta)p)$, who ideally would like to procrastinate consumption for a single period, end up *never* buying the durable good (even though they are fully aware of their time inconsistent behaviour). Note, however, that all incarnations of some of these types, namely those with valuations $v \in ((1-\delta)/(1-\delta+\beta\delta)p, (1-\beta\delta)p)$, would be strictly better off by consuming the good in all periods rather than never consuming it; see the welfare discussion below.[10]

Proposition 2 suggests that a monopolist, who chooses the primary supply q so as to maximise her profit, may have an incentive to prevent future trade in the durable good. Suppose, for simplicity, the monopolist has a constant marginal cost of production $c \geqslant 0$. Her decision problem in period 0 is then to set q so as to maximise $[p_0(T,q) - c]q$, where $p_0(T,q)$ is given by Eq. (4). Since the price $p_0(T,q)$ is of the form $k(T)(1-q)$, the solution to this problem is given by

$$q^*(T) = \frac{k(T) - c}{2k(T)},$$

where $k(T) \equiv [1-\delta+\delta(\beta\delta)^{T+1}(1-\beta)]/[(1-\beta\delta)(1-\delta)]$. If $c = 0$, the monopolist's equilibrium supply is $1/2$, independently of the closure time of secondary markets. If $c > 0$, however, the monopolist's supply is decreasing with closure time T. The equilibrium price in period 0 is given by $p_0^*(T) = [k(T) + c]/2$, and the monopolist's equilibrium profit by

$$\pi^*(T) = \frac{[k(T) - c]^2}{2k(T)}.$$

[10] Phrasing our result in the words of Gul and Pesendorfer (2000, 2001), the only "temptation" which matters in our model is the temptation of certain types and incarnations, who in the absence of secondary markets would buy the good, to procrastinate so that these types end up not consuming in any period. The opposite "temptation" to buy the good at some point, although the consumer would refrain from buying in the absence of secondary markets, does not arise.

Both $p_0^*(T)$ and $\pi^*(T)$ are decreasing in closure time T. The earlier the monopolist can close down further trade, the better she is off. We thus obtain the following additional nonneutrality result.

Proposition 3. *A profit maximising monopolist optimally chooses a smaller primary supply, the later secondary markets close down.*

Qualitatively, the same results hold in a Cournot oligopoly with a fixed number of firms. The effect on quantity is reinforced (ignoring integer effects) in a free entry equilibrium where each firm in the market has to incur a sunk cost. Proposition 3 obtains since the introduction of secondary markets shifts the demand curve in period 0 towards the origin.

To conclude this section, we carry out an intrapersonal welfare analysis for a given stationary price. Observe that *a priori* it may not be possible to Pareto-rank the different allocations since the different incarnations of the same consumer have potentially conflicting interests. A great advantage of our stationary set up is that it allows us to focus on stationary equilibria, which can be Pareto-ranked. However, the following problem remains: an incarnation's utility has been defined only over current and future consumption, but not over past consumption. The reason is that an incarnation cannot control past consumption; hence, the utility over past consumption does not matter for a positive analysis. However, an individual may care about the past: memories of good events are likely to be preferred to memories of bad events. That is, when making utility comparisons also past consumption has to enter the utility. One way of formalising this idea is to view the past as a mirror of the future: consumption which is more distant from today is discounted more (Caplin and Leahy, 1999).[11] Period t incarnation's direct utility is

$$U_t\big(\{x_s\}_{s=0}^{\infty}, \{y_s\}_{s=0}^{\infty}; v\big) = u(x_t, y_t; v) + \beta \sum_{s=0,\ s\neq t}^{\infty} \delta^{|t-s|} u(x_s, y_s; v).$$

His conditional indirect utility from buying in all periods $0, 1, 2, \ldots$ can then be expressed as

[11] There appear to be two natural alternative formulations. The first is that past consumption does not enter current utility. The second is to give earlier periods a greater weight than later periods. Specifically, period t incarnation's utility is given by

$$U_t\big(\{x_s\}_{s=0}^{\infty}, \{y_s\}_{s=0}^{\infty}; v\big) = \delta^t \left[u(x_t, y_t; v) + \beta \sum_{s=-t,\ s\neq 0}^{\infty} \delta^s u(x_{t+s}, y_{t+s}; v) \right].$$

In the case of exponential discounting ($\beta = 1$), this simplifies to

$$U_t\big(\{x_s\}_{s=0}^{\infty}, \{y_s\}_{s=0}^{\infty}; v\big) = \sum_{s=0}^{\infty} \delta^s u(x_s, y_s; v),$$

which is independent of t, i.e., the various incarnations of the same consumer agree on the evaluation of consumption streams. The conclusion of our welfare analysis carries over to both alternative formulations.

$$V_t\big(p; v \mid \{x_s = 1\}_{s=0}^{\infty}\big) = v + \beta\delta^t(v - p) + \beta\sum_{s=1}^{t-1}\delta^s v + \beta\sum_{s=1}^{\infty}\delta^s v$$
$$= \frac{v}{1-\delta}\big(1 - \delta + \beta\delta(2 - \delta^t)\big) - \beta\delta^t p,$$

which is strictly increasing in t.

If the consumer's period 0 incarnation prefers this consumption stream to never buying, so will all other incarnations. This makes it possible to Pareto-rank the consumption streams $\{x_t = 0\}_{t=0}^{\infty}$ (never buying) and $\{x_t = 1\}_{t=0}^{\infty}$ (always buying): if $v \in ((1-\delta)/(1-\delta+\beta\delta)p, (1-\beta\delta)p)$ then all incarnations would prefer to consume in all periods rather than never to consume. Nevertheless, along the (intrapersonal Markov) equilibrium path, they will never consume.

The result on Pareto-ranked consumption streams implies that the intrapersonal Markov equilibrium is not stable with respect to mutual deviations by all incarnations of the consumer.

4. Non-Markovian equilibria

In this section we fully characterise the set of stationary equilibrium prices. To the extent that the infinite time model is merely seen as an approximation of the finite time model with a long time horizon, the Markov perfect equilibrium is the appropriate equilibrium of the intrapersonal game. However, there is a qualitative difference between finite and infinite time. Non-Markovian strategies in which actions may depend on the whole history of play, cannot be optimal in a finite horizon model. If consumers are never sure that secondary markets will cease to exist, one should also analyse non-Markovian equilibria. Apart from the Markovian equilibrium, we are particularly interested in an equilibrium in which each incarnation expects to end up in an eternal no-consumption situation if he does not buy the good himself because this situation corresponds to the equilibrium in the absence of secondary markets. Such strategies will allow incarnations to collude over time. In a collusive intrapersonal equilibrium, a current deviation to no consumption has a long-run impact so that the trade-off between purchasing the good today and not purchasing it today is qualitatively different from the trade-off in a Markov intrapersonal equilibrium.

4.1. Neutrality after all? Collusive strategies

We consider collusive strategies which enable a consumer to mimic the outcome in the absence of secondary markets. Remember that the absence of secondary markets forces a "now or never" purchasing decision in the primary market. To restore the same trade-off in the presence of secondary markets, a deviation from $x_t = 1$ to $x_t = 0$ in any period t must trigger a switch from $x_s = 1$ to $x_s = 0$ for all future incarnations $s > t$.[12]

Implementing such strategies for constant price paths can be done merely by looking at past actions x_s, $s < t$, because the environment for the consumer remains stationary.

[12] Note the similarity to grim trigger strategies in the literature on repeated games (see, e.g., Friedman, 1971).

The basic idea of a collusive strategy is to keep consuming if and only if all previous incarnations decided to consume as well. If some previous incarnation decided not to consume, then the present incarnation, following the collusive strategy, does not consume either. Those (high) types who consume in every period even when using Markov strategies (i.e., those with valuations $v \geqslant (1-\beta\delta)p$) are exempted from this punishment.

Definition 1. Given the stationary price p, the *collusive strategy profile* $\Sigma^C(p; v)$ in consumer type v's intrapersonal game is defined as follows:

$$x_0 = \begin{cases} 1 & \text{if } v \geqslant \dfrac{1-\delta}{1-\delta+\beta\delta} p, \\ 0 & \text{otherwise,} \end{cases}$$

$$x_t = \begin{cases} 1 & \text{if } v \geqslant (1-\beta\delta)p, \\ 1 & \text{if } x_{t-s} = 1 \text{ for all } s \in \{1, \dots, t\} \\ & \quad \text{and } v \in \left(\dfrac{1-\delta}{1-\delta+\beta\delta} p, (1-\beta\delta)p\right), \quad \text{for } t \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 3. *For any stationary price p, the collusive strategy profile $\Sigma^C(p; v)$ forms an SPE in consumer type v's intrapersonal game.*

Suppose all types use collusive strategies, i.e., they buy in period t if the consumption path starting at t, $\{x_s = 1\}_{s \geqslant t}$, gives a higher utility than $\{x_s = 0\}_{s \geqslant t}$. Otherwise, they do not buy (except if their valuation is sufficiently high so that $v \geqslant (1-\beta\delta)p$). The indirect utility, conditional on $\{x_s = 1\}_{s \geqslant t}$, is given by

$$V_t\big(p; v \,\big|\, \{x_s = 1\}_{s \geqslant t}\big) = v - p + \beta \sum_{s=1}^{\infty} \delta^s v,$$

which is nonnegative if $v(1-\delta+\beta\delta)/(1-\delta) \geqslant p$. If all consumers with valuations above $\hat{v}$ buy the good, we must have $1-\hat{v} = q$ for market clearing. Consequently, the competitive equilibrium price is

$$p^C = \frac{1-\delta+\beta\delta}{1-\delta}(1-q),$$

provided all incarnations of all consumers use the collusive strategy.[13]

Proposition 4. *Given initial supply q, the set of collusive strategy profiles $\Sigma^C(p^C; v)$, parameterised by v, and the stationary price path with*

$$p_t = p^C = \frac{1-\delta+\beta\delta}{1-\delta}(1-q) \quad \textit{for all } t$$

form a competitive equilibrium in the durable goods market.

[13] Recall that if secondary markets close in finite time, then the unique competitive equilibrium price in the last period of trade is equal to p^C; see Eq. (3).

Note that the collusive equilibrium restores the equilibrium outcome in the absence of secondary markets, both in terms of the equilibrium price path and the equilibrium allocation.

As pointed out in the previous section, there exist Pareto improvements over the intrapersonal equilibrium outcome reached by Markovian consumers. The collusive strategy profile implements such a Pareto improvement: for a given stationary price p, all incarnations are (weakly) better off in the collusive intrapersonal equilibrium than in the Markov intrapersonal equilibrium. (This may be viewed as an argument against the Markov intrapersonal equilibrium, where the different incarnations of a consumer fail to coordinate on more sophisticated strategies.[14]) Note, however, that this situation is reminiscent of a prisoner's dilemma: if all incarnations of all consumers coordinate on the collusive (rather than the Markovian) intrapersonal strategy, then (for a given primary supply q) all consumers are worse off: the allocation is the same as in the competitive Markov equilibrium, but consumers have to pay a higher price.

4.2. Lower and upper bound on prices; allocative inefficiency

In this subsection, we provide tight upper and lower bounds on prices that can be sustained in a stationary equilibrium. We show that the Markovian price $p^M = (1-q)/(1-\beta\delta)$ is the lower bound on prices and that the collusive price $p^C = (1-q)(1-\delta+\beta\delta)/(1-\delta)$ is the upper bound on prices. Furthermore, we construct a parameterised family of strategy profiles which "implements" any price in $[p^M, p^C]$ as a stationary equilibrium price; the resulting equilibrium allocation of the durable good is inefficient for any price in (p^M, p^C). That is, it is possible to construct *inefficient competitive equilibria*.

The Markov price p^M is the largest lower bound for the set of stationary equilibrium prices. Before spelling this out in a proposition, it is useful to see how, in a competitive equilibrium with a price $p < p^M$, subgame perfection implies restrictions on the punishment strategies that can be used in the intrapersonal game. Note that in the construction of the collusive strategy we used the punishment "never consume again" for a deviation from the equilibrium path; we will now see that such a severe punishment cannot be used in the intrapersonal game when $p < p^M$. To sustain such a low price, present consumption has to be discouraged. In particular, there must exist some type-$\tilde{v}$ consumers with $\tilde{v} - p^M + \beta\delta p^M \geqslant 0$ who do not buy in equilibrium (in some period t). Consider any subgame starting in period t. The worst punishment for buying in this period is that all future incarnations $s \geqslant t+1$ do not consume, i.e., $\{x_s\}_{s \geqslant t+1} = \{0, 0, 0, \ldots\}$. Such a punishment is not subgame perfect: given the sequence $\{x_s\}_{s \geqslant t+1} = \{0, 0, 0, \ldots\}$, period $t+1$ incarnation has an incentive to deviate and consume the durable good if

[14] Suppose that a strategy is inherited by an incarnation from its previous incarnation. That is, a strategy is a genetic pattern and the composition of strategies in the population possibly changes over time. One may then consider the evolution of a population of consumers who inherit the strategy of the previous consumers' incarnation (subject to some mutation). Note that with sufficiently few mutations, the collusive strategy has a higher "fitness" (for certain types) than the Markov strategy in that it gives a higher payoff to those inheriting this strategy. In such an evolutionary context, the collusive and not the Markovian outcome is predicted.

$\tilde{v} - p + \beta\delta p > 0$. Hence, by contradiction, one cannot support prices less than p^M with strategies that use the worst punishment against a deviant incarnation.

Proposition 5. *The Markov price p^M is the lowest price that can be supported in any stationary competitive equilibrium.*

Proof. In Section 2, we have already shown that p^M can be supported in a stationary equilibrium by Markov strategies. Hence, it remains to be shown that a lower price cannot be supported in a stationary competitive equilibrium.

For a price $p < p^M$ to be supportable in a stationary equilibrium, there must be some type $v \geqslant 1 - q$ who does not buy the good in all periods at this price. We now want to show that there exists a profitable deviation for some incarnation of such a type.

Given the stationarity of both the game and the candidate equilibrium price, it is sufficient to consider (mixed) strategy profiles such that the expected utility (prior to randomisation) is the same for all incarnations of a given consumer. (Intuitively, if this were not the case, then some incarnations would have stronger incentives to deviate than others. By equalising the incentives to deviate for all incarnations, we make the "strongest case" in favour of the candidate equilibrium.)

The only way to equalise expected utility across incarnations is to have a constant probability α of consuming the good in each period. Since any punishment in the intrapersonal game must satisfy subgame perfection, we have to consider deviations from punishment paths as well. Hence, there must exist a sequence of probabilities $\{\alpha_i\}_{i=0}^{\infty}$ such that the following strategy profile forms an SPE in the intrapersonal game: each incarnation consumes the good with some probability, α_i say; if a past incarnation has deviated from this strategy (e.g., by consuming the good although, according to the used randomisation device, the incarnation should not have consumed), then all future incarnations consume the good with some (other) probability, α_{i+1} say, until another incarnation deviates.

Using the randomisation device, the "punishment phase $i + 1$" is triggered in period $t + 1$ if $x_t = 1$ and $z_t \notin Z_i \subseteq Z$ (or if $x_t = 0$ and $z_t \in Z_i$), where the probability of the realised random variable z_t being in Z_i is given by $\mu(Z_i) = \alpha_i$.

Let us now consider deviations of the following kind. After having observed the outcome of the randomisation device, the incarnation decides to deviate from its mixed strategy and consume the durable good. Suppose the different incarnations mix with probability α_0 and get "punished" with probability α_1. Then, an incarnation's conditional indirect utility from deviating (conditional on consuming today) is

$$v - (1 - \beta\delta)p + \beta \sum_{t=1}^{\infty} \delta^t \alpha_1 \big[v - (1 - \delta)p\big].$$

Let $a \equiv v - (1 - \beta\delta)p$ and $a' \equiv v - (1 - \delta)p$, and note that $a' > a > 0$ since $v \geqslant 1 - q$ and $p < p^M(q)$. Then, this deviation is unprofitable if and only if

$$a - (\alpha_0 - \alpha_1)\frac{\beta\delta}{1 - \delta}a' \leqslant 0.$$

More generally, we need

$$\alpha_i - \alpha_{i+1} \geqslant \left(\frac{1-\delta}{\beta\delta}\right)\frac{a}{a'} \quad \text{for all } i = 0, 1, 2, \ldots.$$

Clearly, since $1 \geqslant \alpha_i \geqslant 0$ for all i, this cannot hold. Hence, the proposed strategy profile does not form an SPE in the intrapersonal game of consumer type $v \geqslant 1-q$. □

The collusive price p^C is the lowest upper bound for the set of stationary equilibrium prices. Before spelling this out in a proposition we provide the argument for *pure* intrapersonal strategies.

Let us first show that there does not exist an equilibrium with constant price $p \in (p^C, (1-q)/(1-\delta))$. For the price to be sustainable in any equilibrium, there must exist an equilibrium where the marginal consumer of type $v = 1-q$ buys the good at this price. To prove our claim, we have to show that the current incarnation of a consumer of type

$$v \in \left((1-\delta)p, \frac{1-\delta}{1-\delta+\beta\delta}p\right) \tag{5}$$

cannot be induced to buy the good. For such a consumer type, it is easy to see that period t incarnation's most preferred consumption stream is $\{x_s\}_{s\geqslant t} = \{0, 1, 1, 1, \ldots\}$, and the worst is $\{1, 0, 0, 0, \ldots\}$. Hence, the best reward for consumption in the present period is consumption in all future periods: $\{x_s\}_{s\geqslant t+1} = \{1, 1, 1, \ldots\}$, and the worst possible punishment for not consuming today is never to consume again: $\{0, 0, 0, \ldots\}$. Hence, period t incarnation can only be induced to buy today if the utility from consumption path $\{x_s\}_{s\geqslant t} = \{1, 1, \ldots\}$ is higher than from consumption path $\{0, 0, \ldots\}$. This inequality holds if and only if

$$v - p + \beta\delta\sum_{s=0}^{\infty}\delta^s v \geqslant 0, \quad \text{i.e.,} \quad v \geqslant \frac{1-\delta}{1-\delta+\beta\delta}p,$$

which is in contradiction to (5). This completes the proof of the first claim.

We now claim that there does not exist a competitive equilibrium (in pure intrapersonal strategies) with stationary price p such that $p \geqslant (1-q)/(1-\delta)$. Since we are interested in the behaviour of the marginal consumer $\hat{v} = 1-q$, we can confine attention to types v such that

$$v \leqslant (1-\delta)p.$$

Observe that any incarnation of such a type would never like to consume in that the contribution to his utility from consumption in any period is nonpositive.

Consider now the behaviour of period T incarnation of type v. His (indirect) utility is bounded from below by

$$\beta\delta\left(\frac{v}{1-\delta} - p\right),$$

as period T incarnation may decide not to buy in T, and the worst possible punishment is consumption in all future periods. The contribution of no consumption in T and consumption thereafter to period $T-1$ incarnation's indirect utility is

$$\beta\delta^2\left(\frac{v}{1-\delta}-p\right).$$

Now, period T incarnation may decide to buy at T and at dates $\{T+t_k\}_k$, which gives him utility of

$$v-(1-\beta\delta)p+\beta\sum_k\delta^{t_k}\big[v-(1-\delta)p\big]\geqslant\beta\delta\left(\frac{v}{1-\delta}-p\right),$$

where the inequality follows from the fact that he may decide not to buy in T. The contribution of this consumption stream (from T onwards) to period $T-1$ incarnation's utility is

$$\begin{aligned}&\beta\delta\big[v-(1-\delta)p\big]+\beta\delta\sum_k\delta^{t_k}\big[v-(1-\delta)p\big]\\&\quad\geqslant\beta\delta^2\left(\frac{v}{1-\delta}-p\right)+\delta(1-\beta)(p-v)\\&\quad>\beta\delta^2\left(\frac{v}{1-\delta}-p\right)\quad\text{if } v<p \text{ (as assumed).}\end{aligned}$$

Repeating this exercise for all previous incarnations, we obtain that by not buying in $t=0$, period 0 incarnation can ensure himself a utility level of at least

$$\beta\delta^{T+1}\left(\frac{v}{1-\delta}-p\right),$$

which converges to zero as $T\to\infty$. In contrast, if period 0 incarnation decided to buy in $t=0$, his utility would be bounded from above by

$$v-(1-\beta\delta)p<0.$$

This concludes the argument for *pure* intrapersonal strategies.

Proposition 6. *The "collusive" price p^C is the highest price that can be supported in any stationary competitive equilibrium.*

Proof. We have already shown that p^C can be supported in a stationary competitive equilibrium by collusive strategies. Hence, it remains to be shown that a higher price cannot be supported in a stationary equilibrium.

For a price $p>p^C$ to be supportable in a stationary equilibrium, there must be some type $v\leqslant 1-q$ who buys the good in some period at this price. We then have to show that there exists a profitable deviation for some incarnation of such a type. The proof proceeds along the same lines as the proof of the previous proposition. □

With Propositions 5 and 6, we have established that the set of prices of all stationary equilibria must be a subset of $[p^M, p^C]$.

Corollary 1. *Suppose p is the price in a stationary competitive equilibrium. Then, $p \in [p^M, p^C]$.*

Note that $\lim_{\beta\to 1} p^M = \lim_{\beta\to 1} p^C$: the set of stationary equilibrium prices shrinks to a single price as consumers' discounting becomes exponential.

It is possible to sustain any stationary price $p \in [p^M, p^C]$ by assuming that a fraction $\lambda(p)$ of consumers play the collusive strategy in their intrapersonal game, while all others use Markov strategies. This requires that the population is heterogeneous with respect to their type v and with respect to their "personality," expressed by their intrapersonal strategies. For any $\lambda \in (0, 1)$, no trade between consumers occurs along the equilibrium path and the allocation of the durable good is inefficient: there is some low type v' with collusive intrapersonal strategies, who always buys the good along the equilibrium path, and a higher type $v'' > v'$ with a Markovian strategy profile, who never buys the good.

Consider instead the following population which is ex ante only heterogeneous with respect to their type v: at each point in time an incarnation chooses i.i.d. the collusive strategy with probability λ and the Markov strategy otherwise. Based on the randomisation device, a consumer with realisation $z_t \in Z^C \subseteq Z$ with $\mu(Z^C) = \lambda$ follows the collusive strategy, where a past deviation from collusion in period $s < t$ is only punished if $z_s \in Z^C$. We require that the realised population mean corresponds to this probability λ in each period. Being collusive means here to condition one's actions only on the actions of those past incarnations who also used collusive strategies.

Proposition 7. *The set of equilibrium prices which results from all equilibria with a probabilistic mix between collusive strategies and Markov strategies is the set $[p^M, p^C]$.*

Proof. First, find the type $\hat{v}$ who is indifferent between buying and not buying when following the collusive strategy. Clearly, this type's current incarnation does not buy the good when he follows the Markovian strategy. This marginal collusive type is given by

$$\hat{v} - (1-\beta\delta)p + \frac{\beta\delta\lambda}{1-\delta}\big(\hat{v} - (1-\delta)p\big) = 0,$$

i.e.,

$$\hat{v} = \frac{(1-\delta)((1-\beta\delta)+\beta\delta\lambda)}{(1-\delta)+\beta\delta\lambda}p.$$

Hence, types $v \in [\hat{v}, (1-\beta\delta)p)$ buy only in those periods in which the current incarnation has drawn a collusive strategy, and types $v \in [(1-\beta\delta)p, 1]$ buy the good in all periods. Market clearing implies

$$q = 1 - (1-\beta\delta)p + \lambda p\left((1-\beta\delta) - \frac{(1-\delta)((1-\beta\delta)+\beta\delta\lambda)}{(1-\delta)+\beta\delta\lambda}\right) \equiv f(\lambda).$$

Since

$$\lim_{\lambda\to 1} f(\lambda) = 1 - \frac{1-\delta}{1-\delta+\beta\delta}p \quad \text{and} \quad \lim_{\lambda\to 0} f(\lambda) = 1 - (1-\beta\delta)p,$$

for any price $p \in [p^M, p^C]$, we can find a $\lambda \in [0, 1]$ such that this price is supported in equilibrium. Similarly, for any $\lambda \in [0, 1]$, there exists a stationary equilibrium with price $p \in [p^M, p^C]$. □

The example serves well to make another point. In the previous example (where the population of consumers is heterogeneous with respect to their strategy profiles), we observed that the allocation of the durable good is inefficient. This result also holds in the present example. In addition, there is nondegenerate trade of the durable good: some hitherto Markovian consumers who switch to a collusive strategy purchase the good, while hitherto collusive consumers who switch to a Markovian strategy sell the good. This is in contrast to the pure Markovian and collusive strategies ($\lambda = 0$ and $\lambda = 1$, respectively), where all trade in secondary markets is degenerate (in that the same consumers repurchase the goods they sell).

Proposition 8. *In any competitive equilibrium which is induced by intrapersonal equilibria with a probabilistic mix between collusive strategies and Markov strategies, $\lambda \in (0, 1)$, some units of the durable good change hands and the allocation of the durable good is inefficient.*

Recall that the equilibrium allocation in the absence of secondary markets is efficient. Hence, in our model with time-inconsistent consumers, an increase in the number of markets may generate allocative inefficiency.

Remark 1. In our model, we can also consider finite punishment. In particular, we can analyse the set of prices that can be sustained in competitive equilibrium for a given maximal length of the punishment phase. To this end, the strategy profile has to be rewritten. Informally, a period t incarnation must be able to tell whether an action by some incarnation $s < t$ which is different from the action along the equilibrium path is a deviation which has to be punished or whether it is a punishment to some earlier deviation which itself is not to be punished. For a stationary price p the marginal consumer type with a punishment of k periods, v^k, solves

$$v^k - p + \beta \sum_{t=1}^{k} \delta^t v + \beta \delta^k p = 0.$$

If a deviation which occurred within the last k periods triggers a punishment of k periods, the collusive price p^k is

$$p^k = \frac{1 + \beta\delta(1-\delta^k)/(1-\delta)}{1 + \beta\delta^{k+1}}(1-q).$$

Clearly, $p^0 = p^M$ and $p^\infty = p^C$. Note that $p^k < p^{k+1}$, i.e., a longer punishment can support a higher price.

5. Conclusion

In this paper, we have analysed perfectly competitive secondary markets for a durable good. In our market environment, secondary markets are neutral when consumers are exponential discounters:

(i) the price in the primary market does not depend on whether or not the durable good can subsequently be traded in secondary markets;
(ii) the incentives for the provision of primary supply by a monopolist (or oligopolist) are not affected by the existence of secondary markets; and
(iii) no trade occurs in the secondary markets.

With exponential discounting the allocation is efficient: consumers with valuations above a certain threshold buy the good in equilibrium, whereas those with lower valuations do not. When consumers are hyperbolic discounters, these results no longer hold. In the absence of secondary markets, a purchase of the durable good in the primary market implies a commitment to consume the good in all future periods. When the good can be traded in secondary markets, such a commitment is no longer possible and a consumer may procrastinate. We have obtained the following nonneutrality results:

(1) The price in the primary market is decreasing with the number of periods in which secondary markets are open (Section 3, Proposition 2).
(2) The primary supply by a monopolist (with positive marginal costs) is the smaller, the later secondary markets shut down (Section 3, Proposition 3). This result carries over to the case where the primary supply is provided by a group of oligopolistic producers.
(3) When secondary markets never close, there are inefficient competitive equilibria: consumers with a relatively low willingness to pay buy the durable good whereas others with a higher willingness to pay do not (Section 4, Proposition 8).
(4) When secondary markets never close, there are equilibria in which trade in the durable good occurs in each period (Section 4, Proposition 8).

We have characterised the set of stationary equilibrium prices in the case where secondary markets never close. Equilibrium prices are bounded from below by the Markovian price, and bounded from above by the collusive price. The latter coincides with the unique equilibrium price when secondary markets never open. Apart from stationary equilibria, there also exist equilibria with increasing, decreasing, and cycling price paths, despite the stationarity of the market environment. We analyse such equilibria in our discussion paper (Nocke and Peitz, 2001).

While we consider the present setup useful for studying the effects of hyperbolic discounting, there may be other interesting durable good environments. For instance, it may be worthwhile to study the case where consumers' willingness to pay changes over time. One may also want to analyse the case where one or several firms provide additional supply of the durable good over time. This introduces a Coasian commitment problem on the side of the supplier(s).

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