Online Appendix - Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger Policy

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1 Formal Model, Bargaining, and Computation

In this section, we provide a more detailed description of parts of the main paper: the model presented in Section 2, the calculations of merger bargaining outcomes, the proof of Proposition 1, and our computational algorithm.

1.1 Formal Model Description

We follow the timing displayed in Figure 1 and the notation established in Section 2 of the main paper. Let the number of firms be $n \geq 2$, the set of firms be $\mathcal{I} = \{1, \cdots, n\}$, and the industry state be the vector of their capital stocks $\mathbf{K} = (K_1, \cdots, K_n)$. Firms are restricted to an integer number of possible capital levels, with the maximal capital level $\overline{K}$ chosen to be non-binding. Since a firm may have zero capital, let $\mathcal{S} \equiv \{0, 1, 2, ..., \overline{K}\}$ be the admissible values of K_i and let $\mathcal{S}^n$ be the state space. The industry's state at the beginning of a period is its *ex ante state* while its state just after the entry stage and before the Cournot competition stage is its *interim state*.

The logic of backward induction guides our presentation of the model. The firms take their environment and the antitrust policy $\{a_{ij}(\cdot)\}_{ij \in \mathcal{J}}$ as given, where $\mathcal{J} \equiv \{ij | i, j \in \mathcal{I}, i \neq j\}$ is the set of pairs of firms, and $a_{ij}(\mathbf{K})$ is the probability that the authority approves merger M_{ij} when proposed in ex ante state $\mathbf{K}$. Therefore we first derive conditions for their symmetric Markov perfect equilibrium behavior given that their goal is to maximize the expected net present value (ENPV) of their future cash flows. We then turn to the antitrust authority's problem of maximizing welfare. We consider authorities that vary in their goals and their ability to commit.

Firm i's ex ante value function in ex ante state $\mathbf{K}$, $V_i(\mathbf{K})$, is the beginning-of-period ENPV of its future cash flows. Similarly, firm i's interim value $\overline{V_i}(\mathbf{K})$ gives the ENPV of its future cash flows starting from interim state $\mathbf{K}$. The transition from the ex ante state to the interim state is the outcome of the merger bargaining game between firms and, if a merger has been proposed, the merger approval decision of the antitrust authority. The transition from the interim state to next period's ex ante state is the outcome of firms' investment decisions and the subsequent (stochastic) depreciation of capital.

Throughout we assume that the firms play Markov perfect equilibrium strategies that the antitrust policy and the specified merger protocol induce. To understand the firms' Markov perfect equilibrium, fix the antitrust authority's merger approval policy and let $\{V_i(\cdot)\}_{i\in\mathcal{I}}$ be the value functions that give the ENPV of the firms' future cash flows at the beginning of period $t+1$ as a function of the ex ante state $\mathbf{K}$. Given these value functions, each firm i uses backward induction to calculate, for each interim state $\mathbf{K}' \in \mathcal{S}^n$, its optimal period-$t$ investment decision, which must be a best reply to its competitors' investment policy choices. Given this Nash equilibrium in investment policies conditional on the beginning of period $t+1$ value functions $\{V_i(\cdot)\}_{i\in\mathcal{I}}$, each firm can calculate for all interim states $\mathbf{K}'$ its interim values $\overline{V}_i(\mathbf{K}')$ conditional on $\{V_i(\cdot)\}_{i\in\mathcal{I}}$.

Based on this vector of interim values and given the antitrust authority's approval policy, the firms negotiate over mergers. These negotiations, conducted in accordance with the protocols specified in Section 1.2 below, determine for each ex ante state $\mathbf{K}$ the probability of each possible merger M_{ij} being proposed, as well as the ex ante values $\{\hat{V}_i(\cdot)\}_{i\in\mathcal{I}}$ in period t. If $\{\hat{V}_i(\cdot)\}_{i\in\mathcal{I}} = \{V_i(\cdot)\}_{i\in\mathcal{I}}$, then the ex ante value functions, the interim value functions, the investment functions, and the equilibrium merger bargaining outcomes together form a Markov perfect equilibrium for the industry with respect to the fixed merger policy.

We now present the model and our notion of Markov perfect equilibrium in more detail. Following the logic of backward induction, we begin by describing firms' investment policies.

1.1.1 Firms' Investment Policies

At the investment stage, each firm i, after privately learning the K_i independent draws of its capital augmentation costs $(c_1, ..., c_{K_i})$ and the single independent draw of its greenfield cost c_g, unilaterally decides how many units of capital (if any) to add.[1]

Firm i's investment policy is denoted $\xi_i(\cdot|\mathbf{K}) : \{0, 1, \cdots, \overline{K} - K_i\} \times \mathcal{S}^n \to [0, 1]$. Prior to the realization of its cost draws, policy ξ_i gives the probability $\xi_i(k_i|\mathbf{K})$ of firm i adding $k_i \in \{0, 1, ..., \overline{K} - K_i\}$ units of capital in interim state $\mathbf{K}$. Recall that at the end of each period each unit of capital depreciates with probability d, so if firm i enters the depreciation stage

[1] Each firm also decides on the quantity it produces. This decision is embedded in firm i's single-period profit function $\pi(K_i, \mathbf{K}_{-i})$ because we assume competition in the product market is static Cournot.

with K_i units of capital, then the probability it exits the stage with K_i' units of capital is

$$\kappa(K_i'|K_i) = \begin{cases} \binom{K_i}{K_i'}(1-d)^{K_i'}d^{K_i-K_i'} & \text{if } K_i' \in \{0,1,...,K_i\} \\ 0 & \text{otherwise} \end{cases}. \quad (1)$$

Given that firm i follows investment policy ξ_i, the probability of firm i in interim state $\mathbf{K}$ leaving the period with $K_i' \in \mathcal{S}$ units of capital is therefore given by the transition function

$$\tau_i(K_i'|\mathbf{K}) = \sum_{m=0}^{\overline{K}-K_i} \xi_i(m|\mathbf{K})\kappa(K_i'|K_i+m). \quad (2)$$

Consider now firm i's optimal investment policy for a given realization $\tilde{c}$ of its (K_i+1)-length vector of cost draws. Let $c_{K_i}(\cdot|\tilde{c})$ denote the resulting cost function where $c_{K_i}(k_i|\tilde{c})$ is the minimum cost to add k_i units of capital with cost draws $\tilde{c}$. Let $\mathcal{C}_{K_i}$ be the set of possible cost draws $\tilde{c}$ and let h_{K_i} be the associated density that the distributions F and G of the cost draws determine. For a given draw $\tilde{c}$, cost function $c_{K_i}(\cdot|\tilde{c})$, ex ante value function $V_i(\cdot)$, and rival transition functions $\boldsymbol{\tau}_{-i}$ (induced by rival investment policies $\boldsymbol{\xi}_{-i}$), firm i chooses k_i so as to maximize its expected continuation value minus its investment cost:

$$\max_{k_i \in \{0,1,...,\overline{K}-K_i\}} -c_{K_i}(k_i|\tilde{c}) + \delta \sum_{\mathbf{K}' \in \mathcal{S}^n} \kappa(K_i'|K_i+k_i) \left[\prod_{j \neq i} \tau_j(K_j'|\mathbf{K})\right] V_i(\mathbf{K}'),$$

where $\delta < 1$ is the discount factor that the firms and the antitrust authority use. Let k_i^* denote the solution to this optimization problem (which, generically, is unique) and define $\omega(k_i|\tilde{c},\mathbf{K})$ to be the indicator function with value 1 if $k_i = k_i^*$ and 0 otherwise. Firm i's investment policy therefore is

$$\xi_i(k_i|\mathbf{K}) = \int_{\mathcal{C}_{K_i}} \omega(k_i|\tilde{c},\mathbf{K})h_{K_i}(\tilde{c})d\tilde{c}, \quad (3)$$

for $k_i \in \{0,1,...,\overline{K}-K_i\}$. This gives rise to firm i's expected investment cost in interim state $\mathbf{K}$:

$$\mathcal{E}c_i(\mathbf{K}) = \int_{\mathcal{C}_{K_i}} \sum_{k_i \in \{0,1,...,\overline{K}-K_i\}} \omega(k_i|\tilde{c},\mathbf{K})c_{K_i}(k_i|\tilde{c})h_{K_i}(\tilde{c})d\tilde{c}. \quad (4)$$

Firm i's interim value in state $\mathbf{K}$ is its static profit less its expected investment cost plus its ENPV in the continuation game; that is,

$$\overline{V}_i(\mathbf{K}) = \pi(K_i,\mathbf{K}_{-i}) - \mathcal{E}c_i(\mathbf{K}) + \delta \sum_{\mathbf{K}' \in \mathcal{S}^n} \left[\prod_{j=1}^{n} \tau_j(K_j'|\mathbf{K})\right] V_i(\mathbf{K}'), \quad (5)$$

where $\pi(K_i,\mathbf{K}_{-i})$ is firm i's single-period profit from static Cournot competition in the product market.[2]

[2]Note that the static profit function is symmetric in that it depends only on the firm's own capital stock K_i and the vector $\mathbf{K}_{-i}$ of its rivals' capital stocks, and any permutation of $\mathbf{K}_{-i}$ does not affect the firm's profit.

1.1.2 Merger Bargaining and Merger Outcomes

We now fold backwards to the merger bargaining, merger approval, and entry stages. If no merger occurs in ex ante state $\mathbf{K}$, then the interim state remains the same as the ex ante state. If merger M_{ij} occurs, then with probability 1/2 the industry transits to interim state $\mathbf{K}'$ in which firm i becomes the merged firm with capital stock $K_i' = K_i + K_j$, an entrant with capital $K_j' = 0$ replaces firm j, and all other firms' capital stocks remain unchanged. With the complementary probability 1/2 firm j becomes the merged firm and firm i is replaced by the entrant. This probabilistic transition rule, in conjunction with the restriction to symmetric equilibrium strategies (as defined below), ensures that the steady state distribution over $\mathcal{S}^n$ is symmetric. The firms, seeking to maximize their ENPVs, negotiate what mergers (if any) occur in accordance with the protocols defined in Section 2 for the model with two firms, and Section 4 for the model with three firms.

Given the authority's approval policy $\{a_{ij}(\cdot)\}_{ij\in\mathcal{J}}$ and the interim value functions $\{\overline{V}_i(\cdot)\}_{i\in\mathcal{I}}$, the firms play subgame perfect strategies in the bargaining stage. As a general matter, given an extensive form merger bargaining protocol[3], the antitrust authority's approval policy $\{a_{ij}(\cdot)\}_{ij\in\mathcal{J}}$, and interim values $\{\overline{V}_i(\cdot)\}_{i\in\mathcal{I}}$, we can solve for subgame perfect equilibrium bargaining strategies, $\{\sigma_i(\cdot)\}_{i\in\mathcal{I}}$. The outcome arising from these strategies determines the probability $\psi_{ij}(\mathbf{K})$ that each possible merger M_{ij} is proposed in a given state $\mathbf{K}$, each firm i's ex ante expected proposal costs denoted by $\mathcal{E}^i[\phi|\mathbf{K}]$, and the firms' ex ante values in that state, $\{V_i(\mathbf{K})\}_{i\in\mathcal{I}}$. As well, these merger proposal probabilities and the antitrust authority's approval policy together determine the transition probability $T_0(\mathbf{K},\mathbf{K}')$ from ex ante state $\mathbf{K}$ to interim state $\mathbf{K}'$. Here, we treat these calculations as a black box. In Section 1.2 of this Online Appendix we explicitly present for $n=2$ and $n=3$ the essential details of these calculations for the merger protocols that we use in the main text. As an illustration, when $n=3$, the formula for the ex ante value of firm i in ex ante state $\mathbf{K}$ is

$$V_i(\mathbf{K}) = \overline{V}_i(\mathbf{K}) + \frac{1}{2}\left\{-\mathcal{E}^i[\phi|\mathbf{K}] + \sum_{\mu\in\{ij,ik\}} \psi_\mu(\mathbf{K})a_\mu(\mathbf{K})\Delta_\mu(\mathbf{K})\right\} + \psi_{jk}(\mathbf{K})X_i^{jk}(\mathbf{K}). \tag{6}$$

Here, $X_i^{jk}(\mathbf{K}) \equiv a_{ij}(\mathbf{K})[\overline{V}(K_i, K_j + K_k, 0) - \overline{V}(K_i, \mathbf{K}_{-i})]$ is the externality of the proposal of merger M_{jk} on outsider firm i, and so the last term on the right-hand side of (6) is the expectation of the externality imposed on firm i from M_{jk}. The interim value $\overline{V}_i(\mathbf{K})$, which is the first term on the right-hand side, is firm i's disagreement value in the bargaining with other firms. The second term is firm i's half of the expected merger gains (net of expected proposal costs) from mergers involving firm i. Note that this formula defines a mapping from interim

[3] Recall that the two-firm Nash bargaining process specified in Section 2 (and used in Section 3) can equivalently be represented as a non-cooperative bargaining game in which one of the two firms is randomly selected to make a take-it-or-leave-it offer to the other, so that it is nested in the three-firm Burguet-Caminal bargaining protocol specified in Section 4.

values $\left\{\overline{V}_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}$ to ex ante values $\left\{V_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}$. More generally, given a bargaining protocol and a merger approval policy, the equilibrium bargaining strategies give rise to a mapping $\mathbf{V}$ from interim values to ex ante values:

$$\left\{V_i\left(\cdot\right)\right\}_{i\in\mathcal{I}} = \mathbf{V}(\left\{\overline{V}_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}) \tag{7}$$

Consequently, (7) and (5) together implicitly define the Bellman equation for the ex ante values $\left\{V_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}$.

1.1.3 Markov Perfect Equilibrium

Definition of Markov perfect equilibrium. Given the authority's merger policy $\{a_{ij}(\cdot)\}_{ij\in\mathcal{J}}$, if the merger bargaining strategies $\{\sigma_i(\cdot)\}_{i\in\mathcal{I}}$ constitute a subgame perfect equilibrium of the bargaining protocol given the interim values $\left\{\overline{V}_i(\cdot)\right\}_{i\in\mathcal{I}}$, and if the firms' investment policies $\{\xi_i(\cdot)\}_{i\in\mathcal{I}}$, the firms' ex ante value functions $\{V_i(\cdot)\}_{i\in\mathcal{I}}$, and the firms' interim values $\left\{\overline{V}_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}$ satisfy equations (3), (5), and (7), then the collection $(\left\{\sigma_i\left(\cdot\right)\right\}_{i\in\mathcal{I}}, \{\xi_i(\cdot)\}_{i\in\mathcal{I}}, \{V_i(\cdot)\}_{i\in\mathcal{I}}, \left\{\overline{V}_i\left(\cdot\right)\right\}_{i\in\mathcal{I}})$ constitutes a Markov perfect equilibrium that policy $\{a_{ij}(\cdot)\}_{ij\in\mathcal{J}}$ induces.

Restriction to symmetric equilibria. Our models focus on symmetric environments, in which a firm's static profit and investment cost distribution depend only on its capital level K_i and the vector of rival capital levels $\mathbf{K}_{-i}$. As well, any permutation of its rivals' capital stocks leaves firm i's static profit and investment cost distribution unchanged. In addition, merger proposal costs and merger blocking costs are independent of the identities of the firms proposing a merger. Finally, the bargaining protocols we specify are symmetric in the sense that a firm's opportunities do not depend on its identity.[4]

Given these symmetric environments, we restrict attention to symmetric (Markovian) merger approval policies. A merger approval policy $\{a_{ij}(\cdot)\}_{ij\in\mathcal{J}}$ is symmetric if for any state $\mathbf{K}$, there exists a single-valued function $a(\cdot)$ such that we can write $a_{ij}(\mathbf{K}) = a((K_i, K_j), \mathbf{K}_{-ij}) = a((K_i, K_j)^p, \mathbf{K}^{p'}_{-ij})$, where $(K_i, K_j)^p$ is any permutation of the capital stocks K_i and K_j of the two merging firms, and $\mathbf{K}^{p'}_{-ij}$ is any permutation of the capital stock vector of their rivals.

In addition, we restrict attention to Markov perfect equilibria for the firms in which a firm's investment policy and value function are symmetric, as are the merger proposal probability functions arising from the merger bargaining protocol's subgame perfect equilibrium. Formally, firm i's investment policy ξ_i is symmetric if $\xi_i(k_i|\mathbf{K}) = \xi(k_i|K_i, \mathbf{K}_{-i}) = \xi(k_i|K_i, \mathbf{K}^p_{-i})$, where $\mathbf{K}^p_{-i}$ is any permutation of its rivals' vector of capital stocks.[5] A similar condition defines symmetry for firm i's ex ante and interim value functions $V_i(\cdot)$ and $\overline{V}_i(\cdot)$. The equi-

[4] For example, in the non-cooperative implememtation of the two-player Nash bargaining solution used in Section 3, each firm has a 1/2 probability of being the proposer.

[5] Observe that a symmtric investment policy function gives rise to a symmetric capital stock transition function for the firm, satisfying $\tau_i(K_i'|\mathbf{K}) = \tau(K_i'|\mathbf{K}) = \tau(K_i'|K_i, \mathbf{K}^p_{-i})$ for any permutation p.

librium outcome of the merger bargaining induces symemtric merger probability functions if $\psi_{ij}(\mathbf{K}) = \psi(K_{ij}, \mathbf{K}_{-ij}) = \psi(K_{ij}^{p}, \mathbf{K}_{-ij}^{p'})$ for all permutations p and p'.[6]

State transition matrix and industry steady state distribution. The investment and depreciation stage transitions $\tau(\cdot)$ combined with the merger bargaining, merger approval, and entry stage transitions $T_0(\cdot)$ determine the transitions from the ex ante state in one period to the ex ante state at the start of the next period. For example, when $n = 2$, given symmetric merger policy $a(\cdot)$ and a symmetric Markov perfect equilibrium that it induces, the probability that the industry transitions from state $\mathbf{K}$ at the beginning of period t to state $\mathbf{K}'$ at the beginning of period $t+1$ is

$$\begin{aligned} T\left[\mathbf{K}, \mathbf{K}'\right] &= (1 - a_{12}(\mathbf{K})\psi_{12}(\mathbf{K}))\left[\tau(K_1'|K_1, K_2, \xi)\tau(K_2'|K_2, K_1, \xi)\right] \\ &\quad + \frac{1}{2}a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})\{\tau(K_1'|0, K_1 + K_2, \xi)\tau(K_2'|K_1 + K_2, 0, \xi) \\ &\qquad + \tau(K_1'|K_1 + K_2, 0, \xi)\tau(K_2'|0, K_1 + K_2, \xi)\}. \end{aligned}$$

To calculate welfare measures and statistics of the industry's dynamics we need the long-run, steady state distribution that results from implementation of merger policy $a(\cdot)$.

For example, consider again the case in which $n = 2$. Let $\mathbf{\Omega} : \mathcal{S}^2 \to \{1, 2, \ldots, (\overline{K} + 1)^2\}$ be an invertible mapping that maps the two-dimensional matrix of states $\mathbf{K}$ into a vector of states. Then, for every pair of states $\{\mathbf{K}, \mathbf{K}'\} \in \mathcal{S}^2 \times \mathcal{S}^2$, define the $\left(\overline{K} + 1\right)^2 \times (\overline{K} + 1)^2$ transition matrix $\hat{\mathbf{T}}$ to have element $\hat{T}(\boldsymbol{\omega}, \boldsymbol{\omega}') = T\left[\mathbf{\Omega}^{-1}(\boldsymbol{\omega}), \mathbf{\Omega}^{-1}(\boldsymbol{\omega}')\right]$ at row $\boldsymbol{\omega}$ (the state at the beginning of the period) and column $\boldsymbol{\omega}'$ (the state at the beginning of the next period) where state $\boldsymbol{\omega} = \mathbf{\Omega}(\mathbf{K})$ and state $\boldsymbol{\omega}' = \mathbf{\Omega}(\mathbf{K}')$.

Let $\hat{\mathbf{P}}$ be a length $(\overline{K} + 1)^2$ row vector whose elements are non-negative and sum to one, i.e., $\hat{\mathbf{P}}$ is a probability distribution on the state space $\mathcal{S}^2$ transformed by $\mathbf{\Omega}$. If $\hat{\mathbf{P}}\,\hat{\mathbf{T}} = \hat{\mathbf{P}}$, then $\hat{\mathbf{P}}$ is a steady state distribution that the policy $a(\cdot)$ induces over the industry's state space. If $\hat{\mathbf{P}}$ is unique, then, for any probability vector $\mathbf{P}$,

$$\hat{\mathbf{P}} = \lim_{t \to \infty} \mathbf{P}\, \underbrace{\hat{\mathbf{T}}\,\hat{\mathbf{T}}\,\hat{\mathbf{T}} \cdots \hat{\mathbf{T}}}_{t \text{ times}}, \tag{8}$$

i.e., no matter what the initial probability distribution $\mathbf{P}$ on states is, the industry converges to the steady state distribution $\hat{\mathbf{P}}$.[7] Rewrite $\hat{\mathbf{P}}$ as a $\left(\overline{K} + 1\right) \times (\overline{K} + 1)$ matrix $\bar{\mathbf{P}}$ where its element in row $(K_1 + 1)$ and column $(K_2 + 1)$,

$$\bar{P}(K_1, K_2) \equiv \hat{P}\left[\mathbf{\Omega}(\mathbf{K})\right], \tag{9}$$

is the steady state probability of the industry being in state $\mathbf{K}$.

[6] A merger bargaining protocol can be said to be symmetric if given any symmetric interim value functions and any symmetric merger approval rule it induces symmetric merger proposal probability functions and symmetric ex ante value functions.

[7] In our model we cannot guarantee that, for some positive integer t, every element of $\hat{T}^t$ is positive, i.e., we cannot guarantee that $\hat{T}$ is a regular Markov transition matrix. If it were regular, then $\hat{P}$ would be unique.

While for $n > 2$ the formulas for $\hat{\mathbf{T}}$ and $\bar{\mathbf{P}}$ are more complex than these $n = 2$ examples, their construction has the same basic structure.

1.1.4 Antitrust Policy and Welfare Metrics

In this section we specify the distinct choice problems that a "commitment" authority faces and that a "no-commitment" authority faces. We then define a variety of consumer value and aggregate value welfare metrics that the antitrust authority may use as its objective function W. Throughout the discussion $\{\psi, \xi, V, \overline{V}\}$ are the symmetric policy functions of the Markov perfect equilibrium that $a(\cdot)$ induces the firms to follow.

Optimal commitment policy. The antitrust authority commits to a pure action $a_{ij}(\mathbf{K}) \in \{0, 1\}$ for each possible merger M_{ij} in each state $\mathbf{K} \in \mathcal{S}^n$ so as to maximize either (i) ex ante welfare $W(\mathbf{K}')$ in a specific state $\mathbf{K}' \in \mathcal{S}^n$ or (ii) some measure W of "average" ex ante welfare across all states $\mathbf{K} \in \mathcal{S}^n$. For example, if $n = 3$, a policy to encourage the development of an infant industry might maximize $W(0, 0, 0)$. On the other hand, a general purpose policy for mature industries might maximize average steady state welfare WSS where the ex ante welfare $W(\mathbf{K})$ of each state $\mathbf{K} \in \mathcal{S}^n$ is weighted by its steady state probability $\bar{P}(\mathbf{K})$. Observe that the infant industry objective is a weighted average with weight one placed on ex ante welfare in state $(0, 0, 0)$. Therefore define $Z(\{W(\mathbf{K})\}_{K \in \mathcal{S}^n})$ to be whatever weighted average the antitrust authority selects as its objective.

Let $\mathcal{A}$ be the class of admissible commitment policies $a'(\cdot)$. Restricting $\mathcal{A}$ is necessary because, even in the computationally easiest case of $n = 2$, the class of all possible symmetric commitment policies contains $2^{\frac{(\overline{K}+1)\overline{K}}{2}}$ elements. For $\overline{K} = 20$, this makes the problem computationally intractable. The optimal commitment policy $a(\cdot)$ is therefore

$$a(\cdot) = \underset{a'(\cdot) \in \mathcal{A}}{\arg\max}\; Z(\{W(\mathbf{K})\}_{\mathbf{K} \in \mathcal{S}^n}) \tag{10}$$

where the value of $Z(\cdot)$ implicitly varies with the Markov perfect equilibrium that $a'(\cdot)$ induces the firms to play.

If the merger bargaining strategies $\{\sigma_i(\cdot)\}_{i \in \mathcal{I}}$ constitute a subgame perfect equilibrium of the bargaining protocol given the interim values $\{\overline{V}_i(\cdot)\}_{i \in \mathcal{I}}$, the firms' investment policies $\{\xi_i(\cdot)\}_{i \in \mathcal{I}}$, the firms' ex ante value functions $\{V_i(\cdot)\}_{i \in \mathcal{I}}$, and the firms' interim values $\{\overline{V}_i(\cdot)\}_{i \in \mathcal{I}}$ satisfy equations (3), (5), and (7) for all states $\mathbf{K} \in \mathcal{S}^n$, and the merger approval policy $a(\cdot)$ satisfies (10), then the collection $\{\sigma, \xi, V, \overline{V}\}$ and $a(\cdot)$ are respectively a Markov perfect equilibrium for the industry and an optimal commitment policy for the "commitment" antitrust authority.

Markov perfect policy. In this case, the antitrust authority acts instead as an additional player that, unable to commit, makes its approval decision in every state $\mathbf{K}$ so as to maximize its welfare criterion going forward, given the firms' Markov perfect equilibrium play in the continuation game. The resulting policy $a(\cdot)$ and the firms' equilibrium actions together determine the welfare criterion's ex ante and interim values, $W(\mathbf{K})$ and $\overline{W}(\mathbf{K})$ for each state $\mathbf{K} \in \mathcal{S}^n$.

A given merger policy $a(\cdot)$ is a Markov perfect merger policy if at every state it satisfies the one-step deviation principle when the firms play the industry Markov perfect equilibrium $\{\sigma, \xi, V, \overline{V}\}$ that approval policy $a(\cdot)$ induces. Given that firms i and j have proposed to merge in state $\mathbf{K}$ and that the authority's realization of its random blocking cost is b_{ij}, welfare in the event that the authority approves the merger is $\overline{W}(K_1 \ldots, K_{i-1}, K_i + K_j, \ldots, K_{j-1}, 0, K_{j+1}, \ldots, K_n) \equiv \overline{W}(K_i + K_j, 0, \mathbf{K}_{-ij})$ while welfare in the event that it blocks it is $\overline{W}(\mathbf{K}) - b_{ij}$. The authority chooses the maximum of the two, approving the merger if and only if

$$b_{ij} \geq -\left(\overline{W}(K_i + K_j, 0, \mathbf{K}_{-ij}) - \overline{W}(\mathbf{K})\right) \equiv -\Delta_{ij}\overline{W}(\mathbf{K}).$$

This results in a state-dependent, history-independent threshold $\widehat{b}_{ij}(\mathbf{K})$ that b_{ij} must exceed for the antitrust authority to approve merger M_{ij} in state $\mathbf{K}$:

$$\widehat{b}_{ij}(\mathbf{K}) = -\Delta_{ij}\overline{W}(\mathbf{K}) \tag{11}$$

We call this a Markov perfect merger policy because in each period the antitrust authority maximizes anew.

Recall that the blocking cost b_{ij} is a random variable with distribution H whose realization is private to the antitrust authority. The firms only know $\widehat{b}_{ij}(\mathbf{K})$ and H. Given the authority's decision rule, this means that in each state K firms know that the probability of merger M_{ij} being approved is

$$a_{ij}(\mathbf{K}) = 1 - H(\widehat{b}_{ij}(\mathbf{K})). \tag{12}$$

If the merger bargaining strategies $\{\sigma_i(\cdot)\}_{i\in\mathcal{I}}$ constitute a subgame perfect equilibrium of the bargaining protocol given the interim values $\{\overline{V}_i(\cdot)\}_{i\in\mathcal{I}}$, the firms' investment policies $\{\xi_i(\cdot)\}_{i\in\mathcal{I}}$, the firms' ex ante value functions $\{V_i(\cdot)\}_{i\in\mathcal{I}}$, and the firms' interim values $\{\overline{V}_i(\cdot)\}_{i\in\mathcal{I}}$ satisfy equations (3), (5), and (7) for all states $\mathbf{K} \in \mathcal{S}^n$, and the merger approval policy $a(\cdot)$ satisfies (12) for all states $\mathbf{K}$, then the collection $\{\sigma, \xi, V, \overline{V}\}$ and a are respectively a Markov perfect equilibrium for the industry and a Markov perfect policy for the "no-commitment" antitrust authority.

Consumer surplus, producer surplus, and aggregate surplus. For the several definitions of welfare and cost measures that follow we restrict the analysis to the $n = 2$ case because, as with the state transition matrix and the industry steady state distribution, the formulas for the general case with $n > 2$ are complicated and contribute little insight.

If the ex ante state is $\mathbf{K} \in \mathcal{S}^2$ and no merger occurs, then the consumer surplus realized is $CS(\mathbf{K})$, where

$$CS(\mathbf{K}) \equiv \int_{P(Q(\mathbf{K}))}^{\infty} D(s)ds,$$

where $D(\cdot)$ is the industry demand function, $P(\cdot) \equiv D^{-1}(\cdot)$ is the inverse demand function, and $Q(\mathbf{K})$ is the total quantity in the Cournot equlibrium at state $\mathbf{K}$. If merger M_{12} occurs in the period, then the consumer surplus realized is $CS\left(K_1+K_2,0\right)$. The expected consumer surplus at the ex ante state $\mathbf{K}$ is therefore

$$\mathcal{E}CS(\mathbf{K})=\left[1-a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})\right]CS\left(\mathbf{K}\right)+a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})CS(K_1+K_2,0)$$

where $a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})$ is the probability merger M_{12} occurs. Similarly, expected producer surplus at ex ante state $\mathbf{K}\in\mathcal{S}^2$ is

$$\mathcal{E}PS(\mathbf{K})=\left[1-a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})\right]PS\left(\mathbf{K}\right)+a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})PS\left(K_1+K_2,0\right)$$

where $PS\left(\mathbf{K}\right)=\pi(K_1,K_2)+\pi(K_2,K_1)$. Aggregate surplus is the sum of consumer surplus and producer surplus: $AS\left(\mathbf{K}\right)=CS\left(\mathbf{K}\right)+PS\left(\mathbf{K}\right)$. Consequently, in ex ante state $\mathbf{K}$ expected aggregate surplus is $\mathcal{E}AS\left(\mathbf{K}\right)=\mathcal{E}CS\left(\mathbf{K}\right)+\mathcal{E}PS\left(\mathbf{K}\right)$.

Consumer value and aggregate value. We generalize these static criteria to their dynamic analogues, CV and AV, whose values are the ENPVs of consumer welfare and of aggregate welfare respectively. Aggregate welfare accounts not only for consumer and producer surplus at the Cournot competition stage, but also for investment costs, merger proposal costs, and blocking costs.

Ex ante consumer value, $CV\left(\mathbf{K}\right)$, is the ENPV of current and future expected consumer surplus. Its Bellman equation is

$$CV\left(\mathbf{K}\right)=\mathcal{E}CS(\mathbf{K})+\delta\sum_{K_1'\in\mathcal{S}}\sum_{K_2'\in\mathcal{S}}T\left[\mathbf{K},\mathbf{K}'\right]CV(\mathbf{K}').$$

Interim consumer value is, for all states $\mathbf{K}$,

$$\overline{CV}(\mathbf{K})=CS\left(\mathbf{K}\right)+\delta\sum_{K_1'\in\mathcal{S}}\sum_{K_2'\in\mathcal{S}}\tau(K_1'|K_1,K_2,\xi)\tau(K_2'|K_2,K_1,\xi)CV(\mathbf{K}')$$

because consumer surplus is realized at the Cournot competition stage after any proposed merger has been consummated.

Ex ante aggregate value $AV\left(\mathbf{K}\right)$ has four components: consumer value $CV\left(\mathbf{K}\right)$, the sum of the incumbent firms' ex ante values $V_1\left(\mathbf{K}\right)+V_2\left(\mathbf{K}\right)$, the ENPV of all future entrants' cash flows $\mathcal{E}EV(\mathbf{K})$, and the ENPV of the antitrust authority's blocking costs $\mathcal{E}BC\left(\mathbf{K}\right)$. Note that the sum $V_1\left(\mathbf{K}\right)+V_2\left(\mathbf{K}\right)$ fully accounts for the incumbents' expected merger proposal costs and expected capital investment costs. But neither $CV\left(\mathbf{K}\right)$ nor $V_1\left(\mathbf{K}\right)+V_2\left(\mathbf{K}\right)$ includes the last two components, $\mathcal{E}EV(\mathbf{K})$ and $\mathcal{E}BC\left(\mathbf{K}\right)$. We discuss each in turn.

Consider the ENPV of future entrants' cash flows, $\mathcal{E}EV(\mathbf{K})$. A new firm 1 (with probability 0.5 it could be firm 2 instead) comes into existence at the entry stage of each period in which a merger occurs. This new firm's interim value is $\overline{V}_1(0,K_1+K_2)$ where K_1+K_2 is the merged firm's capital level. In the ex ante state $\mathbf{K}=(K_1,K_2)$ the Bellman equation of the ex ante ENPV of all future entrants' cash flows is

$$\mathcal{E}EV(\mathbf{K})=a_{12}(\mathbf{K})\psi_{12}(\mathbf{K})\overline{V}_1\left(0,K_1+K_2\right)+\delta\sum_{K_1'\in\mathcal{S}}\sum_{K_2'\in\mathcal{S}}T\left[\mathbf{K},\mathbf{K}'\right]\mathcal{E}EV(\mathbf{K}').$$

In interim state $\mathbf{K}$, the ENPV is

$$\overline{\mathcal{E}EV}(\mathbf{K}) = \delta \sum_{K_1' \in \mathcal{S}} \sum_{K_2' \in \mathcal{S}} \tau(K_1'|K_1, K_2)\tau(K_2'|K_2, K_1)\mathcal{E}EV(\mathbf{K}').$$

Next consider the ENPV of the antitrust authority's blocking costs, $\mathcal{E}BC(\mathbf{K})$. This depends on whether the authority commits or not. The first case is for a commitment authority that selects a policy $a_{12}(\cdot)$ that, in each ex ante state $\mathbf{K}$, specifies either "approve" or "block" with certainty, i.e., $a_{12}(\mathbf{K}) \in \{0, 1\}$. Given this commitment, each firm knows that expending resources proposing a merger when $a_{12}(\mathbf{K}) = 0$ is hopeless because the authority will block with probability 1. Consequently the authority never has to block a proposal, incurs zero blocking costs, and $\mathcal{E}BC(\mathbf{K}) = 0$ for all $\mathbf{K} \in \mathcal{S}^2$.

For the second, no-commitment case, as explained above, in each ex ante state $\mathbf{K}$ the authority sets a threshold $\widehat{b}_{12}(\mathbf{K})$ such that it blocks a proposed merger if and only if the realization of its private, randomly distributed blocking cost b is less than $\widehat{b}_{12}(\mathbf{K})$. Conditional on a merger being proposed, the expected blocking cost in state $\mathbf{K}$ is

$$\mathcal{E}[b|\mathbf{K}] = \int_{\underline{b}}^{\widehat{b}_{12}(\mathbf{K})} b \, dH(b).$$

where H is b's distribution function that has support $[\underline{b}, \overline{b}]$. The Bellman equation for the ex ante ENPV of blocking costs in ex ante state $\mathbf{K}$ is[8]

$$\mathcal{E}BC(\mathbf{K}) = \psi(\mathbf{K})\mathcal{E}[b|\mathbf{K}] + \delta \sum_{K_1' \in \mathcal{S}} \sum_{K_2' \in \mathcal{S}} T\left[\mathbf{K}, \mathbf{K}'\right] \mathcal{E}BC(\mathbf{K}').$$

In interim state $\mathbf{K}$ its value is

$$\overline{\mathcal{E}BC}(\mathbf{K}) = \delta \sum_{K_1' \in \mathcal{S}} \sum_{K_2' \in \mathcal{S}} \tau(K_1'|K_1, K_2)\tau(K_2'|K_2, K_1)\mathcal{E}BC(\mathbf{K}').$$

Given these definitions, ex ante aggregate value in ex ante state $\mathbf{K}$ is

$$AV(\mathbf{K}) = CV(\mathbf{K}) + V(K_1, K_2) + V(K_2, K_1) + \mathcal{E}EV(\mathbf{K}) - \mathcal{E}BC(\mathbf{K}) \tag{13}$$

and interim aggregate value in interim state $\mathbf{K}$ is

$$\overline{AV}(\mathbf{K}) = \overline{CV}(\mathbf{K}) + \overline{V}(K_1, K_2) + \overline{V}(K_2, K_1) + \overline{\mathcal{E}EV}(\mathbf{K}) - \overline{\mathcal{E}BC}(\mathbf{K}) \tag{14}$$

with the caveat that $\mathcal{E}BC(\mathbf{K}) = \overline{\mathcal{E}BC}(\mathbf{K}) = 0$ if the antitrust authority employs a commitment merger policy.

[8] When $n > 2$ the expected blocking costs in state $\mathbf{K}$ is the expectation over expected blocking costs for each possible merger given the various mergers' proposal probabilities.

Steady State Welfare. Given ex ante welfare function $W(\cdot)$ the steady state, ex ante average welfare the antitrust authority achieves under policy $a(\cdot)$ is

$$WSS = \sum_{K_1' \in \mathcal{S}} \sum_{K_2' \in \mathcal{S}} \bar{P}(\mathbf{K}') W(\mathbf{K}')$$

where, as defined in equation (9), $\bar{P}(\mathbf{K}')$ is the industry's steady state probability of being in state $\mathbf{K}'$.

1.2 Details regarding Calculation of Merger Outcomes for the $n = 2$ and $n = 3$ Cases

This section presents the details of the merger negotation calculations for two cases: duopoly industry states $\mathbf{K}$ in which two firms have capital stocks that are non-zero and triopoly industry states $\mathbf{K}$ in which three firms have non-zero capital stocks.

Merger proposals: duopoly industry states. Ex ante state $\mathbf{K}$ is a duopoly state if and only if two firms have capital stocks of at least one unit. Sections 2 specifies that Nash bargaining determines the outcome of merger negotiations in duopoly states. For $ij \in \mathcal{J}$, recall that

$$\Delta_{ij}(\mathbf{K}) \equiv \overline{V}(K_i + K_j, 0) - [\overline{V}(K_i, \mathbf{K}_{-i}) + \overline{V}(K_j, \mathbf{K}_{-j})],$$

is the joint gain from merger M_{ij} gross of the proposal cost ϕ_{ij}, and that

$$S_{ij}(\mathbf{K}, \phi_{ij}) \equiv a_{ij}(\mathbf{K})\Delta_{ij}(\mathbf{K}) - \phi_{ij}$$

denotes the expected bilateral surplus of firms i and j from merging, conditional on the proposal cost realization ϕ_{ij}, and that $S_{ij}^{+}(\mathbf{K}, \phi_{ij}) \equiv \max\left[0, S_{ij}(\mathbf{K}, \phi_{ij})\right]$. The firms propose their merger only if this surplus is positive, i.e., $S_{ij}^{+}(\mathbf{K}, \phi_{ij}) > 0$. Proposal costs ϕ_{ij} are distributed indepedently with distribution function $\Phi(\cdot)$. Consequently, the ex ante probability of merger M_{ij} being proposed in ex ante state $\mathbf{K}$ is

$$\psi_{ij}(\mathbf{K}) \equiv \Phi(a_{ij}(\mathbf{K})\Delta_{ij}(\mathbf{K})) \tag{15}$$

and the ex ante probability of a merger occurring is $\vartheta_{ij}(\mathbf{K}) \equiv a_{ij}(\mathbf{K})\psi_{ij}(\mathbf{K})$. Nash bargaining over the gains from merging implies that firm i's ex ante value is

$$V(K_i, \mathbf{K}_{-i}) = \overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2}\psi_{ij}(\mathbf{K})\left\{a_{ij}(\mathbf{K})\Delta_{ij}(\mathbf{K}) - \mathcal{E}\left[\phi_{ij}|\mathbf{K}\right]\right\} \tag{16}$$

where the interim value $\overline{V}(K_i, \mathbf{K}_{-i})$ is firm i's disagreement value, the term in curly brackets is the merging firms' expected net gain from proposing a merger (which they divide equally), and

$$\mathcal{E}\left[\phi_{ij}|\mathbf{K}\right] \equiv \frac{\int_{\underline{\phi}}^{a(\mathbf{K})\Delta_{ij}(\mathbf{K})} \phi d\Phi(\phi)}{\psi_{ij}(\mathbf{K})}$$

is the expected proposal cost conditional on the merger being proposed. Equation (16) gives a formula for $V(K_i, \mathbf{K}_{-i})$ in terms of $\overline{V}(K_i, \mathbf{K}_{-i})$ and $\Delta_{ij}(\mathbf{K})$ where $\Delta_{ij}(\mathbf{K})$ itself is a function of interim values. Consequently equation (16) together with equation (5) for $\overline{V}(K_i, \mathbf{K}_{-i})$ implicitly define the Bellman equation for the ex ante value $V(K_i, \mathbf{K}_{-i})$.

Merger proposals: triopoly industry states. Ex ante state $\mathbf{K}$ is a triopoly state if three firms have positive capital stocks. Paralleling the discussion of mergers in a duopoly state, to characterize mergers in triopoly states we must derive merger proposal probabilities $\psi_{ij}(\mathbf{K})$ and write a formula for each firm's ex ante value $V(K_i, \mathbf{K}_{-i})$.

Under the static Burguet and Caminal bargaining protocol that guides our analysis in triopoly states firm i is chosen to be the proposer with probability 1/3 and proposal costs $(\phi_{ij}, \phi_{ik}, \phi_{jk})$ are independently drawn from the cumulative distribution function Φ whose support is $[\underline{\phi}, \overline{\phi}]$. Let the joint density of the costs be $\Phi_3(\phi_{ij}, \phi_{ik}, \phi_{jk})$ on the domain $\mathbf{\Lambda} = [\underline{\phi}, \overline{\phi}]^3$.

Proposition 1 implicitly partitions $\mathbf{\Lambda}$ into five regions that determine what merger proposals are made, if any, in state $\mathbf{K}$. Let $\tilde{\boldsymbol{\phi}} = \left(\tilde{\phi}_{ij}, \tilde{\phi}_{ik}, \tilde{\phi}_{jk}\right)$ be the realization of the proposal costs. Define the function $\chi_i\left(\tilde{\boldsymbol{\phi}}, \mathbf{K}\right)$ that outputs the merger, if any, that is proposed to the antitrust authority given the realized proposal costs $\tilde{\phi}$ and the ex ante state $\mathbf{K}$:

$$\chi_i\left(\tilde{\boldsymbol{\phi}}, \mathbf{K}\right) \equiv \begin{cases} M_{jk} & \text{if } S^{+}_{jk}(\mathbf{K}, \tilde{\phi}_{jk}) > \max\{S_{ij}(\mathbf{K}, , \tilde{\phi}_{ij}), S_{ik}(\mathbf{K}, \tilde{\phi}_{ik})\} \\ M_{ij} & \text{if } S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij}) > S^{+}_{ik}(\mathbf{K}, \tilde{\phi}_{ik}) \geq S^{+}_{jk}(\mathbf{K}, \tilde{\phi}_{jk}) \\ M_{ij} & \text{if } S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij}) > S^{+}_{jk}(\mathbf{K}, \tilde{\phi}_{jk}) > S^{+}_{ik}(\mathbf{K}, \tilde{\phi}_{ik}) \;\&\; S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij})/2 > X^{jk}_{i}(\mathbf{K}) \\ M_{jk} & \text{if } S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij}) > S^{+}_{jk}(\mathbf{K}, \tilde{\phi}_{jk}) > S^{+}_{ik}(\mathbf{K}, \tilde{\phi}_{ik}) \;\&\; S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij})/2 < X^{jk}_{i}(\mathbf{K}) \\ M_{\varnothing} & \text{if } S^{+}_{ij}(\mathbf{K}, \tilde{\phi}_{ij}) = S^{+}_{ik}(\mathbf{K}, \tilde{\phi}_{ik}) = S^{+}_{jk}(\mathbf{K}, \tilde{\phi}_{jk}) = 0 \end{cases} .$$

where $M_{\varnothing}$ represent no merger proposed.

On the domain $\{M_{ij}, M_{ik}, M_{jk}, M_{\varnothing}\} \times \mathbf{\Lambda} \times \mathcal{S}^3$ define the indicator function $\upsilon_i(\mu, \tilde{\boldsymbol{\phi}}, \mathbf{K}|\chi_i)$ to have value 1 if $\chi_i\left(\tilde{\boldsymbol{\phi}}, \mathbf{K}\right) = \mu$ and 0 otherwise. Conditional on firm i being the randomly selected proposer, the probability that merger $M_\mu \in (M_{ij}, M_{ij}, M_{jk}, M_{\varnothing})$ will be proposed is

$$\psi^{i}_{\mu}(\mathbf{K}) = \int_{\Lambda} \upsilon_i(\mu, \tilde{\boldsymbol{\phi}}, \mathbf{K}|\chi_i)\Phi_3(\tilde{\boldsymbol{\phi}})d\tilde{\boldsymbol{\phi}}.$$

where, for example, if $\mu = M_{ij}$ we write $\psi^{i}_{ij}(\mathbf{K})$ rather than $\psi^{i}_{M_{ij}}(\mathbf{K})$, etc. The ex ante probability of merger $M_\mu \in (M_{ij}, M_{ik}, M_{jk}, M_{\varnothing})$ occuring in ex ante state K is then

$$\psi_\mu = \frac{1}{3}\textstyle\sum_{i=1}^{3} \psi^{i}_{\mu}(\mathbf{K}). \tag{17}$$

where the $\frac{1}{3}$ coefficient is the probability each firm has of being selected proposer. Proposal costs are incurred whenever a merger is proposed. Therefore, conditional on firm i being the random proposer, expected proposal costs of mergers in which i is involved, are

$$\mathcal{E}^{i}[\phi|\mathbf{K}] = \begin{cases} 0 & \text{if } \mu = M_{\varnothing} \\ \sum\limits_{\mu \in \{M_{ij}, M_{ik}\}} \int_{\Lambda} \upsilon_i(\mu, \tilde{\boldsymbol{\phi}}, \mathbf{K}|\chi_i)\tilde{\phi}_\mu \Phi_3(\tilde{\boldsymbol{\phi}})d\tilde{\boldsymbol{\phi}} & \text{otherwise} \end{cases}$$

where if $\mu = M_{ij}$ we write $\tilde{\phi}_{ij}$, etc. Ex ante, total expected proposal costs across all three firms are

$$\mathcal{E}[\phi|\mathbf{K}] = \frac{1}{2} \sum_{\mu \in \{M_{ij}, M_{ik}, M_{jk}\}} \int_{\mathbf{\Lambda}} \upsilon_i(\mu, \tilde{\boldsymbol{\phi}}, \mathbf{K}|\chi_i) \tilde{\phi}_\mu \Phi_3(\tilde{\boldsymbol{\phi}}) d\tilde{\boldsymbol{\phi}}$$

where the $\frac{1}{2}$ coefficent corrects for double counting.

The expected externality that firm $i \neq k, j$ realizes if merger M_{jk} occurs is

$$X_i^{jk}(\mathbf{K}) \equiv a_{jk}(\mathbf{K}) \left(\overline{V}_i(K_i, K_j + K_k, 0) - \overline{V}_i(K_i, \mathbf{K}_{-i}) \right).$$

The ex ante value of firm i in ex ante state $\mathbf{K}$ is therefore

$$V(K_i, \mathbf{K}_{-i}) = \overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} \left\{ -\mathcal{E}[\phi|\mathbf{K}] + \sum_{\mu \in \{M_{ij}, M_{ik}\}} \psi_\mu(\mathbf{K}) a_\mu(\mathbf{K}) \Delta_\mu(\mathbf{K}) \right\} + \psi_{jk}(\mathbf{K}) X_i^{jk}(\mathbf{K}) \tag{18}$$

where $\overline{V}(K_i, \mathbf{K}_{-i})$ is firm i's disagreement value, the second term is the expected merger gains net of expected proposal costs that firm i shares with its merger partners, and the last term is the expectation of the externality firm i realizes if merger M_{jk} occurs. Equation (16) gives a formula for $V(K_i, \mathbf{K}_{-i})$ in terms of $\overline{V}(K_i, \mathbf{K}_{-i})$ and $\Delta_{ij}(\mathbf{K})$ where $\Delta_{ij}(\mathbf{K})$ itself is a function of interim values. Consequently equation (16) together with equation (5) for $\overline{V}(K_i, \mathbf{K}_{-i})$ implicitly define the Bellman equation for the ex ante value $\overline{V}(K_i, \mathbf{K}_{-i})$.

1.3 Proof of Proposition 1

We begin with the following lemma.

Lemma 1 *Suppose firm i is selected as the proposer in state $\mathbf{K}$. Further, suppose that firm i invites firm j to enter merger negotiations. Then:*

(i) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) > S_{jk}^+(\mathbf{K}, \phi_{jk})$, firm j accepts the invitation and merger M_{ij} gets proposed.*

(ii) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) < S_{jk}^+(\mathbf{K}, \phi_{jk})$, firm j declines the invitation and merger M_{jk} gets proposed.*

(iii) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) = S_{jk}^+(\mathbf{K}, \phi_{jk}) = 0$, no merger gets proposed.*

Proof. If firm j accepts firm i's invitation, its expected continuation value is

$$\overline{V}(K_j, \mathbf{K}_{-j}) + \frac{1}{2} S_{ij}^+(\mathbf{K}, \phi_{ij}).$$

If firm j instead declines the invitation, it enters merger negotiations with firm k, resulting in an expected continuation value of

$$\overline{V}(K_j, \mathbf{K}_{-j}) + \frac{1}{2} S_{jk}^+(\mathbf{K}, \phi_{jk}).$$

Hence, firm j strictly prefers accepting the invitation if $S^+_{ij}(\mathbf{K}, \phi_{ij}) > S^+_{jk}(\mathbf{K}, \phi_{jk})$, and strictly prefers declining it if the inequality is reversed. If $S^+_{ij}(\mathbf{K}, \phi_{ij}) = S^+_{jk}(\mathbf{K}, \phi_{jk}) = 0$, no matter whether firms i and j or j and k enter into merger negotiations, no merger gets proposed as, generically, both $S_{ij}(\mathbf{K}, \phi_{ij}) < 0$ and $S_{jk}(\mathbf{K}, \phi_{jk}) < 0$ in that case. ■

Proof of Proposition 1.

Part (i). Suppose $S^+_{jk}(\mathbf{K}, \phi_{jk}) > \max\{S^+_{ij}(\mathbf{K}, \phi_{ij}), S^+_{ik}(\mathbf{K}, \phi_{ik})\}$. Lemma 1 implies that, no matter whether firm i invites firm j or firm k, that invitation gets declined, and merger M_{jk} gets proposed.

Part (ii). Suppose $S^+_{ij}(\mathbf{K}, \phi_{ij}) > S^+_{ik}(\mathbf{K}, \phi_{ik}) \geq S^+_{jk}(\mathbf{K}, \phi_{jk})$. Lemma 1 implies that if firm i chooses to invite firm j, then merger M_{ij} gets proposed, yielding firm i an expected continuation value of

$$\overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} S^+_{ij}(\mathbf{K}, \phi_{ij}).$$

If firm i chooses to invite firm k and $S^+_{ik}(\mathbf{K}, \phi_{ik}) > S^+_{jk}(\mathbf{K}, \phi_{jk})$, then by the Lemma merger M_{ik} gets proposed, yielding firm i an expected continuation value of

$$\overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} S^+_{ik}(\mathbf{K}, \phi_{ik}) < \overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} S^+_{ij}(\mathbf{K}, \phi_{ij}).$$

If firm i chooses to invite firm k and $S^+_{ik}(\mathbf{K}, \phi_{ik}) = S^+_{jk}(\mathbf{K}, \phi_{jk}) = 0$ (the case $S^+_{ik}(\mathbf{K}, \phi_{ik}) = S^+_{jk}(\mathbf{K}, \phi_{jk}) > 0$ generically does not occur), then by Lemma 1 no merger gets proposed, yielding firm i an expected continuation value of

$$\overline{V}(K_i, \mathbf{K}_{-i}) < \overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} S^+_{ij}(\mathbf{K}, \phi_{ij}).$$

Hence, firm i invites firm j and merger M_{ij} gets proposed.

Parts (iii) and (iv). Suppose $S^+_{ij}(\mathbf{K}, \phi_{ij}) > S^+_{jk}(\mathbf{K}, \phi_{jk}) > S^+_{ik}(\mathbf{K}, \phi_{ik})$. From Lemma 1, if firm i chooses to invite firm j, then merger M_{ij} gets proposed, yielding firm i an expected continuation value of

$$\overline{V}(K_i, \mathbf{K}_{-i}) + \frac{1}{2} S^+_{ij}(\mathbf{K}, \phi_{ij}).$$

Similarly, if firm i chooses to invite firm k, then merger M_{jk} gets proposed, yielding firm i an expected continuation value of

$$\overline{V}(K_i, \mathbf{K}_{-i}) + I_{\{S^+_{jk}(\mathbf{K}, \phi_{jk}) > 0\}} X^{jk}_i(\mathbf{K}) = \overline{V}(K_i, \mathbf{K}_{-i}) + X^{jk}_i(\mathbf{K}),$$

where the equality follows from the fact that, by assumption, $S^+_{jk}(\mathbf{K}, \phi_{jk}) > 0$, implying that merger M_{jk} would get proposed if firm i were to invite firm k (as firm k would reject and invite firm j with whom firm k has a larger and positive surplus). Hence, if $S^+_{ij}(\mathbf{K}, \phi_{ij})/2 > X^{jk}_i(\mathbf{K})$, then firm i invites firm j and merger M_{ij} gets proposed; if the inequality is reversed, then firm i invites firm k and merger M_{jk} gets proposed.

Part (v) is immediate. ■

1.4 Computation

The algorithm that we use numerically to solve for equilibria is a version of the well-known Pakes-McGuire (1994) algorithm. It is a straightforward iterative process. For a given merger policy $a(\cdot)$ the procedure works as follows. Pick an initial guess for the investment function $\xi^{(0)}$ and the ex ante value function $V^{(0)}$. Then compute an updated estimate of the investment policy function $\xi^{(1)}$ using equation (3). As this is a difficult integral to evaluate, we use Monte Carlo integration at each state $\mathbf{K}$. Specifically, for a given vector $\tilde{c}$ of random cost draws, the ex ante value function $V^{(0)}$, and the rival's investment policy function $\xi^{(0)}$ [which determines the rival's transition probabilities via equation (2)], we calculate firm i's optimal investment decision k_i for that instance of $\tilde{c}$. Repeating this with many cost draws we use the proportion of cost draws for which k_i is optimal as our estimate of $\xi^{(1)}\left(k_i|K_1, K_2, V^{(0)}\right)$.

We then use equation (5) to calculate the interim value function $\overline{V}^{(1)}$. Using this interim value function and merger policy $a(\cdot)$, we compute the merger proposal function $\psi^{(1)}$ using equation (15). Finally we calculate an updated ex ante value function $V^{(1)}$ using equation (16).

Computation of the Markov perfect policy involves an additional step where we update the antitrust authority's merger policy $a^{(1)}(\cdot)$ using equation (12). We calculate W based on the authority's objective function and the state transitions induced by the firms' investment policy function $\xi^{(1)}$, merger proposal function $\psi^{(1)}$, and the authority's intitial merger policy $a^{(0)}(\cdot)$.

We iterate this process using the updated investment function $\xi^{(1)}$ and the updated ex ante value function $V^{(1)}$ as our starting point. We continue this iterative procedure until $\left\|V^{(\ell+1)} - V^{(\ell)}\right\| \leq \varepsilon$ for some small $\varepsilon > 0$.[9]

Computation for the model with three firms is analogous except for updating the merger proposal function ψ. Because the solution to the bargaining process involves integrals which are difficult to evaluate, we use Monte Carlo integration, simulating proposal costs and the selection of the intitial proposer.

A copy of the MatLab code and a document that describes the code and this algorithm in more detail is available online.

2 Merger Policy in the Small and Large Markets

[9] The distance metric we use combines absolute differences and percentage differences. For values less than one we use absolute differences, while for values greater than one we use percentage differences. This is because, for an $\varepsilon = 0.0001$, we want a value of 0.001 and 0.0009 to be considered the same even though they have a percentage difference of 0.1 and we want a value of 1000 and 1000.1 to be considered the same even though they have an absolute difference of 0.1.

(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.5	1.4	1.1	1.0	0.8	0.7	0.6	0.5	0.4	0.4
k1=2	-	1.4	1.3	1.1	0.8	0.6	0.5	0.4	0.3	0.2	0.1
k1=3	-	1.1	1.1	0.8	0.5	0.3	0.1	(0.0)	(0.1)	(0.2)	(0.3)
k1=4	-	1.0	0.8	0.5	0.2	(0.0)	(0.3)	(0.4)	(0.5)	(0.6)	(0.7)
k1=5	-	0.8	0.6	0.3	(0.0)	(0.3)	(0.6)	(0.7)	(0.9)	(1.0)	(1.1)
k1=6	-	0.7	0.5	0.1	(0.3)	(0.6)	(0.8)	(1.0)	(1.2)	(1.3)	(1.4)
k1=7	-	0.6	0.4	(0.0)	(0.4)	(0.7)	(1.0)	(1.3)	(1.4)	(1.6)	(1.7)
k1=8	-	0.5	0.3	(0.1)	(0.5)	(0.9)	(1.2)	(1.4)	(1.7)	(1.8)	(2.0)
k1=9	-	0.4	0.2	(0.2)	(0.6)	(1.0)	(1.3)	(1.6)	(1.8)	(2.0)	(2.2)
k1=10	-	0.4	0.1	(0.3)	(0.7)	(1.1)	(1.4)	(1.7)	(2.0)	(2.2)	(2.3)

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.8	1.9	1.7	1.4	1.2	1.1	1.0	0.8	0.8	0.7
k1=2	-	1.9	2.0	1.8	1.5	1.3	1.1	0.9	0.8	0.6	0.6
k1=3	-	1.7	1.8	1.5	1.2	1.0	0.7	0.5	0.4	0.3	0.2
k1=4	-	1.4	1.5	1.2	0.9	0.6	0.4	0.1	(0.0)	(0.2)	(0.3)
k1=5	-	1.2	1.3	1.0	0.6	0.3	0.0	(0.2)	(0.4)	(0.6)	(0.7)
k1=6	-	1.1	1.1	0.7	0.4	0.0	(0.3)	(0.6)	(0.8)	(1.0)	(1.1)
k1=7	-	1.0	0.9	0.5	0.1	(0.2)	(0.6)	(0.8)	(1.1)	(1.3)	(1.5)
k1=8	-	0.8	0.8	0.4	(0.0)	(0.4)	(0.8)	(1.1)	(1.3)	(1.6)	(1.8)
k1=9	-	0.8	0.6	0.3	(0.2)	(0.6)	(1.0)	(1.3)	(1.6)	(1.8)	(2.0)
k1=10	-	0.7	0.6	0.2	(0.3)	(0.7)	(1.1)	(1.5)	(1.8)	(2.0)	(2.2)

Figure 1: Static change in aggregate surplus for (a) the small market and (b) the large market. Negative numbers are in parentheses.

In this section, we describe our results for the optimal merger policy in the small ($A = 3, B = 22$) and large ($A = 3, B = 30$) markets, and compare them to our results for the intermediate ($A = 3, B = 26$) market found in the main paper. The static welfare effects of mergers are very similar in the three markets: in all of them only a merger in state $(1, 1)$ increases static consumer surplus, and in all of them, a merger in state (K_1, K_2) increases static aggregate surplus unless both K_1 and K_2 are "large," with the set of statically aggregate surplus-increasing mergers being larger in larger markets. Figure 1 shows the set of aggregate surplus-increasing mergers in the small and large markets.

Figures 2 through 7 show the steady state distributions and five-period transitions for the small, intermediate and large markets when no mergers are allowed. When the antitrust authority pursues instead an AV goal and cannot commit, the Markov perfect merger policy results in mergers only in near-monopoly states in which the incumbent is sufficiently large. The larger the market, the more restrictive is the antitrust authority in equilibrium. Figures 8 and 10 show the steady state distribution and probabilities that a merger happens in the small and large markets, while Tables 1 and 2 provide some summary statistics of these equilibria. The average merger probability is 30.6% in the small market, but only 3.0% in the large market (versus 16.1% in the intermediate market). In the small market the industry is almost always (98.6% of the time) in a monopoly state at the Cournot competition stage, compared to 49.4% in the intermediate market, and only 8.2% in the large market. The equilibria involve larger

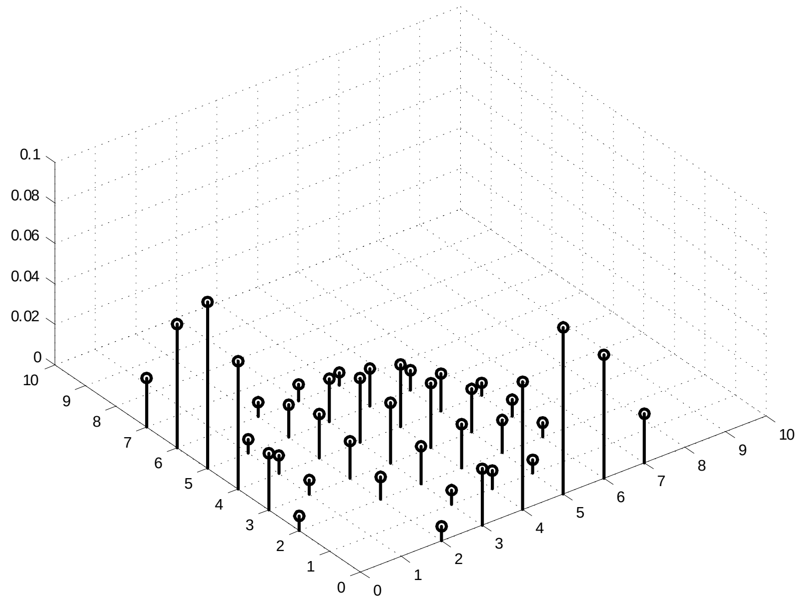

Figure 2: Beginning-of-period steady state distribution of the equilibrium generated with no mergers in the small market. The height of each pin indicates the steady state probability of that state.

capital levels as the market size grows.

Driving these differences are the larger returns from capital additions that increased market size provides. Figures 5 through 7 illustrate the strength of this effect in the no-mergers-allowed equilibria as the market size increases. In these figures each arrow represents the average movement over five periods starting in each state. The almost non-existent movement toward duopoly from state $(5, 0)$ in the small market evident in Figure 5, changes to robust movement towards duopoly from state $(5, 0)$ in the large market in Figure 7. Entry, without the carrot of entry for buyout, is much more attractive and thefore a more effective check on monopoly in large markets. The antitrust authority therefore has an incentive to be more aggressive in blocking mergers.

As in the intermediate market, if the antitrust authority pursues a CV goal and cannot commit, the Markov perfect merger policies in the small and large markets are essentially equivalent to the no-mergers policy.[10] The same is true if it adopts the static consumer surplus-based policy. In contrast, pursuing the static aggregate surplus-based policy is essentially equivalent in outcome to allowing all mergers.

[10]In the large market, the authority would approve mergers in states $(1, 1)$, $(2, 1)$, and $(1, 2)$ but such mergers are not value-enhancing for the firms and therefore never proposed.

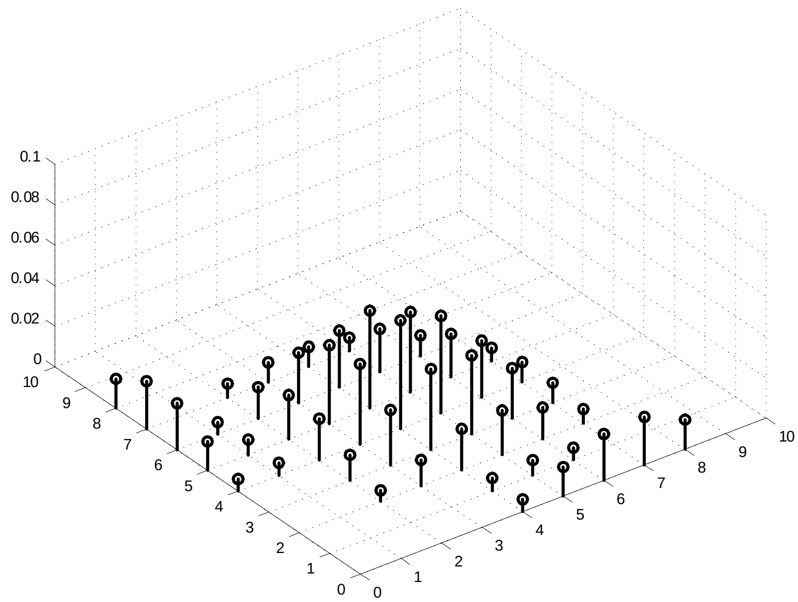

Figure 3: Beginning-of-period steady state distribution of the equilibrium generated with no mergers in the intermediate market. The height of each pin indicates the steady state probability of that state.

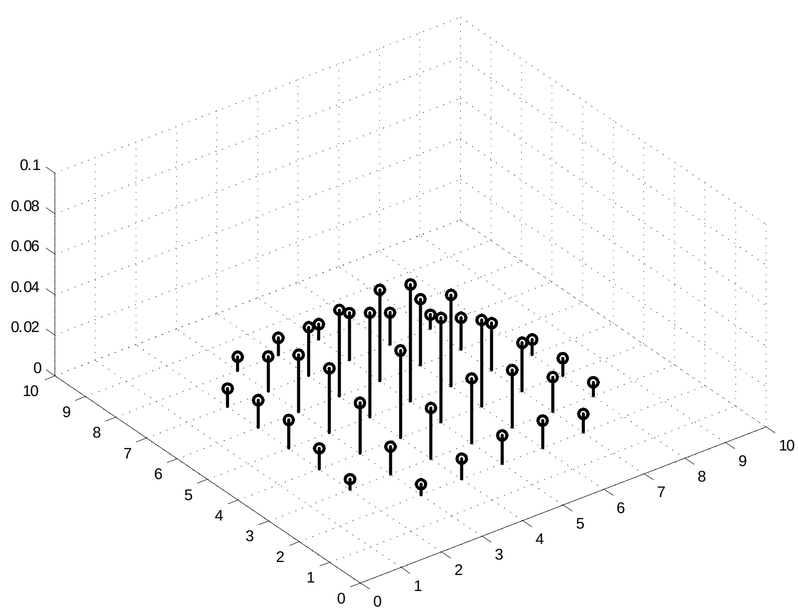

Figure 4: Beginning-of-period steady state distribution of the equilibrium generated with no mergers in the large market. The height of each pin indicates the steady state probability of that state.

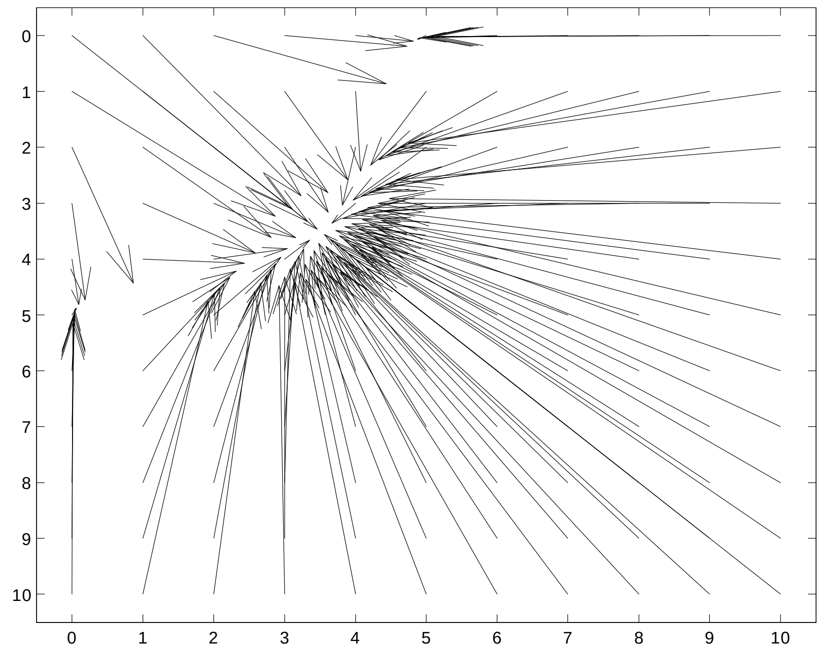

Figure 5: Arrows show the expected transitions over 5 periods in the small market with no mergers allowed.

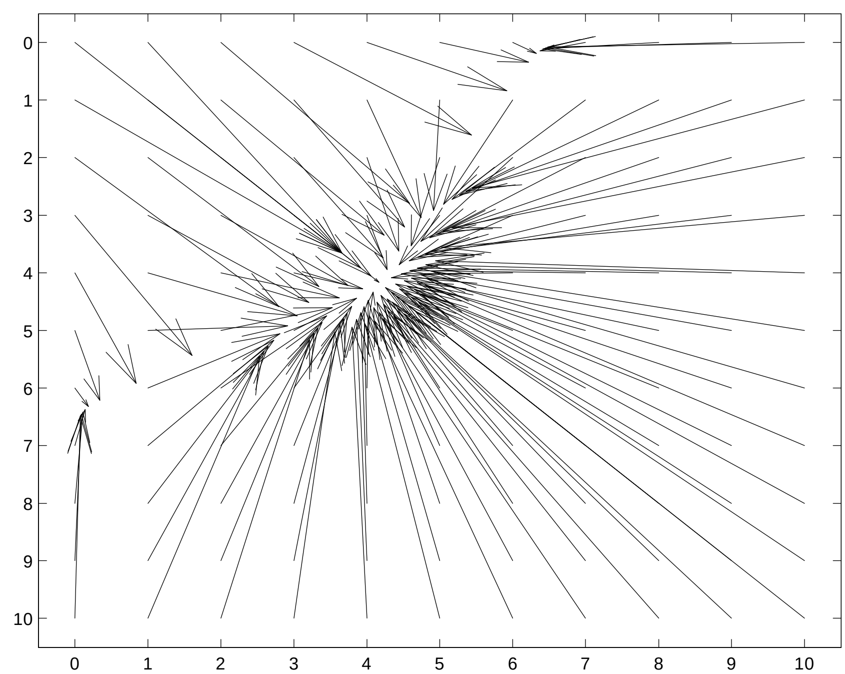

Figure 6: Arrows show the expected transitions over 5 periods in the intermediate market with no mergers allowed.

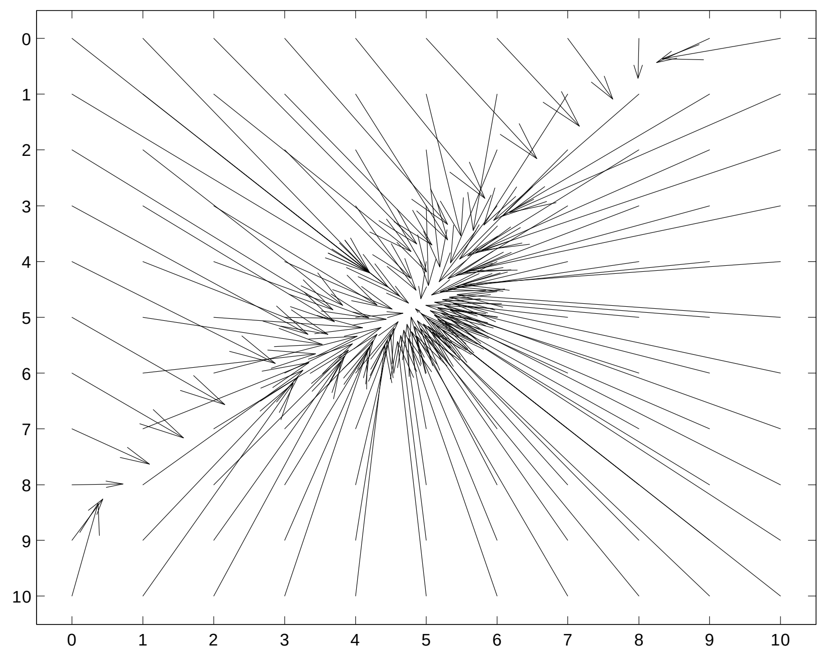

Figure 7: Arrows show the expected transitions over 5 periods in the large market with no mergers allowed.

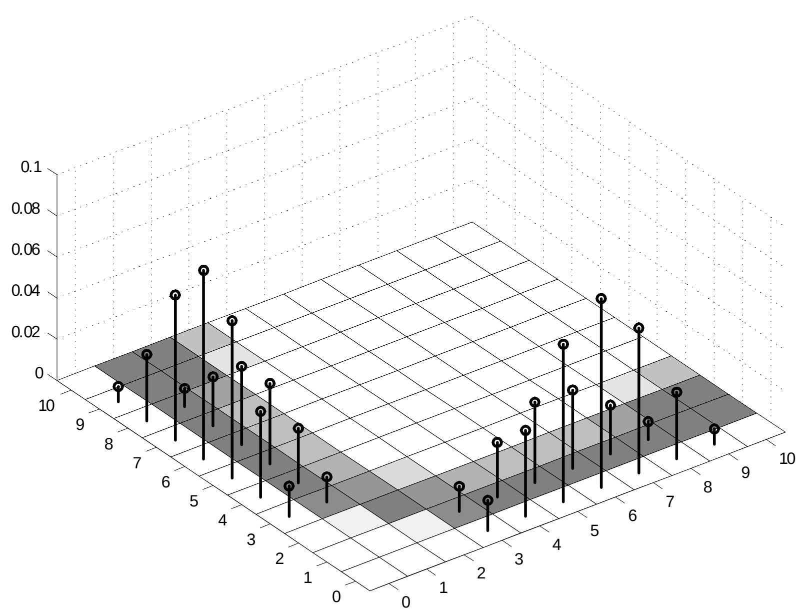

Figure 8: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

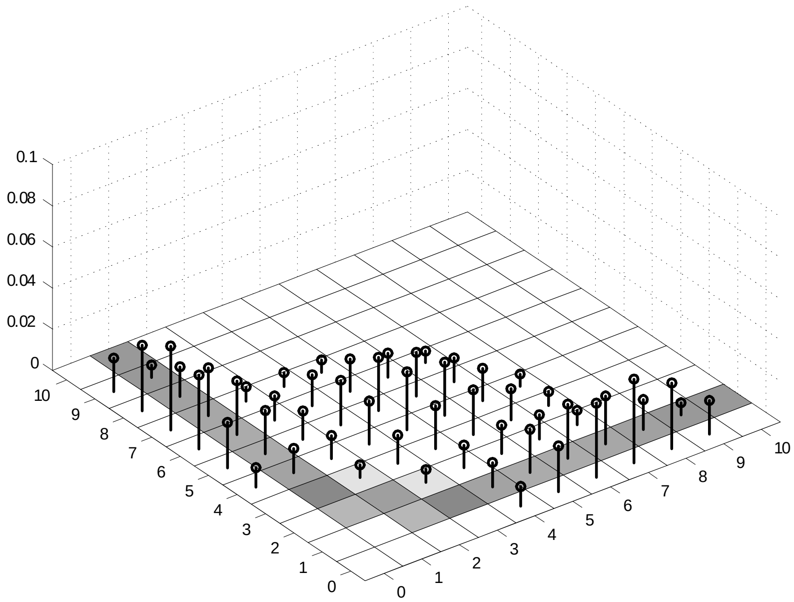

Figure 9: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the intermediate market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

Table 1: Performance Measures for the Small Market under Various Policies

Performance Measure[11]	*No-Mergers/ MPP-CV*	*Static- AS*	*All- Mergers*	*MPP- AV*	*Comm.- AV*	*Comm.- CV*
Avg. Consumer Value	31.8	28.8	28.8	29.1	32.9	33.2
Avg. Incumbent Value	57.8	56.6	56.3	58.0	61.0	57.8
Avg. Entrant Value	0.0	1.1	1.1	0.8	0.0	0.1
Avg. Blocking Cost	0.0	0.0	0.0	0.0	0.0	0.0
Avg. Aggregate Value	89.6	86.5	86.2	87.9	94.0	91.1
Avg. Price	2.25	2.28	2.28	2.28	2.23	2.23
Avg. Quantity	16.5	15.8	15.8	15.9	16.9	16.9
Avg. Total Capital	5.8	5.9	5.9	6.0	6.6	6.2
Merger Frequency	0.0%	33.2%	33.9%	30.6%	6.8%	11.6%
% in Monopoly	58.2%	95.7%	95.2%	98.6%	68.6%	60.8%
% $\min\{K_1, K_2\} \geq 2$	35.9%	0.1%	0.1%	0.3%	17.4%	32.3%
State (0,0) CV	24.0	18.9	18.8	19.4	21.8	24.1
State (0,0) AV	28.4	26.3	26.3	26.8	27.7	28.3

[11] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage. "Static-CS" amd "Static-AS" refer, respectively, to the equilibria under the optimal static consumer surplus-based and aggregate surplus-based merger policies. "MPP-CV" and "MPP-AV" refer, respectively, to the equilibria when the antitrust authority cannot commit (resulting in a Markov

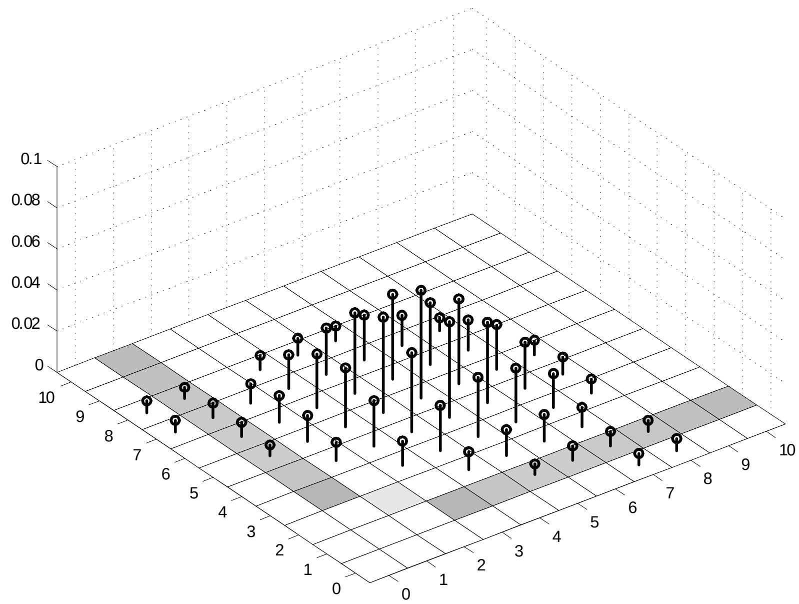

Figure 10: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

Table 2: Performance Measures for the Large Market under Various Policies

Performance Measure[12]	*No-Mergers/ MPP-CV*	*Static- AS*	*All- Mergers*	*MPP- AV*	*Comm.- AV*	*Comm.- CV*
Avg. Consumer Value	61.3	44.2	44.1	60.1	61.4	61.4
Avg. Incumbent Value	81.0	81.2	80.8	81.1	81.1	80.8
Avg. Entrant Value	0.0	2.2	2.2	0.1	0.0	0.0
Avg. Blocking Cost	0.0	0.0	0.0	0.0	0.0	0.0
Avg. Aggregate Value	142.3	127.7	127.2	141.3	142.5	142.3
Avg. Price	2.10	2.23	2.24	2.11	2.10	2.10
Avg. Quantity	27.0	23.0	22.9	26.7	27.0	27.0
Avg. Total Capital	9.6	8.3	8.3	9.5	9.6	9.6
Merger Frequency	0.0%	34.3%	33.6%	3.0%	0.0%	0.1%
% in Monopoly	2.3%	70.4%	68.4%	8.2%	2.3%	1.1%
% $\min\{K_1, K_2\} \geq 2$	94.4%	3.6%	3.8%	87.9%	94.5%	95.5%
State (0,0) CV	36.4	30.0	29.9	35.5	36.5	36.4
State (0,0) AV	45.6	42.4	42.3	45.2	45.6	45.6

perfect policy) under consumer value and aggregate value welfare criteria. “Comm.-CV” and “Comm.-AV” refer, respectively, to the equilibria when the antitrust authority commits to the optimal merger policy (within the class described in Section 3 of this Online Appendix) for maximizing consumer value and aggregate value..

[12] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which

3 Commitment Policy

In our analysis in the main text we assumed that the antitrust authority, like each of the firms, acts as a player in a stochastic dynamic game, being unable to commit to its future policy. To provide a benchmark for comparison, and also because it is of independent interest, we now consider the optimal commitment policy, a state-dependent merger approval rule to which the authority pre-commits before the game starts. For simplicity, we focus on the case $n = 2$. We assume that the antitrust authority seeks to maximize the steady state level of expected welfare, either CV or AV depending on the welfare criterion.[13] In contrast to the Markov perfect policy, the planner in the commitment case considers the impact his policy has on firms' strategies and, in particular, considers how firms' investment behavior is affected by the prospects of future merger approvals.[14] In our discussion, we will focus on the intermediate market; the results for the small and large markets are summarized toward the end of the section.

3.1 Feasible Policies

Formally, we assume that the antitrust authority pre-commits to a pure action $a_{ij}(\mathbf{K}) \in \{0, 1\}$ for each state $\mathbf{K}$ where $a_{ij}(\mathbf{K}) = 1$ if the merger is approved and 0 if it is blocked. Observe that there are 2^{100} possible deterministic symmetric merger policies. Thus, for computational reasons, we restrict the space of admissible commitment policies to two classes.[15]

Herfindahl-based policy. Under this type of policy, a proposed merger in state $\mathbf{K}$ is approved if and only if the induced change in the capital stock-based Herfindahl index is below a threshold $\overline{\Delta H}$:

$$\Delta H(\mathbf{K}) \equiv H([K_1 + K_2, 0]) - H(\mathbf{K}) \leq \overline{\Delta H}$$

where $H(\mathbf{K})$ is the capital stock-based Herfindahl index in state $\mathbf{K}$ and $\overline{\Delta H}$ is the authority's

are at the Cournot competition stage. "Static-CS" amd "Static-AS" refer, respectively, to the equilibria under the optimal static consumer surplus-based and aggregate surplus-based merger policies. "MPP-CV" and "MPP-AV" refer, respectively, to the equilibria when the antitrust authority cannot commit (resulting in a Markov perfect policy) under consumer value and aggregate value welfare criteria. "Comm.-CV" and "Comm.-AV" refer, respectively, to the equilibria when the antitrust authority commits to the optimal merger policy (within the class described in Section 3 of this Online Appendix) for maximizing consumer value and aggregate value.

[13] This policy will generally differ from the policy that would be optimal given that the industry is starting in a particular state (K_1, K_2). In addition to our primary analysis focusing on steady state welfare, we also consider the commitment policy that maximizes the expected welfare of a "new" industry at state (0,0); see footnote 19.

[14] A second difference is that under commitment the antitrust authority considers the impact its policy has on proposal costs, while without commitment those costs are considered to be sunk at the time a merger is reviewed. [A similar point arises in Besanko and Spulber (1992).]

[15] The particular form these simple commitment policies take is partly motivated by which mergers are AV-increasing as one-shot deviations. Note that to limit the number of feasible policies, we do not consider random approval rules.

policy variable.[16,17] For illustration, Figure 11(a) shows the policy $\overline{\Delta H} = 0.35$ where states with $a_{ij}(\mathbf{K}) = 1$ are shaded (only states with $\max\{K_1, K_2\} \leq 10$ are shown), while Figure 11(b) shows the policy $\overline{\Delta H} = 0.2$.

1. *Capital-stock-based policy* Under this type of policy, a proposed merger in state $\mathbf{K}$ is approved if and only if $K_1 + K_2 \notin (\underline{K}, \overline{K})$ and $\min\{K_1, K_2\} \geq \underline{K}_i$ where $\underline{K}$, $\overline{K}$, and $\underline{K}_i$ are the authority's policy variables.[18] Figure 11(c), for example depicts the policy $(\underline{K}, \overline{K}, \underline{K}_i) = (4, 10, 1)$ where states with $a_{ij}(\mathbf{K}) = 1$ are shaded (only states with $\max\{K_1, K_2\} \leq 10$ are shown), while Figure 11(d) shows the policy $(\underline{K}, \overline{K}, \underline{K}_i) = (10, 21, 1)$.

As observed earlier, under a commitment policy the antitrust authority never incurs any blocking costs since if it commits to block a merger in state $\mathbf{K}$ the merger will not be proposed in the first place.

3.2 Optimal Commitment Policy

In the intermediate market, the optimal commitment policy — for either a CV or AV standard — is the Herfindahl-type policy $\overline{\Delta H} = 0.225$. For states in which each firm has no more than 10 units of capital, this policy involves approving a merger only when the smaller firm has one unit of capital and the larger firm has at least seven units. Wherever a merger is approved under this policy, it is also highly profitable to the merging firms and is proposed with probability one. With mergers occurring only 3% of the time, this policy is fairly close to the no-mergers-allowed policy.

Figure 12 shows the steady state distribution of the equilibrium induced by the optimal commitment policy. Table 6 shows steady state averages of various performance measures for this policy. The ability to commit leads to a 4% gain in AV compared to the Markov perfect policy with the AV criterion, and a 2.5% gain in CV compared to the Markov perfect policy with the CV criterion.[19]

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[16] To retain computational tractability we discretize the policy space: $\overline{\Delta H} \in \{0.075, 0.075 + \Delta, 0.075 + 2\Delta, ..., 0.4 - \Delta, 0.4\}$, where $\Delta = 0.025$.

[17] Because there are only two firms, the post-merger Herfindahl indices always equal one: $H(K_1 + K_2, 0) = H(0, K_1 + K_2) = 1$, so $\Delta H(K_1, K_2) = 1 - H(K_1, K_2)$. Therefore a merger is approved if and only if $H(K_1, K_2) \geq 1 - \overline{\Delta H}$. Thus, under the Herfindahl-based policy mergers are only approved if the beginning-of-period Herfindahl is sufficiently high.

[18] To retain computational tractability we discretize the policy space: $\underline{K} \in \{2, 4, ..., 10, 12\}$, $\overline{K} \in \{6, 8, ..., 18, 20\}$ and $\underline{K}_i \in \{1, 2, ..., 6, 7\}$.

[19] We also consider the optimal commitment policy for a new industry, which maximizes the welfare level (CV or AV) at state (0,0). In searching for this policy, we identify first the state (0,0) welfare-maximizing policy in the class of Herfindahl-based or capital-stock-based commitment policies, and then allow the authority to optimize fully for the states $\{\mathbf{K} | 0 \leq K_i \leq 4,\ i = 1, 2\}$. The rationale for the second step is that merger policy at states with small capital levels is likely to be particularly important for maximizing welfare starting in state $(0, 0)$.

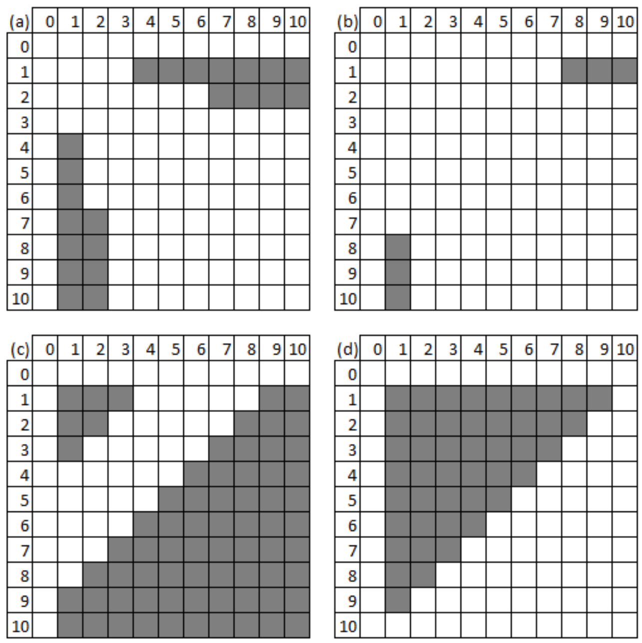

Figure 11: Panels (a) and (b) show Herfindahl-based commitment policies, whereas panels (c) and (d) show capital-stock-based commitment policies. (a) is $\overline{\Delta H} = 0.35$, (b) is $\overline{\Delta H} = 0.2$, (c) is $(\underline{K}, \overline{K}, \underline{K}_i) = (4, 10, 1)$, (d) is $(\underline{K}, \overline{K}, \underline{K}_i) = (10, 21, 1)$. The shaded states are those in which $a_{ij}(\mathbf{K}) = 1$.

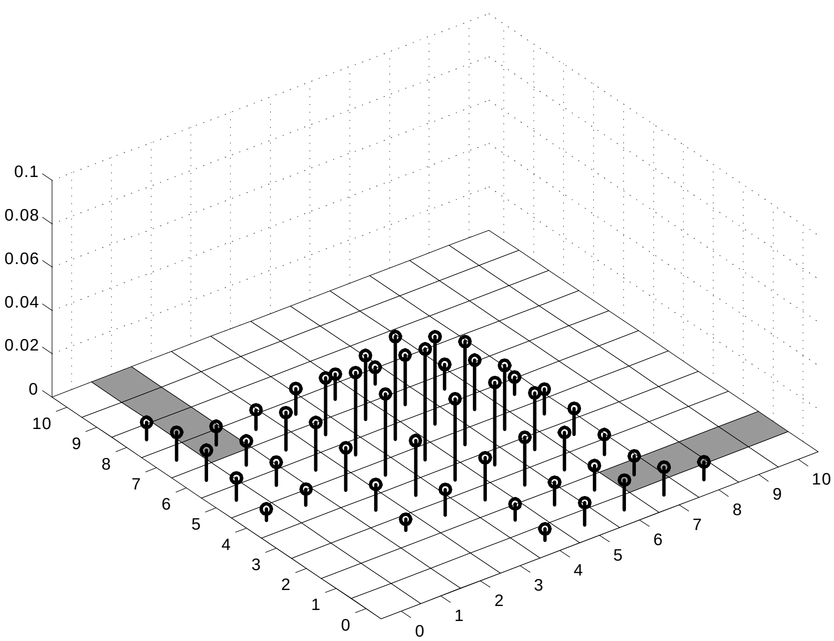

Figure 12: Beginning-of-period steady state distribution of the equilibrium generated by the optimal commitment policy (AV and CV criteria) in the intermediate market. The height of each pin indicates the steady state probability of that state. Cells in which mergers are proposed and approved are darkly shaded.

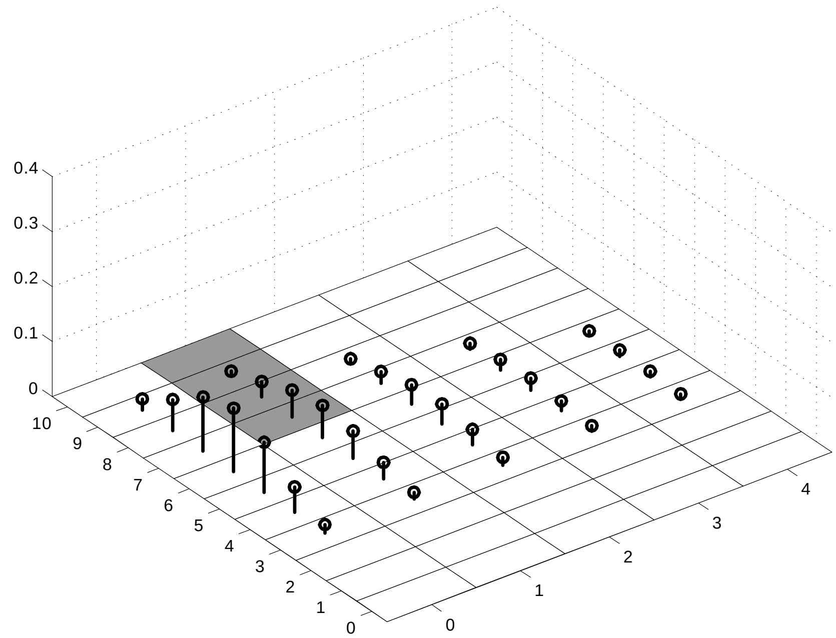

Figure 13: Five-period transitions from state (5,0) under the optimal commitment policy. The height of each pin indicates the probability of the industry being in that state. Cells in which mergers are proposed and approved are darkly shaded.

Strikingly, even though mergers move the industry to a monopoly state, the industry spends less time in a monopoly state (at the Cournot competition stage) with the optimal commitment policy than under the no-mergers-allowed policy (14.3% vs. 18.6%), and capital levels are higher (8.2 vs. 8.0). As can be seen in Figures 13 and 14, the reason there is less monopoly is that the prospect of merger induces entrants to invest, but the limited set of states in which mergers are allowed results in the industry often moving to symmetric duopoly positions following these investments. Indeed, the probability that the industry is in a monopoly state after five periods starting from state $(5, 0)$ is much lower than under the no-mergers policy: 0.45 vs. 0.84. The greater movement to symmetric, duopolistic states from monopoly ones can also be seen by comparing Figure 15 to Figure 6.

While full commitment to a policy may be difficult to achieve, an alternative is to endow the antitrust authority with an objective that may not be the true social objective. In this regard, note that the steady state level of AV under the Markov perfect merger policy when the antitrust authority has a CV objective (essentially the no-mergers-allowed outcome) is

The optimal commitment policy starting from state (0,0) allows mergers in very few states. For the AV objective, the authority allows mergers only in states $\mathbf{K}$ such that $K_i \in \{1, 2\}$, $i = 1, 2$. However, as a merger in state $(1, 1)$ is never [and in states $(1, 2)$ and $(2, 1)$ only rarely] profitable, this is almost equivalent to allowing mergers only in state $(2, 2)$. The resulting AV (resp. CV) level is 37.1 (26.6), whereas under the no-mergers policy it is 36.7 (30.3). For the CV objective, the state (0,0) optimal commitment policy is a no-mergers policy.

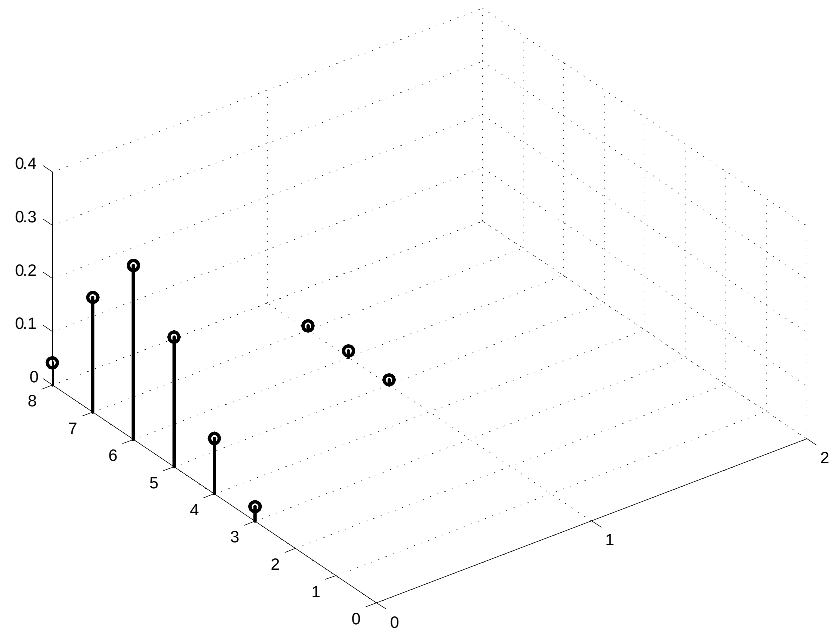

Figure 14: Five-period transitions from state (5,0) under the no-mergers-allowed policy. The height of each pin indicates the probability of the industry being in that state.

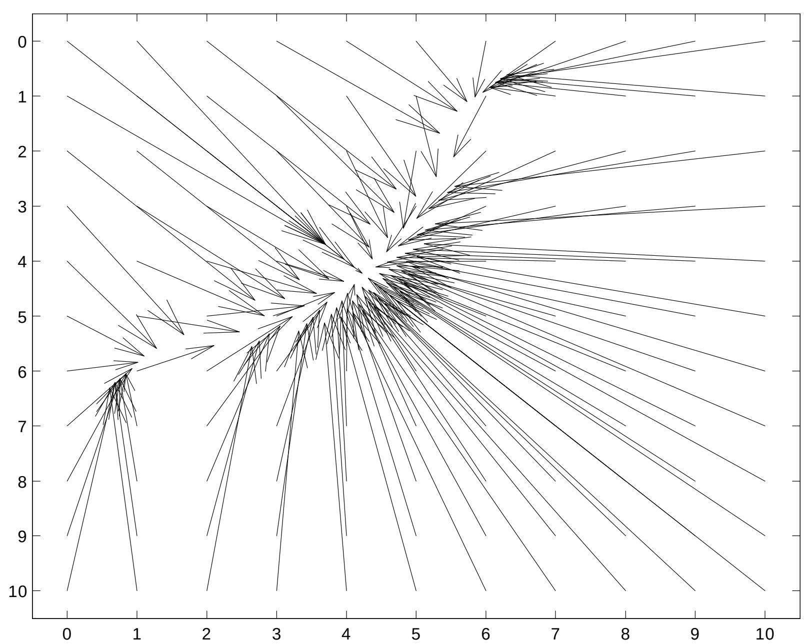

Figure 15: Arrows show the expected transitions over 5 periods under the optimal commitment policy.

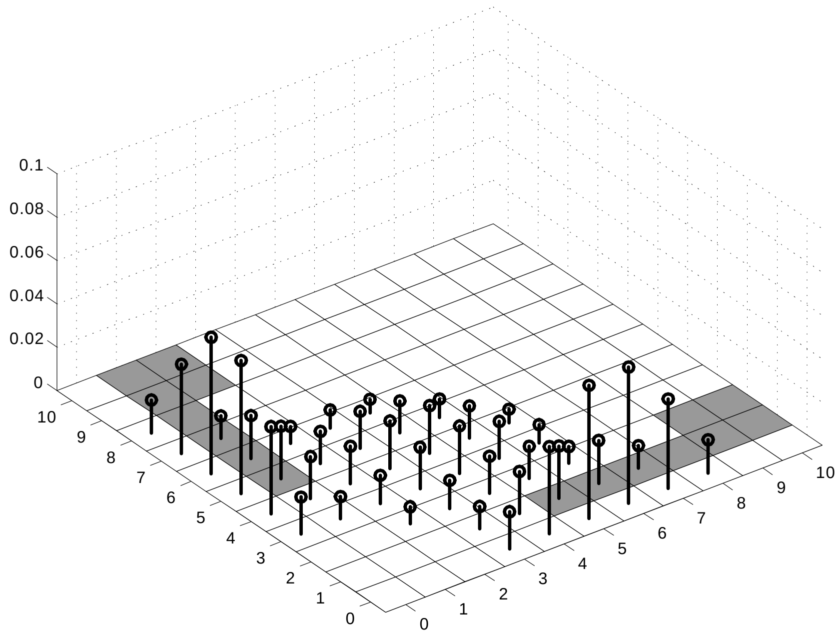

Figure 16: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (CV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

higher than that when it has an AV objective. Thus, when the antitrust authority cannot commit, a CV-maximizing antitrust authority is better for AV in this market than an AV-maximizing authority. This is consistent with a suggestion of Lyons (2002), but arises because of the policy's effect on investment, rather than by inducing a socially more desirable choice of merger partner as in Lyons (2002).

3.3 Commitment Policy in the Small and Large Markets

We now briefly summarize our results for the optimal commitment policy in the small ($A = 3, B = 22$) and large ($A = 3, B = 30$) markets, and compare them to our results for the intermediate ($A = 3, B = 26$) market.

If the antitrust authority pursues a CV goal, then the optimal commitment policy in all three markets involves approving mergers only in near-monopoly states in which the incumbent is sufficiently large. This policy is more restrictive the larger is the market, with the merger probabilities ranging from 0.1% in the large market to 11.6% in the small market (see Tables 1 and 2). Figures 16 and 17 show the steady state distributions and optimal merger policy for the small and large markets.

If the antitrust authority pursues an AV goal instead, its optimal commitment policy is

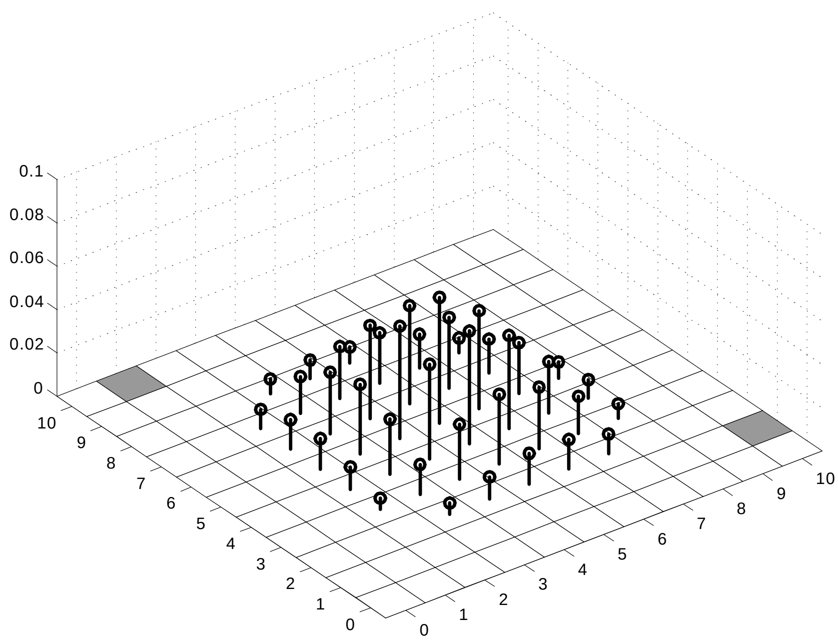

Figure 17: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (CV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

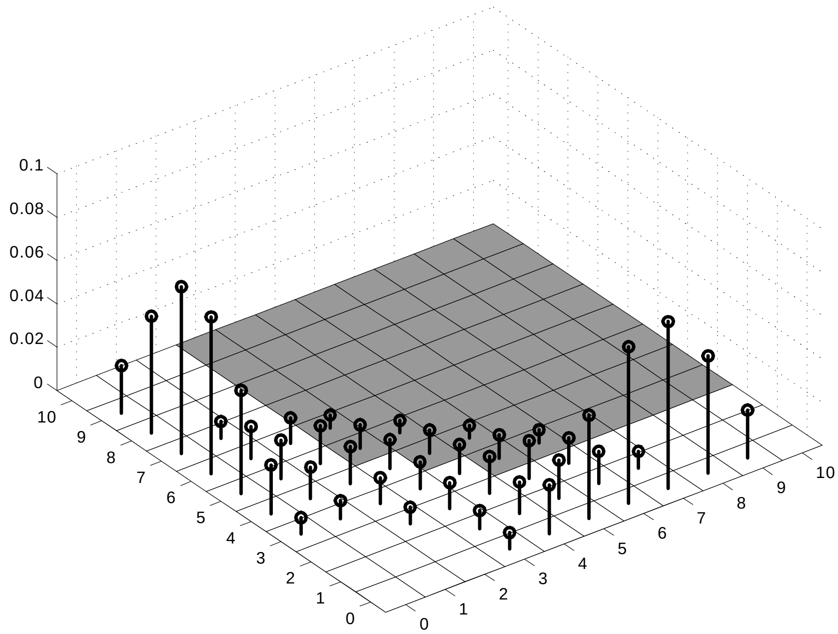

Figure 18: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (AV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

essentially to approve no mergers in the large market. In the small market, however, it does approve mergers in states in which both firms are sufficiently large (resulting in a merger probability of 6.8%), which boosts firms' investment incentives (resulting in an almost 10% higher capital level compared to the AV-maximizing Markov perfect policy). Figures 18 and 19 show the steady state distributions and optimal merger policies for the two markets. Observe that the optimal commitment policy is more restrictive in larger markets even though the set of states in which mergers increase static aggregate surplus is larger in larger markets.

Independently of whether the authority pursues a CV or AV objective, the advantage that commitment has over no commitment is decreasing (both in absolute as well as in relative terms) with the size of the market. For example, compared to the AV-maximizing Markov perfect policy, the AV-maximizing commitment policy induces a steady state average AV that is 6.7% higher in the small market but only 0.8% higher in the large market.

4 Extensions and Robustness

In this section, we investigate several extensions and robustness issues. Section 4.1 investigates how changes in the ease of entry affect the optimal merger policy. Section 4.2 examines the

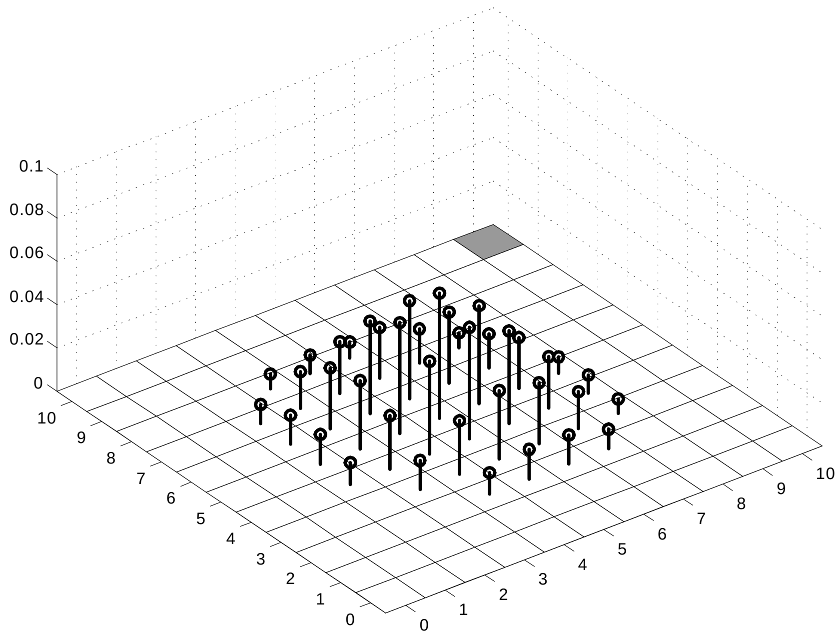

Figure 19: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (AV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

effects of reducing the difference in investment costs between incumbents and entrants. Section 4.3 considers the equilibrium when a planner controls investment and merger decisions. Section 4.4 considers a modification to the model where the entrant is the previously bought-out firm's owner. Section 4.5 examines changing bargaining power from an equal weighting to a capital-weighted bargaining power. Section 4.6 looks at the robustness of our results for various production scale parameters. Finally, Section 4.7 looks at the robustness of our results for various ranges of investment costs. Throughout this section, we focus on the duopoly case ($n = 2$).

4.1 Ease of Entry

It is generally perceived that the potential anticompetitive effects of horizontal mergers are mitigated when entry into the industry is easy. For instance, the current (2010) U.S. Horizontal Merger Guidelines (which are largely based on a consumer welfare standard) state:

> A merger is not likely to enhance market power if entry into the market is so easy that the merged firm and its remaining rivals in the market, either unilaterally or collectively, could not profitably raise price or otherwise reduce competition compared to the level that would prevail in the absence of the merger. Entry is that easy if entry would be timely, likely, and sufficient in its magnitude, character, and scope to deter or counteract the competitive effects of concern.

To study how the ease of post-merger entry affects optimal merger policy and the resulting performance of the industry, we extend the baseline model in two ways: first by introducing a probability $e \geq 0$ that a new entrant arrives at the entry stage whenever the current state of the industry has a single active firm, and second by introducing a minimum scale $\underline{\Delta K}^g > 1$ for greenfield investment. We focus on the intermediate market and the AV criterion. Contrary to the conventional view, we find that in both cases optimal merger policy may become more permissive when entry becomes more difficult.[20]

4.1.1 Timeliness of Entry

Consider, first, the timeliness of entry following a merger. Table 3 reports the performance measures of the intermediate ($A = 3, B = 26$) market under the Markov perfect policy with an AV welfare criterion for different levels of the entry probability e.[21] Despite the inefficiencies

[20] For a similar observation in a static context, see Whinston (2007). While new entry is generally viewed as being price-reducing and thus beneficial to consumers, it may be excessive from an aggregate welfare point of view [Mankiw and Whinston (1986)].

[21] Formally, this requires extending the state space to $\mathcal{S}' \equiv \{-1, 0, 1, ..., 20\}^2$, where $K_i = -1$ means that firm i is an entrant who has not yet arrived. The firms' expected gain from merging is therefore now given by

$$\Delta_G(K_1, K_2) = e\overline{V}(K_1 + K_2, 0) + (1 - e)\overline{V}(K_1 + K_2, -1) - [\overline{V}(K_1, K_2) + \overline{V}(K_2, K_1)],$$

associated with entry for buyout, welfare declines as entry becomes less timely: the steady state levels of CV and AV fall from 43.3 and 113.6, respectively, to 28.0 and 98.5 as e decreases from 1 to 0. The reason for this finding is that, as e decreases, the industry spends more and more time in a monopoly state: the steady state probability of monopoly increases from 49.4% at $e = 1$ to 100% at $e = 0$. This hurts consumers and society a lot in the short run (for a given level of capital) but even more so in the long run because a monopolist has little incentive to build capital in the absence of a threat of entry: the average total capital level decreases from 7.7 to 5.3 as e decreases from 1 to 0.

Table 3: Timeliness of Entry and Markov Perfect Policy Outcomes (Intermediate Market, AV Criterion)

Performance Measure[22]	*e=1.0*	*e=0.8*	*e=0.6*	*e=0.4*	*e=0.2*	*e=0.0*
Avg. Consumer Value	43.3	41.6	37.4	33.1	30.6	28.0
Avg. Incumbent Value	69.9	70.3	70.7	69.6	69.7	70.5
Avg. Entrant Value	0.5	0.5	0.9	1.8	1.4	0.0
Avg. Blocking Cost	-0.1	0.0	0.0	0.0	0.0	0.0
Avg. Aggregate Value	113.6	112.4	109.0	104.6	101.7	98.5
Avg. Price	2.19	2.21	2.25	2.29	2.32	2.35
Avg. Quantity	21.0	20.6	19.6	18.5	17.7	16.9
Avg. Total Capital	7.7	7.5	7.1	6.4	5.9	5.3
Merger Frequency	16.1%	19.5%	30.3%	36.7%	20.0%	0.0%
% in Monopoly	49.4%	58.9%	83.6%	99.4%	100%	100%
% $\min\{K_1, K_2\} \geq 2$	44.2%	35.3%	13.1%	0.1%	0.0%	0.0%

Table 3 also reveals that the frequency of mergers is non-monotonic in the timeliness of post-merger entry: as e decreases from our base case of $e = 1$, the probability that a merger occurs in a randomly selected period first increases (from 16.1% at $e = 1$ to 36.7% at $e = 0.4$) and then decreases. As a merger is infeasible in states in which there is only one active firm, this steady state weighted merger probability is equal to the probability that there are two active firms times the probability of a merger conditional on two firms being active, and is bounded from above by the entry probability e.[23] This explains why the merger frequency converges to zero as the entry probability e becomes small.

where the first (second) term on the right-hand side is the probability of new entry (no new entry) occurring times the continuation value of the merged firm in that event.

[22] All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the Cournot competition stage.

[23] The steady state weighted merger probability is maximized when the probability of a merger, conditional on there being two active firms, is equal to one. In that case, the probability that there are two active firms is equal to the entry probability e, implying that the steady state weighted merger probability is equal to e as well.

To understand why the merger frequency *increases* as e decreases from 1 to 0.4, consider the merger probability conditional on two firms being active, which is the product of two probabilities: the probability that the two active firms propose a merger and the probability that a proposed merger is approved.

Consider first states in which both firms have at least one unit of capital. As e decreases, mergers become more profitable in such states as the merged firm spends more time in a monopoly state before a new entrant appears. Moreover, the AV-maximizing Markov perfect policy tends to become *less* restrictive as e declines, reflecting the reduced entry for buyout behavior. When the entry probability e is high, the Markov perfect policy approves mergers only in states in which at least one of the firms is sufficiently small (as we have seen for $e = 1$ in Section 3.3 of the main paper). As e decreases, this approval region increases. For example, a proposed merger in state (3,3) is never approved if $e \geq 0.6$ but always approved if $e = 0.4$.

Consider now states in which an entrant has arrived yet has no capital. When the entry probability is one, the authority would always approve a proposed merger in such a state: approving the merger has no effect on AV, but blocking is costly. However, when $e = 1$, such a merger would not be proposed as it is not profitable.[24] When the post-merger entry probability is sufficiently small, such a merger becomes profitable as the arrival of a new entrant following a merger takes time, allowing the merging firms to reap monopoly profits in the meantime. As e decreases, firms are therefore more likely to propose mergers between an entrant and an incumbent. At the same time, while the antitrust authority starts to block mergers, it allows some proposed mergers. Hence, the probability of a merger between an entrant and an incumbent becomes positive for $e \leq 0.6$.[25]

4.1.2 Minimum Scale for Greenfield Investment

We now explore a different way in which entry may become more difficult. Specifically, we extend the model by introducing a minimum size for greenfield investment, $\underline{\Delta K}^g$, focusing again on the intermediate market. As incumbents rarely use the greenfield technology, this essentially amounts to introducing a minimum scale of entry.

Table 4 shows the same performance statistics for $\underline{\Delta K}^g$ ranging from 1 (our base case) to 5. As with reductions in the timeliness of entry, a larger minimum scale of greenfield entry raises the likelihood of being in a monopoly state, and has a non-monotonic effect on the probability of merger. Similar to cases in which e approaches zero, as $\underline{\Delta K}^g$ grows large the probability of merger declines because the likelihood of post-merger entry grows small; nonetheless, as $\underline{\Delta K}^g$

[24] When $e = 1$, a merger in state $(K_1, 0)$ or $(0, K_2)$ does not affect producer value because the old entrant gets immediately replaced by a new entrant. As the value of the new entrant is strictly positive, this implies that the merger must decrease the joint continuation values of the merging firms.

[25] If the antitrust authority adopts a CV standard instead of an AV standard, the relationship between the timeliness of entry and the steady state probability of a merger remains non-monotonic: as e decreases, the merger frequency first increases and then decreases.

grows, both the set of states in which mergers are permitted by the antitrust authority and the set of states in which mergers are proposed grow larger. However, in contrast to a reduction in the timeliness of entry, aggregate value shows relatively small and non-monotonic changes as $\underline{\Delta K}^g$ rises. The reason for this difference is that the aggregate capital in the market does not fall as $\underline{\Delta K}^g$ gets larger, in contrast to the case when e gets small. This occurs because while the likelihood of entry grows smaller as $\underline{\Delta K}^g$ grows, when entry does occur it is at a larger scale, and the incumbent monopolist is incented to invest to reduce this possibility and get better merger terms when entry does occur.

Table 4: Minimum Scale of Greenfield Investment and Markov Perfect Policy Outcomes (Intermediate Market, AV Criterion)

Performance Measure[26]	$\underline{\Delta K}^g = 1$	$\underline{\Delta K}^g = 2$	$\underline{\Delta K}^g = 3$	$\underline{\Delta K}^g = 4$	$\underline{\Delta K}^g = 5$
Avg. Consumer Value	43.3	39.6	37.8	38.5	35.6
Avg. Incumbent Value	69.9	73.1	73.8	76.8	76.6
Avg. Entrant Value	0.5	0.3	0.2	0.1	0.0
Avg. Blocking Cost	-0.1	-0.1	0.0	0.0	0.0
Avg. Aggregate Value	113.6	113.0	111.8	115.4	112.3
Avg. Price	2.19	2.22	2.24	2.23	2.26
Avg. Quantity	21.0	20.2	19.7	19.9	19.2
Avg. Total Capital	7.7	7.6	7.6	7.9	7.1
Merger Frequency	16.1%	16.4%	15.6%	8.8%	3.5%
% in Monopoly	49.4%	82.3%	97.0%	99.5%	99.8%
% $\min\{K_1, K_2\} \geq 2$	44.2%	17.5%	2.6%	0.5%	0.2%

4.2 Entrant Investment Efficiency

In our analysis of the welfare effects of various merger policies, “entry for buyout” plays a prominent role. When mergers are allowed a new entrant’s private benefit from investing significantly exceeds the incremental aggregate value that results from those investments, while the incremental aggregate value from an incumbent’s investment exceeds its private benefit to the incumbent. As a result, the entrant invests too much and the incumbent invests too little. The entrant’s high cost greenfield investment substitutes for the incumbent’s lower cost investment done through capital augmentation and directly causes waste.

In practice, however, entrants’ investments are not always less efficient than incumbents’ investments, and may sometimes even be more efficient.[27] In this subsection, we explore

[26] All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the Cournot competition stage.

[27] Henderson (1993) provides evidence of this in the photolithographic alignment equipment industry where several generations of entrants supplanted incumbents by more efficiently using their knowledge capital.

this point by changing the model's parameters to close the gap between the investment costs entrants and incumbents face.

Focusing on the intermediate market, we examine whether this change largely eliminates the waste that entry for buyout causes by studying the effect of a change from the no-mergers-allowed policy to the all-mergers-allowed and Markov perfect policies when the antitrust authority's criterion is AV maximization. Overall, we find that (i) entry for buyout behavior continues to be prevalent, (ii) its social costs are greatly reduced; (iii) the antitrust authority is much more willing to allow mergers in the Markov perfect policy; and (iv) with this change, consumer value falls somewhat more when moving from no-mergers-allowed to the Markov perfect policy.

Recall that capital augmentation each period enables a firm with K units of capital, if it wishes, to double each unit j at a cost c_j drawn independently and uniformly from the interval $[\underline{c},\bar{c}]$. If it wants to more than double its current stock of capital, then it can purchase additional greenfield units at constant unit cost c_g, where c_g is uniformly drawn from $[\bar{c},\bar{c}_g]$. Let $s = \bar{c} - \underline{c}$ and $s_g = \bar{c}_g - \bar{c}$ be the spread of capital augmentation costs and greenfield costs respectively. In the baseline industry analyzed in the previous sections the values are $\underline{c} = 3, \bar{c} = 6$, $\bar{c}_g = 7$, $s = 3$, and $s_g = 1$. To close the gap between entrant and incumbent investment costs we reduce s to 1 and s_g to 0.25. Since this change, if $\underline{c}$ were held fixed, would reduce firms' investment costs, leading to less monopoly and very different merger behavior, we simultaneously raise $\underline{c}$ to 4.645, which keeps the frequency of monopoly unchanged when no mergers are allowed. Thus, we have $\underline{c} = 4.645$, $\bar{c} = 5.645$, $\bar{c}_g = 5.895$; we refer to these modified parameter values as the "efficient entry environment."

Table 5 shows the results when we switch from our baseline environment to the efficient entry environment. The table reports the same performance statistics as before, with the addition of one new measure: "Avg. Monop. to Merger Time." This statistic measures the expected number of periods the industry takes to transition from a monopoly state to a state in which the incumbents merge.[28] Comparing the two environments, we see that entry for buyout behavior actually increases when we move to the efficient entry environment; for example, when all mergers are allowed, the monopoly to merger time falls from 2.6 to 2.1. However, the costs of this behavior are greatly reduced: AV now falls only 0.6% when all mergers are allowed (from 87.9 with no mergers to 87.4 when all mergers are allowed), compared to 10.0% in our baseline case (from 117.5 with no mergers to 105.8 with all mergers allowed). Because of the reduction in the inefficiency of *pre-merger* investment behavior, allowing mergers is much more attractive for the antitrust authority, and the Markov perfect policy results in far more mergers in the efficient entry environment: the probability of merger is now 42.6% in each period, compared to only 16.1% in our baseline case. Indeed, the equilibrium is essentially equivalent to the case in which all mergers are allowed. Finally, this increased merger activity results in a much greater likelihood of the industry being in a monopoly state (79.4% of the

[28] We use the steady state distribution over monopoly states as weights, and exclude state $(0, 0)$.

time in the efficient entry environment vs. 49.4% in our baseline case). As a consequence, there is a somewhat greater reduction in consumer value when moving from no mergers being allowed to the Markov perfect policy (a reduction of 13.6%, from 34.9 to 30.5 in the efficient entry environment, vs. a reduction of 10.0%, from 48.1 to 43.3).[29]

Table 5: Performance Measures for the Efficient Entry Environment in the Intermediate Market

	Baseline Environment			Efficient Entry Environment		
Performance Measure[30]	*No-Mergers*	*All-Mergers*	*MPP-AV*	*No-Mergers*	*All-Mergers*	*MPP-AV*
Avg. Consumer Value	48.1	35.8	43.3	34.9	30.5	30.5
Avg. Incumbent Value	69.4	68.1	69.9	53.1	54.9	54.9
Avg. Entrant Value	-	1.9	0.5	-	2.0	2.0
Avg. Blocking Cost	-	-	-0.1	-	-	0.0
Avg. Aggregate Value	117.5	105.8	113.6	87.9	87.4	87.4
Avg. Price	2.15	2.26	2.19	2.27	2.32	2.32
Avg. Quantity	22.2	19.2	21.0	18.9	17.7	17.7
Avg. Total Capital	8.0	7.0	7.7	5.6	5.7	5.7
Merger Frequency	0.0%	37.7%	16.1%	0.0%	42.6%	42.6%
% in Monopoly	18.6%	86.0%	49.4%	18.6%	79.4%	79.4%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	44.2%	68.6%	4.2%	4.2%
Avg. Monop. to Merger Time	-	2.6	6.1	-	2.1	2.1

4.3 The Planner's Solution

We consider the second-best dynamic problem where the planner controls both firms' merger decisions (independent of their private profitability) as well as their investment decisions (assuming the planner has perfect information about firms' private cost draws), taking as given only that, in every period, firms compete in a Cournot fashion. The analysis provides a

[29] The greater percentage reduction in consumer value in the Markov perfect policy compared to when no mergers are allowed depends on market size. In results not reported here, we find that it remains true in the large market, but in the small market there is no reduction in consumer value from allowing mergers in the efficient entry environment. The other effects we report here for the intermediate market (continued entry for buyout behavior, reduced cost of that behavior, and greater frequency of mergers) hold as well in the small and large markets. Figures 26-28 in this Online Appendix show these effects for a broader range of investment costs for incumbents and entrants, focusing on the intermediate market. Specifically, for each combination of $s \in [1, 3]$ and $s_g \in [0.25, 1]$ we find the level $\underline{c}(s, s_g)$ at which the percentage of time in a monopoly state is 18.6% in the no-mergers-allowed equilibrium. The figures show, respectively, the average time from monopoly to merger in the all-mergers-allowed equilibrium (Figure 26), $(AV_{No} - AV_{all})/AV_{No}$ (Figure 27), and the probability of merger in the Markov perfect policy (Figure 28) for each economy $(s, s_g, \underline{c}(s, s_g))$.

[30] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage.

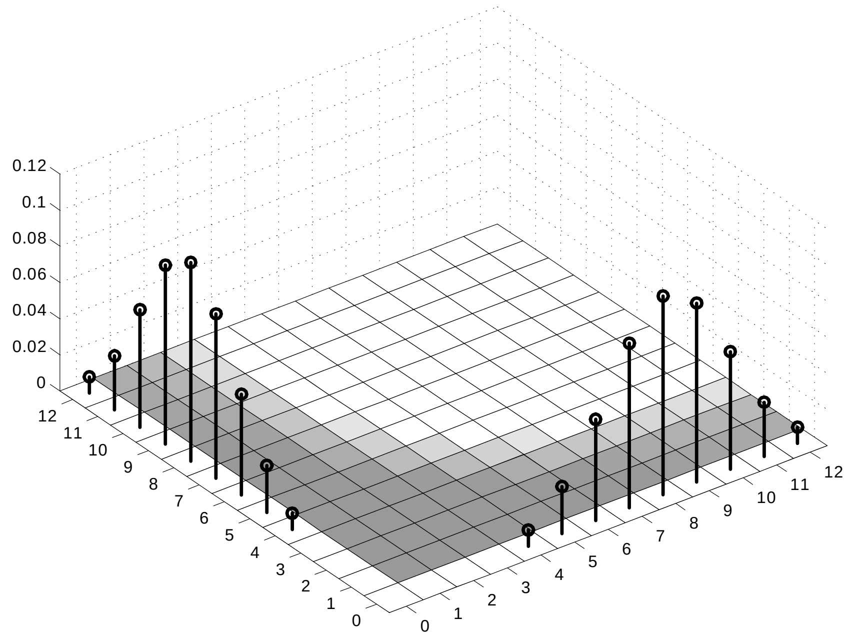

Figure 20: Solution to the planner's second-best problem (AV criterion) in the intermediate market. The height of each pin gives the probability of the corresponding state in the steady state generated by the planner's optimal policy. The shading of the cells indicates the merger probabilities, with a darker shading corresponding to a higher merger probability.

benchmark for how optimal merger policy would look absent concerns about the efficiency of investment behavior. In our analysis, we confine attention to the AV criterion.[31]

Figure 20 shows the steady state distribution for the solution of this second-best problem in the intermediate market: the height of each pin gives the beginning-of-period probability of the corresponding state in the steady state generated by this policy; the cells in which mergers are approved are darkly shaded. As the planner controls not only merger decisions but also firms' investment decisions, the planner does not face a time inconsistency problem; i.e., the solution is independent of whether or not the planner can commit to his future decisions.[32]

As Figure 20 shows, in the steady state generated by the planner's solution, the industry is always in a monopoly state. A merger is implemented in many states, unless these states involve high capital levels for both firms. In fact, the set of states in which mergers happen is almost identical to the set of states in which a merger is statically aggregate surplus-increasing

[31] The second-best solution is not well-defined for the CV criterion as consumers always benefit from larger capital stocks.

[32] The existence of blocking costs is irrelevant for the solution to the second-best problem as it can never be optimal from the planner's point of view to propose a merger and subsequently block it in the event blocking costs are sufficiently low.

(for reasons that will be discussed below).[33]

Table 6: Performance Measures for the Intermediate Market under Various Policies

Performance measure[34]	*No-Mergers/ MPP-CV*	*All-Mergers*	*MPP-AV*	*Commitment (CV and AV)*	*Planner*
Avg. Consumer Value	48.1	35.8	43.3	49.3	39.2
Avg. Incumbent Value	69.4	68.1	69.9	68.8	82.1
Avg. Entrant Value	0.0	1.9	0.5	0.0	0.0
Avg. Blocking Cost	0.0	0.0	-0.1	0.0	0.0
Avg. Aggregate Value	117.5	105.8	113.6	118.1	121.3
Avg. Price	2.15	2.26	2.19	2.14	2.23
Avg. Quantity	22.2	19.2	21.0	22.5	20.1
Avg. Total Capital	8.0	7.0	7.7	8.2	8.1
Merger Frequency	0.0%	37.7%	16.1%	3.0%	0.0%
% in Monopoly	18.6%	86.0%	49.4%	14.3%	100.0%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	44.2%	78.8%	0.0%
State (0,0) CV	30.3	23.9	25.6	30.4	25.3
State (0,0) AV	36.7	34.0	35.5	36.7	41.8

The fact that in the second-best solution the industry is always in a monopoly state may be surprising at first. After all, when mergers are not allowed the industry seems to be a workable duopoly. The reason is closely related to the fact that mergers are frequently aggregate surplus increasing, given our chosen parameters. To understand this point, suppose first that the planner could not only control mergers but also costlessly undo previously approved mergers. Suppose also that there were no merger proposal costs. What would the planner's optimal policy be in that case? In any state (K_1, K_2), the planner would optimally implement a merger if and only if the merger increases static aggregate surplus as this is statically optimal and also does not impede dynamic optimality as the planner controls investment, the investment technology is merger neutral, and the planner can costlessly undo any previously approved merger. In Figure 5 of the main paper we saw that a merger increases static aggregate surplus in every state in which $K_1 + K_2 \leq 10$ (except in state $(5, 5)$ in which the gain is approximately zero) and, also, in several additional states in which $K_1 + K_2 > 10$. So, unless the planner wants to spend a large amount of time in states with more than 10 units of capital, the steady state generated by the planner's policy will visit only monopoly states even if the planner cannot undo previously approved mergers and there are proposal costs — which is what is going on here.[35] Finally, note that this reasoning also explains why the set of states in which

[33] For comparison with the optimal merger policy (with and without commitment), performance measures of the planner's solution are provided in the last column of Table 6.

[34] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage.

[35] In the steady state generated by the planner's solution, the industry is sometimes (8.3% of the time) in a monopoly state with more than 10 units of capital, the joint frequencies of states $(11, 0)$ and $(0, 11)$ being 6.1%.

the planner implements mergers almost coincides with the set of statically aggregate surplus-increasing mergers. They do not coincide fully because of the presence of merger proposal costs, which the static criterion does not take into account.

The results in the small and large markets are very similar: in both markets, the AV-maximizing second-best solution involves monopoly all of the time.

4.4 Entrant Identity

A key restriction in the duopoly model analyzed in Section 3 of the main paper is that no more than two firms can be active at any one time. Throughout this restriction has been posed exogenously. Our baseline assumption is that the entering firm after a merger is owned by an entrepreneur who has never before been active within the industry. This assumption begs the question as to why he did not enter previously before the merger took place.

An alternative to the exogenous restriction we have used is to assume that only two entrepreneurs have the necessary skill and knowledge set to compete in the industry. If that is the case and both entrepreneurs are active in the industry, then the owner/manager of the acquired firm would become the new entrant following a merger. (We assume there is not a "no-compete" clause in the acquisition agreement.) Equation (1) in the main paper giving the joint value gain from merging then becomes

$$\Delta_{12}(K_1, K_2) \equiv \left\{ \left[\overline{V}(K_1 + K_2, 0) + \overline{V}(0, K_1 + K_2) \right] - \left[\overline{V}(K_1, K_2) + \overline{V}(K_2, K_1) \right] \right\}.$$

New to the definition is the entrant's ex ante value $\overline{V}(0, K_1 + K_2)$. It must be included because the entrepreneur who is bought out intends to re-enter. In other words, the two entrepreneurs will agree to merge—one buying out the other—if it pays them jointly to create temporarily a monopoly situation in the industry until that time the bought-out entrepreneur successfully returns to the industry. Since $\overline{V}(0, K_1 + K_2) \geq 0$ this weakly increases the merger frequency (holding the policy and value function constant). Table 7 shows a side-by-side comparison for the intermediate market of the equilibria for these two different assumptions concerning entry. When all mergers are allowed, this change increases the frequency of mergers. (Although note that in the AV-maximizing Markov perfect policy the merger frequency ends up lower than before.) Inspection shows that, overall, our results are not qualitatively different from our earlier results.

But these are both states that are reachable by aggregate surplus increasing mergers.

Table 7: Performance Measures when the Bought Firm is the Entrant in the Intermediate Market

Performance Measure[36]	New Entrant			Bought is Entrant		
	No-Mergers	*All-Mergers*	*MPP-AV*	*No-Mergers*	*All-Mergers*	*MPP-AV*
Avg. Consumer Value	48.1	35.8	43.3	48.2	35.6	45.4
Avg. Incumbent Value	69.4	68.1	69.9	69.4	68.7	69.6
Avg. Entrant Value	-	1.9	0.5	-	-	-
Avg. Blocking Cost	-	-	-0.1	-	-	-0.0
Avg. Aggregate Value	117.5	105.8	113.6	117.6	104.3	115.0
Avg. Price	2.15	2.26	2.19	2.15	2.26	2.17
Avg. Quantity	22.2	19.2	21.0	22.2	19.2	21.5
Avg. Total Capital	8.0	7.0	7.7	8.0	7.0	7.8
Merger Frequency	0.0%	37.7%	16.1%	0.0%	49.2%	11.8%
% in Monopoly	18.6%	86.0%	49.4%	18.3%	94.0%	35.9%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	44.2%	76.0%	0.1%	57.0%
Avg. Monop. to Merger Time	-	2.6	6.1	-	2.6	8.5

4.5 Capital-weighted Bargaining Power

In the main paper, we have assumed that firms split the surplus from merging equally when $n = 2$. Here, we explore the case where the surplus division in Nash bargaining is proportional to the merging firms' capital stocks, i.e., in state (K_1, K_2), firm i gets a share $K_i/(K_i + K_{-i})$.

When a firm expects to merge in the future, capital-weighted bargaining power provides it with an additional incentive to add capital, holding fixed the rival's investment. Consider, for example, state $(5, 5)$ under the all-mergers-allowed policy. Moving from equal bargaining weights to capital-weighted bargaining power increases each firm's expected investment from 1.0 to 1.4. In monopoly states, however, the entrant faces a countervailing incentive because (i) it will capture only a small fraction of the surplus from merging and (ii) the incumbent invests more than under equal bargaining weights. As a result in monopoly states in which the incumbent has more than five units of capital, which represents nearly all of the steady state at the investment stage, the change in the division of bargaining power decreases the entrant's expected investment. The distortion in firms' investment incentives due to entry for buyout is thus mitigated when the division of bargaining power is proportional to firms' capital stocks.

Table 8 provides the performances measures for the intermediate market under the no-mergers, all-mergers-allowed and AV-oriented Markov perfect policies. As before, the no-mergers policy achieves the highest average aggregate value while the all-mergers-allowed policy performs worst. However, because of the changed investment incentives under the capital-weighted division of bargaining power, the latter policy does not perform quite as

[36] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage.

badly as before: Compared to the case of equal bargaining weights, the average capital stock is considerably larger (7.5 instead of 7.0), resulting in a higher average aggregate value (111.2 instead of 105.8).

Because of the improved investment incentives in monopoly states under the capital-weighted division of bargaining power is that the AV-oriented Markov perfect policy allows mergers in a much larger set of states. In fact, the approval probability is less than 50% only in states in which both firms have at least five units of capital. As a result, the average merger frequency increases from 16.1% to 29.6%, which is not much lower than the 32.1% merger frequency when all mergers are allowed. The performance of the Markov perfect policy is therefore close to that of the all-mergers-allowed policy: the average AV is 112.1, compared to 111.2 under the latter policy (and 113.6 under the Markov perfect policy with equal bargaining weights).

Table 8: Performance Measures for the Intermediate Market under Various Policies and Capital-weighted Bargaining Power

Performance Measure[37]	*No-Mergers*	*All-Mergers*	*MPP-AV*
Avg. Consumer Value	48.1	37.2	37.3
Avg. Incumbent Value	69.4	72.9	74.0
Avg. Entrant Value	0.0	1.1	0.8
Avg. Blocking Cost	0.0	0.0	0.0
Avg. Aggregate Value	117.5	111.2	112.1
Avg. Price	2.15	2.25	2.24
Avg. Quantity	22.2	19.6	19.6
Avg. Total Capital	8.0	7.5	7.5
Merger Frequency	0.0%	32.1%	29.6%
% in Monopoly	18.6%	96.0%	49.4%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.2%	44.2%
State (0,0) CV	30.3	25.4	25.9
State (0,0) AV	36.7	35.1	35.3

4.6 Outcomes for Various Scale Parameter Values

In Section 3.4 of the main paper, we examined the extent to which several of the features of the equilibria in our small, intermediate, and large markets extend across a wider range of demand parameters B and A. Here we do a similar analysis across demand parameter B, the size of the market, and production parameter θ, the scale parameter. Our analysis in this section shows the same patterns as are shown in the main paper. It suggests that changing

[37] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage.

the production scale parameter θ leads to similar comparative statics as changing the demand function choke price A.

Figure 21 reports on the difference in aggregate value between the no-mergers-allowed and all-mergers-allowed equilibria. The figure depicts contour lines showing the parameters at which the aggregate value difference $(AV_{No} - AV_{All})/AV_{No}$ achieves a given percentage value (each contour line is labelled). Also shown in the figure are three dots representing the parameters of our small, intermediate, and large markets, as well as dashed lines showing markets that spend 5%, 20%, and 60% of the time in monopoly when no mergers are allowed (these are roughly the monopoly percentages in our large, intermediate, and small markets). As can be seen in the figure, aggregate value with no mergers allowed is greater than with all mergers allowed provided that the market is large enough. This pattern is nearly identical to the pattern seen in the main paper.

Figure 22 shows the percentage difference in entry probabilities in the no-mergers-allowed and all-merger-allowed equilibria, $[\Pr(Entry)_{All} - \Pr(Entry)_{No}]/\Pr(Entry)_{All}]$.[38] Consistent with the entry for buyout we observed earlier, the level of entry is always weakly greater in the all-mergers-allowed equilibrium, although the difference declines to zero in very large markets where the probability of entry rises to 1 under either merger policy.

Figure 23 shows the probability of a merger occurring under the Markov perfect policy. We see the same pattern as we saw in the main paper. Moving from the Southwest corner, the probability of merger increases as the market gets larger. Continuing in the same direction, however, the probability of merger begins decreasing once the market is large enough that the Markov perfect policy of the antitrust authority allows fewer mergers.

Figure 24 shows the percentage difference in aggregate value between the Markov perfect policy and the no-mergers-allowed equilibrium $(AV_{MPP} - AV_{No})/AV_{MPP}$. Again, we see the same pattern as we saw in the main paper. In small markets, the Markov perfect policy leads to higher aggregate value than when no mergers are allowed. The no-mergers policy outperforms the Markov perfect policy provided the market is large enough. However, for the largest markets in the Northeast corner, the Markov perfect policy leads to the same equilibrium as the no-mergers policy because mergers are never consummated.

Figure 25 shows the same AV comparison but relative to the outcome with the static aggregate surplus based policy, $(AV_{MPP}-AV_{Static})/AV_{MPP}$. The figure shows that the Markov perfect policy outperforms the static agrgegate surplus based policy provided the market is large enough.

[38] $\Pr(Entry)_x$ is calculated by weighting the probability of entry in each monopoly stateunder merger policy x by the probability of that state in the all-mergers-allowed equilibrium.

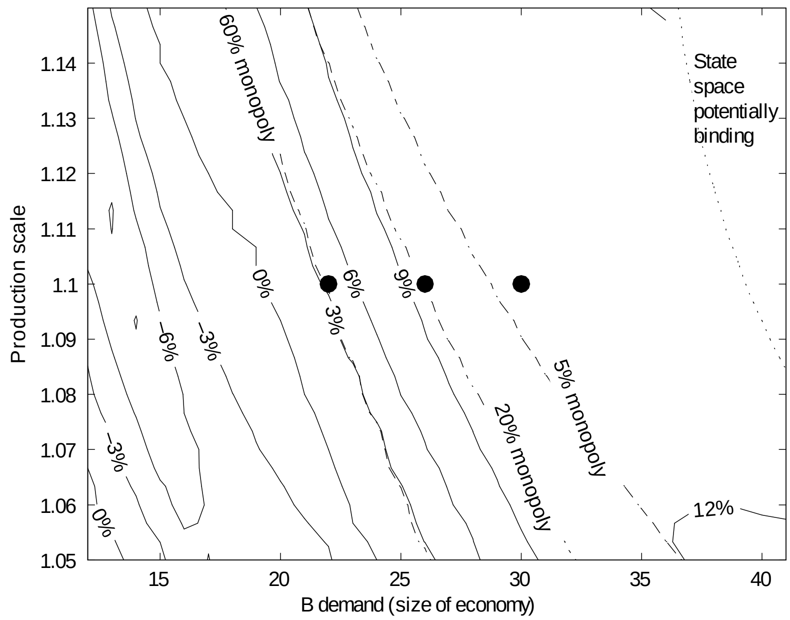

Figure 21: Contour lines of the percentage difference between the steady state aggregate value of the no-mergers and all-mergers-allowed equilibria, $(AV_{No} - AV_{All})/AV_{No}$.

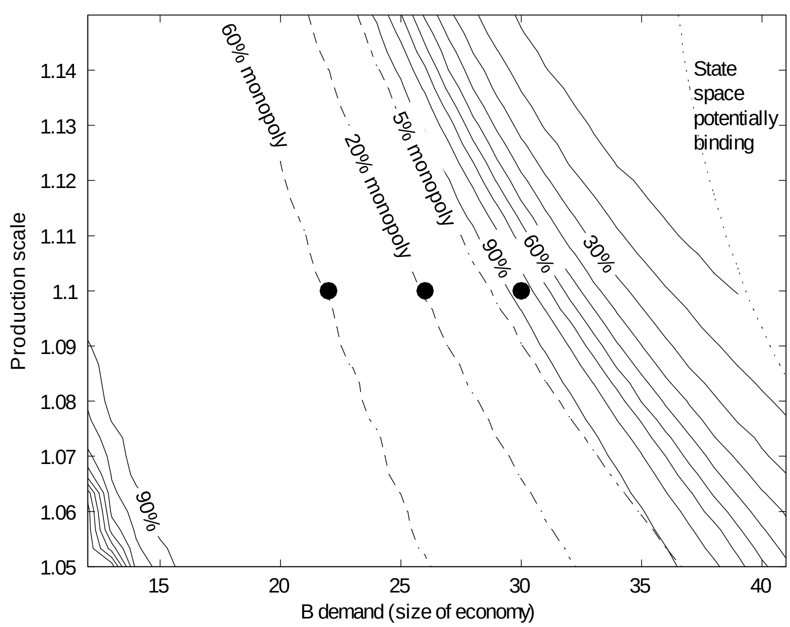

Figure 22: Contour lines of the percentage of entry probabilities between the no-mergers and all-mergers-allowed equilibria, $[\Pr(Entry)_{All} - \Pr(Entry)_{No}]/\Pr(Entry)_{All}]$.

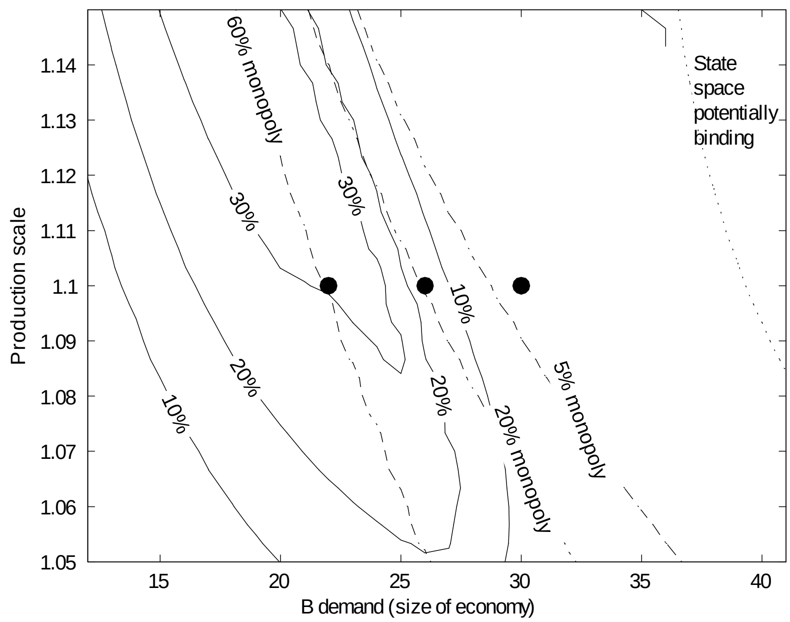

Figure 23: Contour lines of the steady state probability of merger in the MPP-AV equilibria.

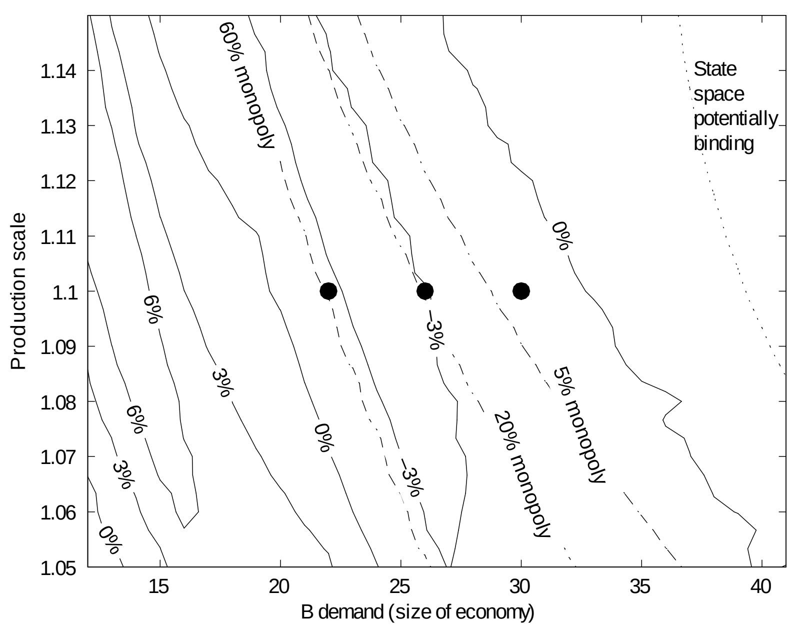

Figure 24: Contour lines of the percentage difference between the steady state aggregate value of the MPP-AV and no-mergers equilibria, $(AV_{MPP} - AV_{No})/AV_{MPP}$.

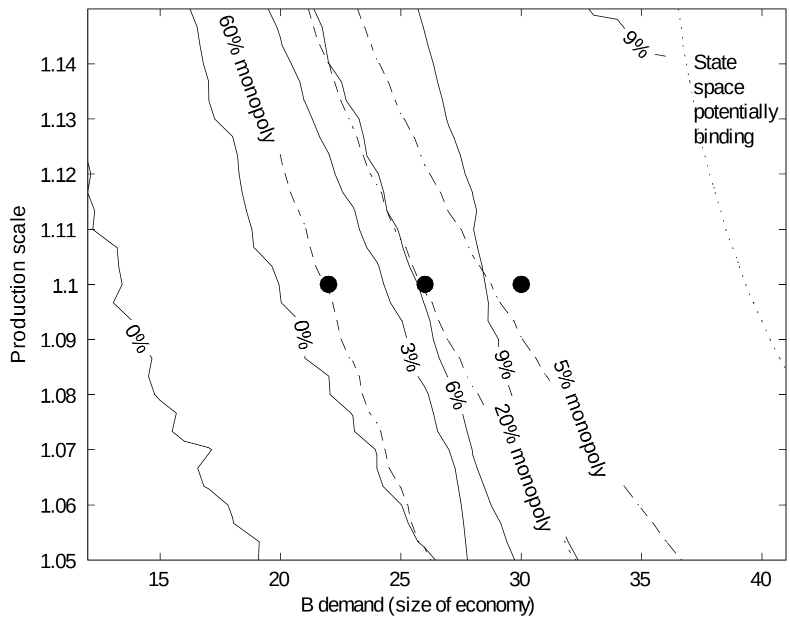

Figure 25: Contour lines of the percentage difference between the steady state aggregate value of the MPP-AV and static-AS policy equilibria, $(AV_{MPP} - AV_{Static})/AV_{MPP}$.

4.7 Outcomes for Various Ranges of Investment Costs

In this section we examine outcomes for a broader range of investment costs for incumbents and entrants, focusing on the intermediate market. Specifically, for each combination of $s \in [1, 3]$ and $s_g \in [0.25, 1]$ we find the level $\underline{c}(s, s_g)$ at which the percentage of time in a monopoly state is 18.6% in the no-mergers-allowed equilibrium. Recall that our baseline intermediate economy corresponds to $(s, s_g) = (3, 1)$ and our efficient entry environment discussed in Section 4.2 corresponds to $(s, s_g) = (1, 0.25)$. These points represent, respectively, the Northeast and Southwest corners of the contour plots below.

Figure 26 shows the average time from monopoly to merger in the all-mergers-allowed equilibrium. As can be seen, there is quicker entry for buyout (i.e. average time from monopoly to merger goes down) when the spread of augmentation draws decreases and the entrant's investments become more efficient relative to the incumbent's. Figure 27 shows the difference in aggregate value between the no-mergers-allowed and all-mergers-allowed equilibria. As noted in Section 4.2, increasing entrant efficiency (moving in the Southwest direction) helps to mitigate the reduction in aggregate value from allowing mergers. Figure 28 shows the probability of merger in the Markov perfect policy. The probability of merger increases as we increase entrant efficiency due to the fact that the antitrust authority allows more mergers when entrants are more efficient.

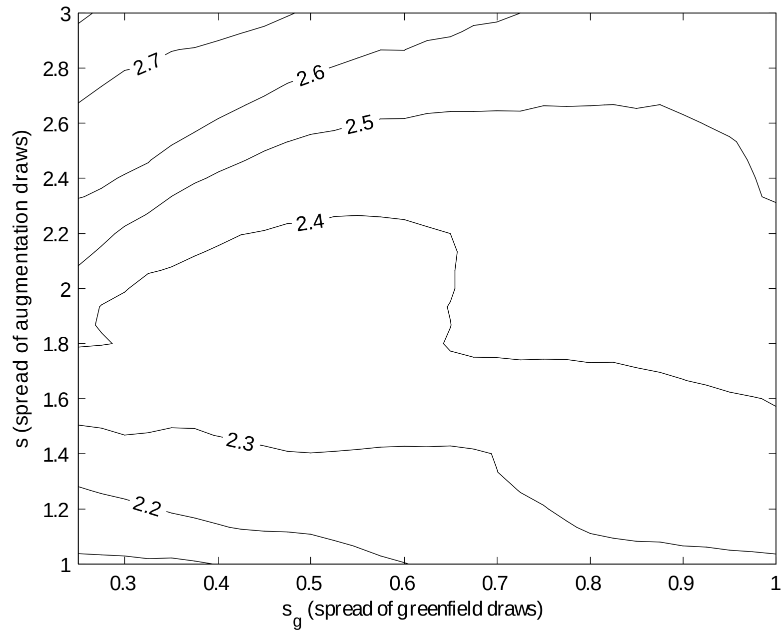

Figure 26: Contour lines showing the steady state weighted average time from monopoly to merger in the all-mergers-allowed equilibrium. The minimum augmentation draw, $\underline{c}$, is set as a function of s and s_g to achieve 18.6% monopoly in the no-mergers equilibrium.

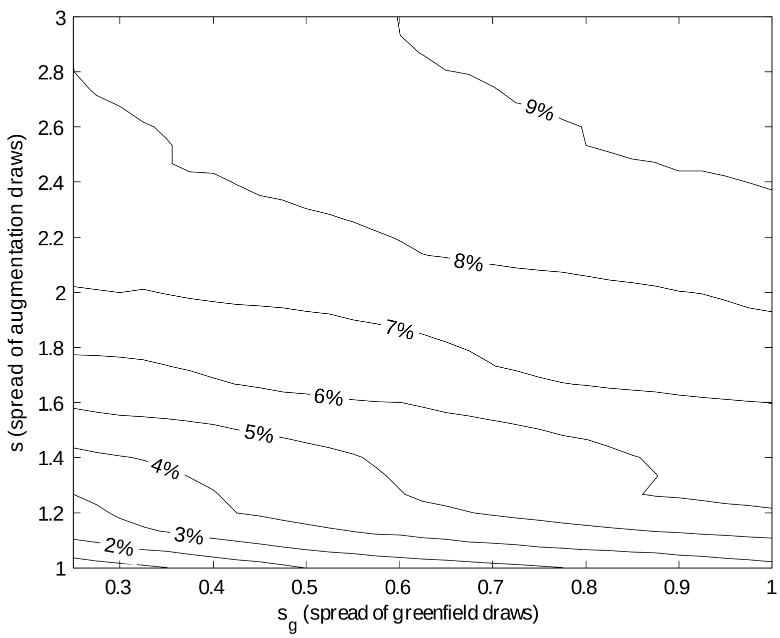

Figure 27: Contour lines of the percentage difference between the steady state aggregate value of the no-mergers and all-mergers-allowed equilibria, $(AV_{No} - AV_{All})/AV_{No}$. The minimum augmentation draw, $\underline{c}$, is set as a function of s and s_g to achieve 18.6% monopoly in the no-mergers equilibrium.

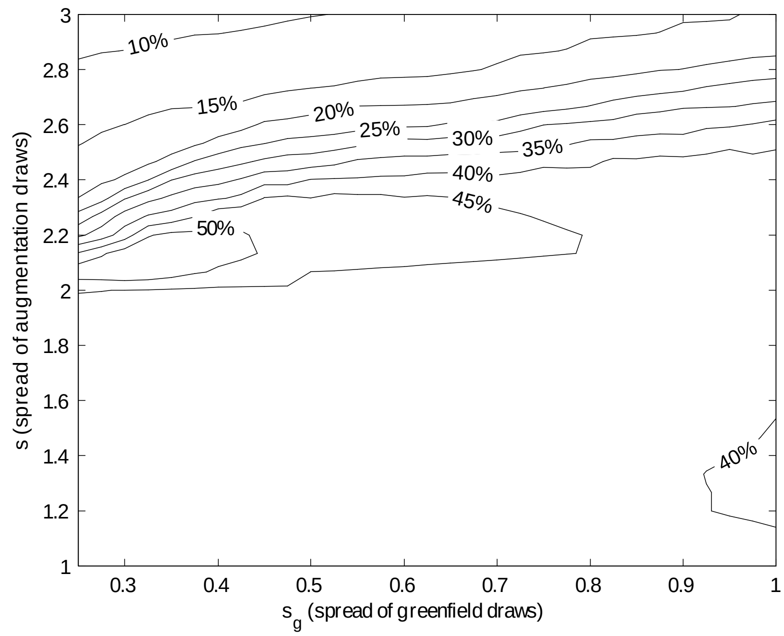

Figure 28: Contour lines of the steady state probability of merger in the MPP-AV equilibria. The minimum augmentation draw, $\underline{c}$, is set as a function of s and s_g to achieve 18.6% monopoly in the no-mergers equilibrium.

5 Multiplicity of Equilibria

Dynamic stochastic games with infinite horizons generally have multiple equilibria when players are patient. Within the context of the Ericson and Pakes (1995) model of computable Markov perfect equilibria, Besanko et al. (2010) develop a homotopy-based method for tracing out paths on the equilibrium manifold and systematically finding points in the parameter space for which multiple equilibria exist.[39] It does not, however, provide a guarantee that it will find all equilibria.

The homotopy technique depends on differentiating the equations that implicitly define the model's equilibria. This requirement makes it, as a practical manner, infeasible to apply to our merger model because a key step in numerically solving for equilibria is a Monte Carlo integration. Numerically differentiating this integral with reasonable accuracy is not possible with the computing power to which we have access. Consequently we implemented a cruder search for multiple equilibria that may fail to find cases of multiplicity that the homotopy technique would find if it were feasible.[40]

The idea is straightforward. Along lines through the parameter space, we calculate sequences of equilibria using the equilibrium values of one equilibrium as the starting points for the next equilibrium computation. For example, one line we search is where the demand parameter $B \in \{11, 12, 13, ..., 40, 41\}$ and all other parameters are fixed. We start the equilibrium calculations from both ends of the line and use the equilibrium values calculated for a particular B as the initial values for calculating the equilibrium at the next B. If equilibrium multiplicity exists along the line, then the equilibrium values for a particular B reached from the line's left end *may* not equal the equilibrium values for that same B reached from the line's right end.

We performed this test for 93 total lines of three types. In the first type of line, we vary the demand parameter $B \in \{11, 12, 13, ..., 40, 41\}$ while fixing the demand parameter $A \in \{1.5, 1.6, 1.7, ..., 4.4, 4.5\}$. All other parameters are fixed at their standard values. In the second type of line, we vary the scale parameter $\theta \in \{1.05, 1.053, 1.057, ..., 1.147, 1.15\}$ while fixing the demand parameter $B \in \{11, 12, 13, ..., 40, 41\}$. In the third type of line, we vary the augmentation cost spread parameter $s \in \{1, 1.13, 1.27, ..., 4.87, 5\}$ while fixing the minimum augmentation cost parameter $\underline{c} \in \{2, 2.1, 2.1, 4.9, 5\}$. For the no-mergers policy we find multiplicity for some parameter values. For the all-mergers-allowed policy and the Markov perfect policy based on the AV criterion we find no multiplicity, but we do find regions in which our algorithm failed to calculate an equilibrium for the Markov perfect policy.[41]

[39] See Borkovsky, Doraszelski, and Kryukov (2010, 2012) for further discussion and illustration of how to use this homotopy technique.

[40] We thank Uli Doraszelski for suggesting this technique to us.

[41] The regions where we cannot find Markov perfect policy equilibria are where the equilibria transition from being almost entirely monopoly in the steady state to being only 85% monopoly. At this point, small changes in the antitrust authority's policy result in large changes in equilibrium behavior so it is difficult to find a Markov

Each instance of multiplicity that we find for the no-mergers policy has a common structure. The distinguishing strategic difference in the two equilibria is the investment behavior at state (1,0) and in some cases state (2,0). Total investment is approximately the same, but in one equilibrium, the incumbent invests more, and in the other equilibrium, the entrant invests more. Each firm wants to have an aggressive investment policy if the other firm has a passive investment policy, and a passive policy if the other firm has an aggressive policy. Almost certainly a third equilibrium exists that is unstable and not computable with our algorithm.[42]

An example of the no-mergers multiplicity is when $(B, A) = (33, 2.8)$ where the investment at state (1,0) is the difference in the equilibria. In "equilibrium 1" the incumbent builds, in expectation, 2.0 units of capital while the entrant builds 1.1 units of capital. In "equilibrium 2" the behavior reverses: the incumbent builds 1.2 units of capital while the entrant builds 2.2 units of capital. Table 9 shows that the performance measures for these two equilibria are quite close since investment behavior in state (1,0) does not have much impact on steady state behavior.

Finally, we point out that we find no multiplicity for our baseline parameters.

Table 9: Performance Measures for Two Pure Equilibria under No Mergers at $(\mathbf{B}, \mathbf{A}) = (\mathbf{33}, \mathbf{2.8})$

Performance measure[43]	Equil. 1	Equil. 2
Avg. Consumer Value	30.9	31.0
Avg. Incumbent Value	68.3	68.3
Avg. Aggregate Value	99.2	99.4
Avg. Price	2.20	2.20
Avg. Quantity	19.9	19.9
Avg. Total Capital	6.4	6.4
% in Monopoly	79.3%	78.9%
$\% \min\{K_1, K_2\} \geq 2$	19.3%	19.7%

6 Additional Tables and Figures Referenced in the Main Paper

Table 10 displays the equilibrium statistics of our primary duopoly market parameterization from the main paper, only allowing for a third firm. Comparing to Table 2 in the main paper, it can be seen that this market is a "natural duopoly" in that even when three firms are allowed, the no-mergers steady state measures are very similar to when only two firms are

perfect policy which is the best response to the firm behavior it induces. We believe there are equilbria in this region but that they are very unstable and our algorithm can not find them.

[42] See Besanko et al. (2010, section 3.2) for a discussion of the inability of Pakes-McGuire-like algorithms to compute unstable equilibria.

[43] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ which are at the Cournot competition stage.

allowed. One can also see that our insights from the study merger policy in the duopoly case carry over to the triopoly case.

Table 10: Performance Measures for the (A=3,B=26) **Market under Various Policies (Allowing a Third Firm)**

Performance Measure[44]	*No-Mergers/ Static-CS/ MPP-CV*	*All-Mergers*	*Static-AS*	*MPP-AV*
Avg. Consumer Value	48.3	38.0	38.1	47.1
Avg. Incumbent Value	69.3	68.9	69.4	69.6
Avg. Entrant Value	0.0	0.8	0.8	0.1
Avg. Blocking Cost	0.0	0.0	0.0	-0.0
Avg. Aggregate Value	117.6	107.7	108.2	116.7
Avg. Price	2.15	2.24	2.24	2.16
Avg. Quantity	22.2	19.8	19.8	21.9
Avg. Total Capital	8.0	7.6	7.6	8.1
Merger Frequency	0.0%	50.8%	50.2%	13.3%
% in Monopoly	18.0%	85.4%	86.9%	31.3%
% in Duopoly	81.5%	14.6%	13.1%	68.7%
State (0,0,0) CV	32.7	26.0	26.2	29.9
State (0,0,0) AV	34.7	31.9	32.1	34.9

Figure 29 shows the difference between the private and social incentives to invest when all mergers are allowed. The socially insufficient incentive for incumbent firms to invest and the socially excessive incentive for entrants to invest results in the detrimental effects of entry for buyout seen in the main paper.

[44] All values are ex ante (beginning-of-period) values except % in Monopoly and % in Duopoly (showing the percentages of the time that industry capital is in each type of state) which are at the Cournot competition stage. "No-mergers" and "All-Mergers" refer to the no-mergers-allowed and all-mergers-allowed policies, respectively. "Static-CS" and "Static-AS" refer, respectively, to the equilibria under the optimal static consumer surplus-based and aggregate surplus-based merger policies. "MPP-CV" and "MPP-AV" refer, respectively, to the equilibria when the antitrust authority cannot commit (resulting in a Markov perfect policy) under consumer value and aggregate value welfare criteria. "State (0,0) CV" and "State (0,0) AV" are the values of CV and AV, respectively, for a new industry that starts with no capital.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	(0.6)	(0.2)	0.5	1.2	2.2	3.0	3.5	3.9	4.2	4.4	4.6
k1=1	(1.2)	(1.3)	0.3	0.9	1.4	1.6	1.9	2.1	2.2	2.3	2.3
k1=2	(1.7)	(1.3)	(0.4)	0.1	0.4	0.7	0.9	1.0	1.2	1.3	1.3
k1=3	(1.4)	(1.2)	(0.6)	(0.2)	0.0	0.2	0.4	0.5	0.6	0.7	0.8
k1=4	(1.3)	(1.1)	(0.7)	(0.4)	(0.2)	0.0	0.1	0.3	0.3	0.4	0.4
k1=5	(1.3)	(1.0)	(0.7)	(0.4)	(0.3)	(0.1)	(0.0)	0.1	0.2	0.2	0.3
k1=6	(1.2)	(0.9)	(0.7)	(0.5)	(0.3)	(0.2)	(0.1)	(0.0)	0.1	0.1	0.1
k1=7	(1.1)	(0.9)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.0)	0.0	0.1
k1=8	(1.0)	(0.8)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.0)	0.0
k1=9	(0.9)	(0.8)	(0.7)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.0)
k1=10	(1.0)	(0.9)	(0.7)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)

Figure 29: Private incentive of the row firm (firm 1) to invest minus the social incentive for the row firm to invest in the intermediate market with all mergers allowed. Negative numbers are in parentheses.

References

[1] Besanko, D., U. Doraszelski, Y. Kryukov, and M. Satterthwaite. (2010). "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics." *Econometrica* 78: 453-508.

[2] Besanko, D., and D. F. Spulber (1993), "Contested Mergers and Equilibrium Antitrust Policy," *Journal of Law, Economics, and Organization* 9: 1-29.

[3] Borkovsky, R. N., U. Doraszelski, and Y. Kryukov. (2010), "A User's Guide to Solving Dynamic Stochastic Games Using the Homotopy Method," *Operations Research* 58: 1116-1132.

[4] Borkovsky, R. N., U. Doraszelski, and Y. Kryukov. (2012), "A Dynamic Quality Ladder Duopoly with Entry and Exit: Exploring the Equilibrium Correspondence Using the Homotopy Method," *Quantitative Marketing & Economics* 10: 197-229.

[5] Ericson, R. and A. Pakes (1995), "Markov-perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies* 62: 53-82.

[6] Lyons, B. (2002), "Could Politicians be More Right than Economists? A Theory of Merger Standards," *CCP Working Paper* 02-01, University of East Anglia.

[7] Mankiw, N. G. and M. D. Whinston (1986), "Free Entry and Social Inefficiency," *RAND Journal of Economics* 17: 48-58.

[8] Whinston, M. D. (2007), "Antitrust Policy toward Horizontal Mergers," in: *Handbook of Industrial Organization*, vol. 3, eds. M. Armstrong and R. H. Porter, Amsterdam: North Holland.