Optimal Merger Remedies*

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Abstract

We develop a framework to study horizontal mergers when the parties can propose remedies to an antitrust authority. Remedies are modeled as asset divestitures, which make the firm receiving the assets more efficient at the expense of the merged firm. We consider both the case where the merger affects a single market and where it affects multiple markets. Solving for the merging firms' optimal proposal, we investigate when it involves remedies—and if so, which assets should be divested, and to whom, and how this depends on market characteristics such as the level of competitiveness.

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JEL classification: L13, L40, D43

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1 Introduction

Antitrust authorities regularly clear mergers subject to the implementation of remedies. For example in the U.S., more than 60% of the mergers that were challenged by the authorities between 2003 and 2012 were later approved subject to remedies (Kwoka, 2014). Similarly, in Phase 2 investigations between 1990 and 2024, the European Commission was almost five times less likely to prohibit a merger than to clear it subject to remedies.[1] Typically remedies are "structural", in that they involve the merging parties divesting assets; these assets may include manufacturing facilities, personnel, intellectual property and data.[2,3] The aim of these asset divestitures is to strengthen existing competitors, or to facilitate entry of new firms, and thereby prevent a merger from harming consumers.

Surprisingly, despite their importance, the existing literature on merger policy largely ignores the possibility of remedies. Allowing for remedies raises several natural questions. For instance, to what extent can remedies substitute for efficiencies—marginal cost savings which, absent remedies, would be required for a merger to not harm consumers? And how do market characteristics affect whether remedies are required and, if so, which assets the merger partners should optimally divest, and to whom? Moreover, a merger will often affect several different markets; for example, a supermarket merger impacts consumers in different geographic markets, while a pharmaceutical merger impacts consumers in different product (drug) markets. To the extent that an authority is willing to balance gains and losses across different markets, how does this affect the remedies optimally proposed by the merger partners? For example, is it better to divest few assets in many markets, or many assets in few markets?

In this paper, we offer a tractable framework to study optimal merger remedies. In our model, a merger between two firms may generate synergies, as well as a set of assets that could be feasibly divested to another firm. (This asset-receiving outsider may or may

[1] In particular, there were 151 approvals subject to remedies and 33 prohibitions; see `https://www.competition-policy.ec.europa.eu/mergers/statistics_en`. If Phase 1 investigations are included as well, then between 1990 and 2014 there were 343 approvals subject to remedies and only 19 prohibitions (Affeldt, Duso, and Szücs, 2018).

[2] For example, the 2015 merger between the supermarket chains Albertsons and Safeway was approved subject to the parties divesting 168 stores; see `https://www.ftc.gov/news-events/news/press-releases/2015/01/ftc-requires-albertsons-safeway-sell-168-stores-condition-merger`. The 2020 merger between the mobile phone operators T-Mobile and Sprint was approved subject to the divestiture of, among other things, spectrum and cell towers; see `https://www.justice.gov/archives/opa/pr/justice-department-settles-t-mobile-and-sprint-their-proposed-merger-requiring-package`. In 2007 the merger between the UK waste disposal operators Stericycle and Sterile Technologies Group was approved subject to the divestiture of, among other things, plants and employees; see CMA (2023). In 2009 the merger between the herbicide suppliers Nufarm and AH Marks was approved subject to the transfer of data and intellectual property; see CMA (2023).

[3] This is contrast to "behavioral" remedies, which involve the merging parties making commitments about future conduct. Structural remedies are usually preferred because they are easier to implement and do not require monitoring (see, e.g., CMA, 2018).

not already be selling in the market.) We impose minimal assumptions on the effects of asset divestitures; all we assume is that having more assets makes a firm more productive. Specifically, as the merged firm divests more assets, its marginal cost increases and/or its product quality decreases; similarly, as an outsider receives more assets, its marginal cost decreases and/or its product quality increases. In our baseline analysis divesting assets does not generate any revenue for the merged firm, but we relax this later on. We consider the following game. First, the merger partners decide which assets (if any) to divest and to whom, and then propose this to an antitrust authority. Second, using a consumer surplus standard, the authority accepts or blocks this proposal. Finally, firms compete in a Cournot fashion.

We begin by analyzing a merger which affects a single market. First, we characterize the set of mergers and associated divestitures that would be blocked by the antitrust authority, based on their unilateral effects. We demonstrate that as the market becomes less competitive, this set increases. Second, we examine the optimal proposal by the merger partners. We show that if a merger without remedies would not be blocked, then this is what will be proposed. If instead such a merger would be blocked, then if the merger partners choose to propose a merger, it must leave consumer surplus unchanged. We show that for the parties to optimally propose such a merger, it must generate substantial synergies (in a way that we make precise in the text). That is, remedies cannot fully substitute for merger-induced efficiencies. Moreover, we show that as the market becomes less competitive, it is less likely that a merger is proposed and approved—but conditional on one being proposed and approved, it is more likely to entail divesting assets to a firm that was previously not producing anything, thereby creating a new competitor.

We then turn to a merger which affects multiple markets, as is the case for many mergers.[4] Many of the insights from our single-market analysis carry over to this setting, if the antitrust authority adopts a policy that blocks a merger that harms consumers in at least one market. However, such a "market-by-market" policy may not always be appropriate. For instance, if the same consumers are present in many of the affected markets (Crane, 2015), or "if a merger with massive competitive benefits would be made unlawful by unfixable anticompetitive effects in a single tiny market" (Werden, 2017), it may be more appropriate to balance gains in some markets against losses in others. Indeed, this is consistent with antitrust policy in some countries.[5] We therefore focus on the benchmark case where the antitrust authority

[4]For example, among the mergers investigated by the European Commission between 1990 and 2014, the number of affected markets ranged from 1 to 245 with an average of 6 (Affeldt, Duso, and Szücs, 2018).

[5]For example, in the UK, the CMA can clear a merger at Phase 1 if a significant lessening of competition (SLC) and its adverse effects in one market are outweighed by consumer gains "in any market in the United Kingdom (whether or not in the market(s) in which the SLC has occurred or may occur)." (CMA, 2018) Similarly, in Switzerland, a merger that creates or strengthens a dominant position in one market is only blocked if it "does not improve the conditions of competition in another market such that the harmful effects of the dominant position [in the initial market] can be outweighed." (Fedlex, 2023) See also OECD (2016)

clears a merger (and associated remedies) provided that consumer surplus aggregated across all markets is not reduced.

Within this multimarket setting, we show that, conceptually, one can view the merger partners as choosing (through the proposed remedies) a post-merger consumer surplus level and associated profit in each market. In particular, as consumer surplus in any given market is increased, the merged firm's profit in that market decreases. We introduce the concept of a *remedies exchange rate*, which measures how many dollars the merged firm must give up in a market when consumer surplus is increased by one dollar through divestitures. We demonstrate that, under certain conditions, this exchange rate improves as the induced level of consumer surplus increases. As a result, the optimal merger and remedies proposal is "bang bang": it involves no asset divestitures in some markets, and maximal asset divestitures in all other markets. We then show that as a market is made less competitive, the merged firm may be more likely to propose no asset divestitures in that market—in contrast to the prediction of our single-market analysis.

We show our results are robust to two extensions. First, we allow for negotiations between the merger partners and the antitrust authority, concerning which remedies should be implemented. Second, we allow for the merged firm to receive revenues from its divested assets. These revenues are determined endogenously through a bargaining process between the merger partners and the asset-receiving outsider.

Related Literature Much of the literature on the unilateral effects of mergers focuses on a single market, and assumes that the authority must either accept or reject the merger, without allowing for remedies. At the heart of this literature is the Williamson (1968) trade-off, whereby a merger can raise prices due to a market power effect, or lower them due to efficiencies. Farrell and Shapiro (1990) formalize this trade-off in a homogeneous goods Cournot model, and show that for a merger not to harm consumers, it must generate efficiencies in the form of lower marginal costs.[6] Moreover, Nocke and Whinston (2010) show that any merger that does not harm consumers is profitable for the merger partners.

There is a small theoretical literature on structural merger remedies. In a homogeneous goods setting, Vergé (2010), Vasconcelos (2010) and Dertwinkel-Kalt and Wey (2015) assume that firms have the same cost functions, and impose strong functional form assumptions on those cost functions. Vergé (2010) shows, for example, that with three firms and no merger synergies, a merger must harm consumers even accounting for asset divestitures. In a similar vein, focusing on four symmetric firms, Vasconcelos (2010) shows that in equilibrium remedies are never used. Dertwinkel-Kalt and Wey (2015) show that, as the level of merger-induced

for discussion of the so-called "balancing clause" in German merger law. Werden (2017) points out that even the old 2010 U.S. merger guidelines allowed the market-by-market rule to be broken in certain circumstances.

[6]Using this homogeneous goods Cournot model, Reisinger and Zenger (2025) relate the price effect of a merger to the level of such efficiencies, as well as to market shares and the number of active firms.

synergies decreases, it is less likely that a merger is approvable even with remedies. Moreover, they provide conditions under which, conditional on remedies being used, the pre- and post-merger equilibrium prices are the same. In contrast to these papers, we allow for arbitrary firm heterogeneity and merger-induced synergies, and impose only weak restrictions on how divestitures affect costs. In addition, we consider not only a single-market setting, but we also study remedies when a merger affects multiple markets.[7,8]

While in our paper a merger may affect different product or geographic markets, Johnson and Rhodes (2021) and Nocke and Schutz (forthcoming) consider mergers between differentiated multiproduct firms within a single market. The former studies mergers between vertically differentiated firms in a Cournot setting, and finds for example that mergers without synergies can raise consumer surplus, but only when certain necessary observable conditions on the pre-merger industry structure are satisfied. The latter studies mergers between multiproduct firms in a differentiated Bertrand setting, and among other things shows that absent synergies the merger's harm to consumers is proportional to the naively-computed change in the Herfindahl index.[9] However none of these papers consider merger remedies.

The rest of the paper proceeds as follows. Section 2 studies optimal remedies when a merger affects a single market, while Section 3 considers the case where a merger affects multiple markets. Section 4 extends our baseline analysis to allow for bargaining between the antitrust authority and the merger partners, and for the merger partners to earn revenue from divesting assets. Section 5 concludes with a discussion of future avenues for research. All omitted proofs are available in the Appendix.

2 Single-Market Analysis

We first consider the case in which a merger affects only a single market.

2.1 The Setting

There is a set $\mathcal{N}$ of firms producing a homogeneous good with constant returns and competing in a Cournot fashion. The marginal cost of firm i is denoted $c_i \geq 0$. Inverse demand is given by $P(Q)$, where Q denotes total output. We impose standard assumptions on demand ensuring that there exists a unique Nash equilibrium in quantities for any vector of marginal costs:

[7] In a different line of research, Cosnita-Langlais and Tropeano (2012) examine whether merger partners that are privately informed about the magnitude of any synergies, can credibly signal that information through the remedies that they propose to the antitrust authority.

[8] There is also a small empirical literature on merger remedies, including Tenn and Yun (2011), Friberg and Romahn (2015), Alviarez, Head, and Mayer (2025). These papers find that asset divestitures tend to mitigate price increases that would otherwise result from horizontal mergers.

[9] Along these lines, Nocke and Whinston (2022) show that the synergies required for a merger not to harm consumers are increasing in the naively-computed Herfindahl index.

$P(0) > 0$, $\lim_{Q\to\infty} P(Q) = 0$, and for all Q such that $P(Q) > 0$, $P'(Q) < 0$ and $\sigma(Q) < 1$, where $\sigma(Q) \equiv -QP''(Q)/P'(Q)$ is the curvature of inverse demand.

We consider a potential merger between two *active* firms, i.e., firms that produce a strictly positive output pre merger. The set of (potential) merger partners is exogenous and denoted $\mathcal{M} \subset \mathcal{N}$. The set of non-merging outsiders is denoted $\mathcal{O} \equiv \mathcal{N} \setminus \mathcal{M}$; not all of these outsiders need be active pre merger, and the set of active outsiders may be affected by the merger.

The merger partners have a t-dimensional vector of assets, $K \in \mathbb{R}^t_+$, that could feasibly be divested to a single rival firm.[10] We treat the amount of each asset as a continuous variable. A remedy is denoted (k, i), where $k \leq K$ is the vector of assets being divested and $i \in \mathcal{O}$ is the asset-receiving outsider (which may or may not be active pre merger). A merger without remedies is denoted $(0, i)$, for any $i \in \mathcal{O}$.[11] Let $\mathcal{F} \subseteq ([0, K], \mathcal{O})$ denote the set of feasible mergers and associated divestitures. Following a divestiture $(k, i) \in \mathcal{F}$, the merged firm's marginal cost is $\overline{c}_M(-k)$ and the receiving outsider's marginal cost is $\overline{c}_i(k)$. The merger has no effect on any outsider's marginal cost, except through divestitures; that is, $\overline{c}_i(0) = c_i$. We assume that post-merger marginal costs $\overline{c}_M(-k)$ and $\overline{c}_i(k)$ are weakly decreasing (in each argument) and $\mathcal{C}^2$. For now, we assume that the merger partners receive no revenue from divesting assets (but we relax this assumption later on).

We assume that the antitrust authority applies a consumer surplus standard. It therefore blocks the merger and associated remedy (k, i) if and only if the resulting post-merger consumer surplus is strictly lower than the pre-merger level. The game proceeds as follows: The merger partners decide whether or not to propose their merger and any remedy (k, i); if so, the authority then accepts or blocks the proposal; given the resulting market structure, firms compete in a Cournot fashion. Throughout, we focus on equilibria in which a merger is proposed if and only if it would not be blocked and is strictly profitable.[12]

Remark 1. *The homogeneous goods Cournot model is isomorphic to an alternative model of quantity competition where firms offer different qualities and prices adjust accordingly. In the Appendix we consider two variants:*

1. *The inverse demand of firm i is $w_i P(\sum_{j\in\mathcal{N}} w_j \tilde{q}_j)$, where $w_i > 0$ is firm i's quality and $\tilde{q}_i \geq 0$ is its output, and firm i's marginal cost is denoted $\tilde{c}_i$. By redefining $q_i \equiv w_i \tilde{q}_i$ and $c_i \equiv \tilde{c}_i / w_i$, equilibrium outcomes and welfare measures are exactly as in the above homogeneous goods Cournot model.*

2. *The inverse demand of firm i is $P(\sum_{j\in\mathcal{N}} q_j) + w_i$, where w_i is firm i's quality and $q_i \geq 0$ is its output, and firm i's marginal cost is denoted $\tilde{c}_i > w_i$. By redefining*

[10] Different dimensions of K could be thought of as representing different "types" of asset, e.g., patents, data, machines, personnel etc.

[11] Note that, conditional on $k = 0$, the identity of i is irrelevant for the equilibrium outcome.

[12] Equivalently, we could assume that there is an arbitrarily small merger proposal cost.

$c_i \equiv \tilde{c}_i - w_i$, equilibrium outcomes and welfare measures are again exactly as in the above homogeneous goods Cournot model.

In light of the above remark, efficiencies in our model can also be interpreted as the merger enabling the insiders to offer a higher quality product; similarly, asset divestitures can also be interpreted as enabling the asset-receiving outsider to produce higher quality.

2.2 Preliminary Analysis

We begin by briefly recapping standard analysis of Cournot oligopoly, using an aggregative games approach (see, e.g., Nocke, forthcoming). To ease notation we drop the firm subscript.

A firm with marginal cost c chooses output q to maximize its profit $q[P(Q) - c]$. The firm's *output fitting-in function* $r(Q; c)$ is the output level that solves its first-order condition:

$$r(Q; c) = \frac{\max\{P(Q) - c, 0\}}{-P'(Q)}. \quad (1)$$

We can then use the output fitting-in function to rewrite the expression for firm profit, and thereby obtain the firm's *profit fitting-in function*:

$$\pi(Q; c) = -\left(r(Q; c)\right)^2 P'(Q). \quad (2)$$

Notice that given our assumptions on market demand $P(Q)$, both $r(Q; c)$ and $\pi(Q; c)$ are weakly decreasing in Q and c, and strictly so whenever $c < P(Q)$.

The pre-merger equilibrium total output Q^* is then the unique Q which solves

$$Q = \sum_{j \in \mathcal{N}} r(Q; c_j). \quad (3)$$

(Note that for any inactive firm j, $r(Q^*; c_j) = 0$.) Following a merger with remedy (k, i), the equilibrium total output $\overline{Q}^*(k, i)$ is the unique Q which solves

$$Q = r\left(Q; \overline{c}_M(-k)\right) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q; c_j) + r(Q; \overline{c}_i(k)). \quad (4)$$

(Note that the merger and associated remedy may induce some firms to become active or inactive.) Consumer surplus as a function of total output Q is given by

$$v(Q) = \int_0^Q [P(z) - P(Q)]dz, \quad (5)$$

and is thus strictly increasing in Q.

Comparative statics are well behaved. The following result is standard and its proof is therefore omitted.

Lemma 1. *A decrease in the marginal cost of any active firm j leads to a strict increase in equilibrium total output and in firm j's equilibrium profit, and a strict decrease in the equilibrium profit of any other active firm $l \neq j$.*

2.3 Equilibrium Analysis

We now turn to the equilibrium analysis of our two-stage game. We proceed by backward induction: we first characterize the set of merger proposals that would be approved by the antitrust authority, and then derive the merger partners' optimal proposal.

2.3.1 The Authority's Problem

We first introduce some terminology. The *acceptance set* $\mathcal{A} \subseteq \mathcal{F}$ denotes the set of mergers that weakly raise consumer surplus relative to the pre-merger situation; any merger $(k, i) \in \mathcal{A}$ would therefore be approved by the authority. Call $\mathcal{B} = \mathcal{F} \setminus \mathcal{A}$ the authority's *blocking set.*

As a preliminary step, consider a merger without divestitures, $(0, i)$.

Lemma 2. *There exists a $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$ such that a merger without divestitures lies in the acceptance set if and only if $\overline{c}_M(0) \leq \widehat{c}_M$.*

Hence, as was pointed out by Farrell and Shapiro (1990), for a merger among active firms to be approved, it must involve synergies in that $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$.[13] Now consider the more interesting case where $\overline{c}_M(0) > \widehat{c}_M$, so that a merger without divestitures would not be approved. Our next result gives a condition for the merger to be approvable with remedies.

Proposition 1. *Suppose $\overline{c}_M(0) > \widehat{c}_M$, such that a merger without divestitures would be blocked. The acceptance set $\mathcal{A}$ is non-empty if and only if there exists a (k, i) such that*

$$\underbrace{P(Q^*) - \max_{j \in \mathcal{M}} c_j}_{\textit{market power effect}} \leq \underbrace{\min_{j \in \mathcal{M}} c_j - \min\{P(Q^*), \overline{c}_M(-k)\}}_{\textit{efficiency effect (merged firm)}} + \underbrace{\min\{P(Q^*), c_i\} - \overline{c}_i(k)}_{\textit{efficiency effect (outsider)}}. \tag{6}$$

Intuitively, a merger is acceptable to the authority if and only if its market power effect is dominated by its efficiency effect, both evaluated at the pre-merger price $P(Q^*)$. The left-hand side of equation (6) captures the market power effect of a merger. One can view a merger as the shuttering of the less efficient merger partner, and from equation (1) that firm's pre-merger output was proportional to its margin $P(Q^*) - \max_{j \in \mathcal{M}} c_j$. The right-hand side of (6) captures possible efficiencies due to the merger. The first term is for the merged

[13] Note that $\widehat{c}_M$ could be negative, in which case a merger without divestitures is never approved.

firm itself: the more efficient merger partner becomes the merged firm, and its pre-merger cost $\min_{j\in\mathcal{M}} c_j$ is replaced by $\bar{c}_M(-k)$. However, if the merged firm becomes inactive (e.g., due to the divestiture), it behaves at Q^* *as if* its marginal cost is $P(Q^*)$ rather than $\bar{c}_M(-k)$. The second term is for the asset-receiving outsider: it receives assets which reduce its cost from c_i to $\bar{c}_i(k)$. However, if this firm was previously inactive, it behaved *as if* its marginal cost was $P(Q^*)$ instead of c_i. Importantly, as we show in the proof of Proposition 1, given that by assumption a merger without divestitures would be blocked, a necessary condition for merger (k, i) not to harm consumers is that asset-receiving outsider i is active after the merger, i.e., $\bar{c}_i(k) < P(Q^*)$.

Notice from equation (6) that remedies introduce an inherent trade-off: as more assets are divested from the merged firm to an outsider firm, the efficiency effect becomes larger for the asset-receiving firm but smaller for the merged firm. Therefore divesting more assets does not necessarily make a merger more likely to be approved.

We now study how the blocking set depends on the competitiveness of the market, which we define as follows:

Definition 1. *A market becomes less competitive if the pre-merger price $P(Q^*)$ increases due to i) a change in demand or ii) an upward vertical shift in $\bar{c}_j(\cdot)$ for any active firm $j \in \mathcal{O}$.*

Thus we allow for two ways in which a market can become less competitive. The first is *any* change in the demand curve $P(Q)$ that leads to a higher pre-merger price. The second is an increase in some active outsider's pre-merger marginal cost $c_j \equiv \bar{c}_j(0)$, which from Lemma 1 necessarily raises the pre-merger equilibrium price; to ensure that the impact of any divestiture is unaffected, we require that the whole marginal cost curve $\bar{c}_j(\cdot)$ is shifted up vertically.

Lemma 3. *Suppose merger (k, i) is in the blocking set $\mathcal{B}$. Then, as the market becomes less competitive, that same merger remains in the blocking set.*

As the market becomes less competitive, the blocking set increases in the set order—meaning that any given merger is "more likely" to be blocked. Intuitively, an increase in the pre-merger price $P(Q^*)$ raises each merger partner's pre-merger profit margin and thereby the market power effect of the merger. In the proof we show that this (weakly) dominates any possible change in the efficiency effect of the merger.[14] Our next result provides conditions under which the increase in the market power effect is *exactly* offset by an increase in the efficiency effect.

Lemma 4. *Suppose $\bar{c}_M(0) > \widehat{c}_M$. Suppose merger (k, i) is in the acceptance set, where $\bar{c}_M(-k) < P(Q^*)$ and where i is inactive pre merger. Then (k, i) remains in the acceptance set as the market becomes less competitive, provided that firm i is still inactive pre merger.*

[14] Nocke and Whinston (2010) obtained the same result in the special case without divestitures.

This lemma focuses on the case where divestitures are required to approve the merger (i.e., $\overline{c}_M(0) > \widehat{c}_M$), and considers a divestiture such that the merged firm would be active at the pre-merger price (i.e., $\overline{c}_M(-k) < P(Q^*)$). As we show in the next subsection, both assumptions must hold at an optimal merger proposal that entails positive asset divestitures. The first assumption implies that equation (6) gives a necessary and sufficient condition for merger (k, i) to not harm consumers. This assumption also implies that the asset-receiving outsider i must become active post merger, as otherwise (k, i) would not be in the acceptance set. Hence, from equation (6), the efficiency effect of the merger on the outsider is $P(Q^*) - \overline{c}_i(k)$. Meanwhile the second assumption implies that the efficiency effect on the merged firm is $\min_{j\in\mathcal{M}} c_j - \overline{c}_M(-k)$, which is independent of $P(Q^*)$. The result in the lemma then obtains, because as $P(Q^*)$ increases, the increase in the combined efficiency effect of the merger exactly offsets the increase in the market power effect.

From Lemmas 3 and 4, there is a sense in which, as the market becomes less competitive, for a merger to be approved it is more likely that assets need to be divested to a previously-inactive firm. This is because from Lemma 3 the blocking set expands, but from Lemma 4 it does not for those mergers where assets are divested to a previously-inactive firm.

2.3.2 The Merger Partners' Problem

Folding backwards, we now analyze the merger partners' proposal decision. Clearly, if all feasible mergers are in the blocking set, so that $\mathcal{A} = \emptyset$, then no merger is proposed. We now focus on the more interesting case in which not all feasible mergers would be blocked.

Conditional on proposing a merger, the merger partners seek to

$$\max_{(k,i)} \pi(\overline{Q}^*(k, i); \overline{c}_M(-k)) \quad \text{s.t.} \quad v(\overline{Q}^*(k, i)) \geq v(Q^*),$$

that is, choose a merger and remedy (k, i) to maximize post-merger profit, subject to the merger weakly increasing consumer surplus and hence being in the acceptance set $\mathcal{A}$. A merger is then proposed if and only if it yields strictly more profit than the status quo profit $\sum_{j\in\mathcal{M}} \pi(Q^*; c_j)$ from not merging. In order to characterize the solution to this optimization problem, it is useful to introduce the *synergies curve* and the *divestitures curve*.

The synergies curve. The synergies curve represents post-merger outcomes in the absence of divestitures. Specifically, it depicts possible combinations of consumer surplus $v(\overline{Q}^*(0, i))$ and merged firm profit $\pi(\overline{Q}^*(0, i); \overline{c}_M(0))$ arising from different values of the post-merger marginal cost $\overline{c}_M(0)$ (or equivalently, different values of the synergy). As illustrated by the two panels in Figure 1, the synergies curve is upward-sloping because, from Lemma 1, both merged-firm profit and consumer surplus are decreasing in $\overline{c}_M(0)$. Each panel also depicts with a star the pre-merger outcome (v^*, π^*), where $v^* \equiv v(Q^*)$ and $\pi^* \equiv \sum_{j\in\mathcal{M}} \pi(Q^*; c_j)$, as

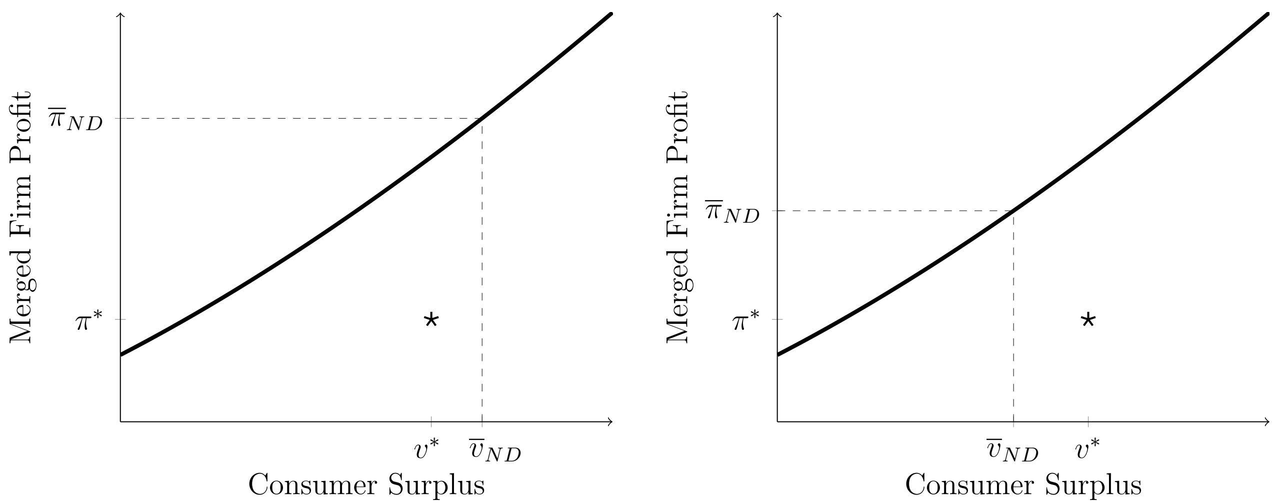

Figure 1: The synergies curve. In the left panel the realized synergy is large enough that a merger without divestitures is approvable, whereas in the right panel the opposite holds.

well as the "no-divestiture" outcome $(\overline{v}_{ND}, \overline{\pi}_{ND})$ corresponding to one specific value of $\overline{c}_M(0)$. The left panel illustrates a case where $\overline{c}_M(0)$ is small (i.e., the synergy is large) and hence the merger would be approved even absent divestitures; the right panel illustrates a case where the opposite holds. Note that in the two panels, the pre-merger outcome lies below the synergies curve; as shown by Nocke and Whinston (2010), a merger without divestitures that leaves consumer surplus unchanged is necessarily profitable.

The divestitures curve. The divestitures curve represents post-merger outcomes in the presence of divestitures. To derive this curve, we first introduce the concept of a *conditional divestitures curve* $d_M(v; i)$. Let $\overline{v}_{\max}(i) \equiv \max_k v(\overline{Q}^*(k, i))$ denote the maximum consumer surplus that can be achieved by divesting assets to outsider i. We then define for each $v \in [\overline{v}_{ND}, \overline{v}_{\max}(i)]$,

$$d_M(v; i) \equiv \max_k \pi(\overline{Q}^*(k, i); \overline{c}_M(-k)) \quad \text{s.t.} \quad v(\overline{Q}^*(k, i)) = v. \tag{7}$$

Fixing the identity of the asset-receiving outsider $i \in \mathcal{O}$, the conditional divestitures curve $d_M(v; i)$ gives for each feasible level of consumer surplus v the maximal profit that the merged firm can achieve. Note that there may be multiple asset combinations k that could be divested to firm i and achieve the same level of consumer surplus v.

Lemma 5. *The conditional divestitures curve $d_M(v; i)$ is weakly decreasing in v, and strictly so at any v such that $d_M(v; i) > 0$.*

A conditional divestitures curve $d_M(v; i)$ is downward-sloping for two reasons. First, consumer surplus is increasing in total output, but the merged firm's profit fitting-in function is decreasing in total output (holding fixed its marginal cost). Second, as consumer surplus increases through optimally-chosen asset divestitures, the merged firm's marginal cost weakly increases. Intuitively, starting from some consumer surplus level v_1, suppose to the contrary there exist divestitures that achieve a higher consumer surplus level $v_2 > v_1$ at a strictly *lower* marginal cost for the merged firm. Then, as we show in the proof of Lemma 5, the merged firm could have achieved the initial consumer surplus level v_1 with fewer divestitures than those required to achieve v_2. But then the merged firm's marginal cost at v_1 would be (weakly) lower than at v_2—a contradiction. Since the merged firm's marginal cost weakly increases as divestitures are used to increase consumer surplus, and since the merged firm's profit fitting-in function is decreasing in its cost, this also leads to a downward-sloping conditional divestitures curve.

Figure 2 depicts the synergies curve, pre-merger outcome, and three different conditional divestitures curves—each one corresponding to a different asset-receiving outsider. (The left panel again illustrates a case where even absent divestitures the merger would not harm consumers, whereas the right panel illustrates the reverse.) The red curve depicts a situation where the asset-receiving outsider is active at $v = \overline{v}_{ND}$, and where asset divestitures reduce its marginal cost strictly more than they raise the merged firm's marginal cost. Hence, as more assets are divested, consumer surplus increases and the merged firm's profit continuously decreases. The green curve depicts a similar situation, except that starting from $k = 0$, initial asset divestitures increase the merged firm's marginal cost by more than they reduce the outsider's marginal cost. As a result, initially divestitures result in both lower consumer surplus and lower merged-firm profit, as depicted by the dotted "backward-bending" part of the curve. However, eventually, divestitures reduce the outsider's marginal cost by more than they increase the merged firm's cost, such that sufficiently large divestitures can induce consumer surplus levels above $\overline{v}_{ND}$. Since the conditional divestitures curve is only defined for $v \geq \overline{v}_{ND}$, it therefore has a downward jump at $v = \overline{v}_{ND}$.[15] Finally, the blue curve depicts a similar situation to that of the red curve, except that the asset-receiving outsider is inactive at $v = \overline{v}_{ND}$. Starting from $k = 0$, as we initially divest assets the outsider remains inactive, but the merged firm's marginal cost increases, resulting in a move down the synergies curve. Eventually the outsider becomes active, after which asset divestitures raise consumer surplus as those assets are used more productively by the outsider than by the merged firm. The blue dashed curve represents outcomes where $v < \overline{v}_{ND}$, whereas the thick blue curve represents outcomes where $v \geq \overline{v}_{ND}$. Hence, as in the case of the green curve, the conditional divestitures curve exhibits a downward jump at $v = \overline{v}_{ND}$.

So far we have considered conditional divestitures curves. The *divestitures curve* $d_M(v)$ is then defined as the upper envelope of the conditional divestitures curves, on the interval $[\overline{v}_{ND}, \overline{v}_{\max}]$ where $\overline{v}_{\max} \equiv \max_{i \in \mathcal{O}} \overline{v}_{\max}(i)$. That is, for any feasible consumer surplus level v,

[15] Note that without imposing regularity conditions, such a discontinuity could also arise at some $v > \overline{v}_{ND}$.

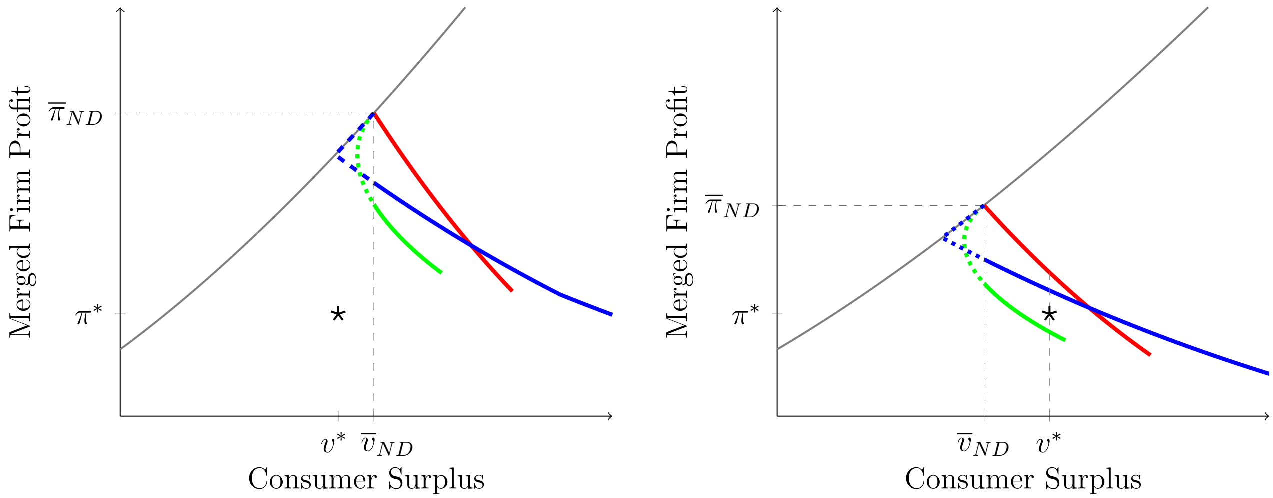

Figure 2: Three conditional divestitures curves. The two panels correspond to the situations depicted in Figure 1.

the divestitures curve gives the maximum achievable profit for the merged firm.

Corollary 1. *The divestitures curve $d_M(v)$ is weakly decreasing in v, and strictly so at any v such that $d_M(v) > 0$.*

This result follows immediately from Lemma 5.

Optimal Merger Remedies. We now solve for optimal merger proposals. We start with the simplest case in which synergies are sufficiently strong so that a merger without remedies would not be blocked when proposed.

Proposition 2. *Suppose that $\overline{c}_M(0) \leq \widehat{c}_M$. Then, it is an equilibrium for the merger partners to propose a merger without divestitures, $(0, i)$, and the antitrust authority approves it.*

The proof has two parts. First, a merger without divestitures that does not harm consumers is privately profitable (Nocke and Whinston, 2010). Intuitively, this is because such a merger generates sufficiently large efficiencies for the merged firm. Second, divesting assets can only hurt the merged firm—both because its own marginal cost increases, and because the marginal cost of the asset-receiving outsider decreases. For example, the left panel of Figure 2 shows that the merged firm's profit is higher at no divestitures than at any divestiture which raises consumer surplus above $\overline{v}_{ND}$.[16]

We now turn to the more interesting case where $\overline{c}_M(0) > \widehat{c}_M$ such that a merger without remedies would be blocked. It is immediate that if the merger partners propose a merger, the

[16]The proposition shows that no divestitures is *an* equilibrium. If there exists a (k, i) with $k \neq 0$ such that $\overline{c}_M(-k) = \overline{c}_M(0)$ and $\overline{c}_i(k) = \overline{c}_i(0) = c_i$, proposing and approving this (k, i) would also be an equilibrium since the payoffs are the same as those without divestitures.

merged firm must be active after the proposed divestiture (as otherwise the merger would not be profitable). Since the divestitures curve $d_M(v)$ is strictly decreasing in v by Corollary 1, it follows that an optimal merger proposal must leave consumer surplus unchanged, such that the authority is indifferent between accepting and blocking the merger. Denote by $\mathcal{U} \subset \mathcal{F}$ the (non-empty) set of mergers that leave consumer surplus unchanged.[17] Let

$$\underline{c}_M \equiv \min_{(k,i)\in\mathcal{U}} \overline{c}_M(-k)$$

denote the lowest post-merger marginal cost associated with any such merger.

Importantly, however, a merger in $\mathcal{U}$ is not necessarily profitable for the merger partners. This is for two reasons. Firstly, because $\overline{c}_M(0) > \widehat{c}_M$ the merger generates relatively few (if any) synergies. As a result, even at the no-divestitures outcome $\overline{v}_{ND}$, the merger may not be profitable. Secondly, in order to raise consumer surplus from $\overline{v}_{ND}$ to its pre-merger level v^*, the merged firm has to offer remedies which move it down its divestitures curve and hence reduce its profit. To this end, let $\tilde{c}_M$ be the critical level of post-merger marginal cost that would leave the merger partners' joint profit unchanged, conditional on the merger belonging to $\mathcal{U}$. That is, $\tilde{c}_M$ solves $\pi(Q^*;\tilde{c}_M) = \sum_{j\in\mathcal{M}} \pi(Q^*;c_j)$.[18] Using equation (2), this yields

$$\tilde{c}_M = P(Q^*) - \sqrt{\sum_{j\in\mathcal{M}} [P(Q^*) - c_j]^2}\ . \tag{8}$$

We are now in a position to characterize the optimal merger proposal:

Proposition 3. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. If $\underline{c}_M < \tilde{c}_M$ then in equilibrium a merger in $\mathcal{U}$ that results in marginal cost $\underline{c}_M$ is proposed and approved. Otherwise, no merger is proposed.*

To illustrate, consider again the right panel of Figure 2. The merger partners offer an asset divestiture k on the red curve which induces the pre-merger consumer surplus level v^*; this is profitable since it lies above the pre-merger profit (represented by the star), and moreover, no other divestiture which is acceptable to the authority gives strictly higher profit.[19]

Using Proposition 3 and the observation that $\tilde{c}_M < \min_{j\in\mathcal{M}} c_j$,[20] we obtain the following:

Corollary 2. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. A necessary condition for a merger to be proposed and approved is $\underline{c}_M < \min_{j\in\mathcal{M}} c_j$.*

[17]Recall that by assumption $\mathcal{A}$ is non-empty and $\overline{c}_M(0) > \widehat{c}_M$, so by continuity $\mathcal{U}$ is non-empty.

[18]Note that since the profit fitting-in function is decreasing in its second argument, $\tilde{c}_M \geq 0$ if and only if $\pi(Q^*;0) \geq \sum_{j\in\mathcal{M}} \pi(Q^*;c_j)$.

[19]Note that whenever a merger with divestitures is proposed, the asset-receiving outsider is strictly better off than before the merger. The reason is that consumer surplus is unchanged, but the asset-receiving outsider i's marginal cost has decreased with the divestitures, implying that $\pi(\overline{Q}^*;\overline{c}_i(k)) = \pi(Q^*;\overline{c}_i(k)) > \pi(Q^*;c_i)$.

[20]The observation $\tilde{c}_M < \min_{j\in\mathcal{M}} c_j$ is related to the main insight in Salant, Switzer, and Reynolds (1983), according to which, absent synergies, a merger between two firms is unprofitable.

Corollary 2 shows that for a merger to both be profitable and not harm consumers, the merged firm must be strictly more efficient than either of the merger partners even after optimal asset divestitures have been implemented: $\min_{(k,i)\in\mathcal{U}} \overline{c}_M(-k) < \min_{j\in\mathcal{M}} c_j$. The intuition is straightforward. If this condition were not satisfied, the merged firm would earn weakly less profit than the more efficient merger partner, because under an optimal merger proposal total output is the same pre and post merger. Since the merger can be thought of as the shuttering of the less efficient merger partner, the merger would be unprofitable.

As discussed earlier, it is natural to ask to what extent asset divestitures can substitute for merger-induced efficiencies. Our analysis shows that they can do so only partially. Indeed, since $\overline{c}_M(0) \leq \underline{c}_M$, Corollary 2 trivially implies that if $\overline{c}_M(0) \geq \min_{j\in\mathcal{M}} c_j$, i.e., there are no synergies in the sense of Farrell and Shapiro (1990), no merger will be proposed.[21]

We now turn to investigating how market competitiveness affects equilibrium merger proposals. Lemma 3 shows that, as the industry becomes less competitive, the acceptance set shrinks—implying that the merger partners may have to propose remedies that they would not have otherwise offered. Not surprisingly, as the next proposition demonstrates, this makes it less likely that a merger will be implemented.

Proposition 4. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. If no merger is proposed and approved in equilibrium, then the same is true after a decrease in the competitiveness of the market.*

As the market becomes less competitive, profitability of mergers is affected through two channels. First, $\underline{c}_M$ weakly increases. Intuitively, as we just noted, the acceptance set shrinks, which makes it costlier to satisfy the antitrust authority. Second, $\tilde{c}_M$ strictly decreases. Intuitively, as $P(Q^*)$ increases, holding outputs fixed, the profit margin of the merged firm increases by the same amount as the pre-merger profit margins of each of the merger partners. However, as any merger in $\mathcal{U}$ leaves total output unchanged, and as the asset-receiving outsider must produce strictly more output than it did before the merger, the merged firm produces strictly less output than the merger partners did jointly beforehand. Hence $\tilde{c}_M$ must strictly decrease to ensure that the profitability of the merger does not change as the market is made less competitive. Since a decrease in market competitiveness leads to higher $\underline{c}_M$ and lower $\tilde{c}_M$, the result then follows from Proposition 3.

Finally, it is also interesting to investigate whether (conditional on divestitures being required) it is optimal to divest assets to a firm that was previously active or inactive.

Proposition 5. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. Suppose also that the merger partners propose merger (k, i), entailing a divestiture to a previously-inactive firm. Then, if the market is*

[21] In a similar vein, Spector (2003) (in the case of Cournot competition) and Caradonna, Miller, and Sheu (forthcoming) (in the case of price competition with CES or logit demand) show that entry cannot fully substitute for a lack of merged-induced synergies. Our result applies both when asset divestitures are used to create a new entrant, and when they are used to make an already-active firm more efficient.

made less competitive and the asset-receiving firm i is still inactive before the merger, the merger partners optimally propose the same merger, or no merger at all.

While we know from Proposition 4 that, as the market becomes less competitive, a merger is less likely to be proposed, Proposition 5 shows that, conditional on one being proposed, it is more likely to involve assets being divested to a previously-inactive firm (hence enabling firm entry). To see the intuition, recall from Lemma 4 that, for a merger involving asset divestitures to a previously-inactive firm, a decrease in competitiveness does not affect the level of divestiture required for the merger not to harm consumers. Hence, if merger (k, i) was initially the most profitable merger, it remains so as the market is made less competitive. However, note that as the market is made less competitive, the merger (k, i) may no longer be profitable, in which case no merger is proposed.

3 Multimarket Analysis

In this section we consider the case in which the merger partners are active in more than one market. To this end, we now assume there is a continuum of independent markets, with unit measure, indexed by $h \in [0, 1]$, and a set $\mathcal{N}$ of firms, each of which is present in a subset of markets. Within each market, competition is as in our earlier single-market setting, with all variables now having a superscript denoting the market. As before, we consider a merger among the two firms in set $\mathcal{M} \subset \mathcal{N}$. A merger proposal now consists of an asset divestiture and an asset-receiving outsider for each market h, and is denoted $(k^h, i^h)^{h \in [0,1]}$. For simplicity, we assume that both merger partners are active in each market before the merger.[22] We consider again a three-stage game in which the merger partners first decide whether or not to propose their merger (and a set of divestitures), the authority then accepts or blocks it, and then firms compete in each market in a Cournot fashion.

We consider two objectives on the part of the antitrust authority. First, preventing consumer harm in each and every market—which we call the "market-by-market" approach. Second, ensuring that consumer surplus aggregated over all markets is not reduced by the merger—which we call the "balancing" approach.

3.1 The Market-by-Market Approach

Under the market-by-market approach, the antitrust authority blocks a merger if and only if that merger reduces consumer surplus in at least one market. Let $\mathcal{B}^h$ denote the set of merger proposals in market h that strictly reduce consumer surplus in that market; we have already characterized this set in our earlier single-market analysis. If the authority follows a

[22] If only one (or no) merger partner were active in a given market, then we implicitly assume that there are no synergies and no divestible assets, so that we can ignore such a market without loss.

market-by-market approach, the blocking set $\mathcal{B}$ consists of those merger proposals for which there is at least one market h where $(k^h, i^h) \in \mathcal{B}^h$.

It then follows trivially that our main insights on the properties of the blocking set carry over from our single-market analysis. For example, if a positive measure of markets becomes less competitive, any given merger proposal is more likely to be blocked (as in Lemma 3), while approvable mergers are more likely to involve divestitures to firms that were previously inactive in the markets that have become less competitive (as in Lemma 4). In addition, it is clear that an optimal merger proposal (should one exist) entails, for each market h, either zero divestitures (if such a proposal is not in $\mathcal{B}^h$) and otherwise divestitures which just satisfy the authority's constraint in that market.

The important difference with our earlier analysis, however, is that now the merger partners are willing to balance possible profit gains and losses across markets. Therefore the merger partners may propose a merger which reduces their profit in some markets, provided those losses are outweighed by profit gains in other markets. Recall that in the single-market analysis, if initially no merger is proposed and approved, this remains true as the market is made less competitive. This result carries over to a multimarket setting with the market-by-market approach, provided that initially in no market is there a merger which is both profitable and approvable in that market. To see why, first note that if initially there is at least one market in which initially no merger is approvable, by Lemma 3 this remains true after some markets are made less competitive—and hence following a market-by-market approach the multimarket merger would continue to be blocked. Next, note that if instead initially in each market there is a merger which is approvable, all such mergers must have been unprofitable in those markets. Hence, by Proposition 2, in each market divestitures would be required, i.e., $\overline{c}_M^h(0) > \widehat{c}_M^h$ for all h. By Proposition 4 as some markets are made less competitive, in each market the merger would either harm consumers (in which case the multimarket merger would be blocked) or continue to reduce the merger partners' profit in that merger (and so even if the multimarket merger would be approved when proposed, it would not be profitable since it would reduce profit in each market).

However in general it is not true that, as some markets are made less competitive, a multimarket merger is less likely to be proposed and approved. To see this, suppose that initially there is a merger which is approvable, but which raises profit in some markets and lowers it in others, and in aggregate is unprofitable. Suppose we now make less competitive some markets in which the merger raises profit. In the Online Appendix we construct an example where the merger becomes even more profitable in those markets, and so the multimarket merger becomes profitable in the aggregate (and remains approvable).

3.2 The Balancing Approach

As discussed in the Introduction, when evaluating a merger, there are good reasons why an antitrust authority may be willing to balance gains and losses in consumer surplus across markets. As a benchmark, we focus on the case where the authority trades these off one-for-one, and hence maximizes consumer surplus aggregated over all markets. Mirroring our single-market analysis, we begin by considering the antitrust authority's problem, before solving for the merger partners' optimal proposal.

3.2.1 The Authority's Problem

The authority's acceptance set $\mathcal{A} \subset (\mathcal{F}^h)^{h\in[0,1]}$ is the set of mergers that satisfy

$$\overline{V}^* \equiv \int_{[0,1]} v^h(\overline{Q}^{h*}(k^h, i^h))dh \geq \int_{[0,1]} v^h(Q^{h*})dh \equiv V^* \tag{9}$$

where V^* and $\overline{V}^*$ are aggregate consumer surplus before and after the proposed merger, respectively.

We saw in the single-market analysis that, as a market becomes less competitive, the acceptance set shrinks, as any given merger (k, i) is more likely to harm consumers (see Lemma 3). This result carries over to the multimarket setting if, initially, the merger either benefits consumers in all markets or harms them in all markets. Specifically, if the merger initially harms consumers in all markets, this is still the case when a subset of markets is made less competitive—and so the merger continues to be blocked. If instead the merger initially benefits consumers in all markets, then as some markets are made less competitive, the merger may harm consumers in those markets—and so the merger may now be blocked. However, in general, in our multimarket setting it is not true that the acceptance set shrinks as some markets are made less competitive. To see this, suppose that initially the merger decreases aggregate consumer surplus, but benefits consumers in a subset of markets. Suppose we now make less competitive some markets where the merger initially harmed consumers. In the Online Appendix we construct an example where the merger-induced harm in those markets becomes smaller (while remaining positive). As a result, the merger may now raise aggregate consumer surplus and hence be approved.

3.2.2 The Merger Partners' Problem

We now turn to the merger partners' problem. They propose a merger if and only if it is in the acceptance set $\mathcal{A}$ and would strictly increase their profit aggregated over all markets, i.e.,

$$\overline{\Pi}_M^* \equiv \int_{[0,1]} \pi^h(\overline{Q}^{h*}(k^h, i^h); \overline{c}_M^h(-k^h))dh > \sum_{j\in\mathcal{M}} \int_{[0,1]} \pi^h(Q^{h*}; c_j^h)dh \equiv \Pi_M^*, \tag{10}$$

where Π_M^* and $\overline{\Pi}_M^*$ are the merger partners' profit aggregated over all markets before and after the proposed merger, respectively.

Two special cases are the following. First, suppose the acceptance set is empty. Then, trivially, in equilibrium no merger is proposed. Second, suppose a merger with zero divestitures in each and every market would be approved if proposed. Then, it is an equilibrium for the merger partners to propose such a merger. This follows from Proposition 2, which shows that in each market such a merger would not only be profitable, but would also be more profitable than any other merger.

Henceforth we focus on the more interesting case where the acceptance set is non-empty, but divestitures are required in a positive measure of markets to ensure that aggregate consumer surplus does not decrease with a merger. Following the logic of our single-market analysis, if the merger partners propose a merger, it will be one that leaves aggregate consumer surplus unchanged; in general, such a merger will increase consumer surplus in some markets but decrease it in others. Amongst those mergers that would leave aggregate consumer surplus unchanged, the merger partners choose one that maximizes their profit. Hence, we can recast the merger partners' problem as choosing in each market h a point on that market's divestitures curve, subject to the constraint that aggregate consumer surplus remains unchanged. The associated Lagrangian is given by

$$\mathcal{L} = \max_{(v^h \in [\overline{v}^h_{ND}, \overline{v}^h_{\max}])^{h\in[0,1]}} \int_{[0,1]} \left[d^h_M(v^h) + \lambda(v^h - V^*) \right] dh, \tag{11}$$

where $\lambda > 0$ is the Lagrange multiplier on the aggregate consumer surplus constraint. (In order for the solution to this Lagrangian to be proposed, it must satisfy the profitability constraint in equation (10).)

Market h's contribution to the above Langrangian is $d^h_M(v^h) + \lambda(v^h - V^*)$. Note that if divestitures are used to slightly increase consumer surplus v^h, the change in market h's contribution is positive if and only if $-d^{h\prime}_M(v^h) < \lambda$. The left-hand side of this inequality is the (absolute value of the) slope of the divestitures curve. It indicates how many dollars the merger partners have to give up in market h in order to increase consumer surplus in that market by one dollar. For this reason, we call $-d^{h\prime}_M(v^h)$ the *remedies exchange rate.*

3.2.3 The Remedies Exchange Rate

Before solving for optimal merger remedies using the above Lagrangian, we investigate the properties of the remedies exchange rate. For notational simplicity, when no confusion arises, we henceforth drop market superscripts.

Redundant assets case. It is instructive to start with the case where assets are *redundant*:

Definition 2. *Assets are said to be redundant if* $\bar{c}_M(-k) = \bar{c}_M(0)$ *for all* $k \leq K$.

The redundant assets case arises, for instance, when the divestible assets are data or intellectual property or are held in duplicate by the merged firm, such that divesting them has no effect on the merged firm's marginal cost.

When assets are redundant, the market-level divestitures curve takes a particularly simple form. To see this, let $Q(x) \equiv v^{-1}(x)$ denote market-level output when consumer surplus equals x. Note that for any merger (k, i) that induces consumer surplus v, the merged firm's profit equals

$$d_M(v; i) = d_M(v) = \pi(Q(v); \bar{c}_M(0)),$$

since $\bar{c}_M(-k) = \bar{c}_M(0)$ by assumption. This implies that the conditional divestitures curves "overlap," in the sense that for any outsiders i and i' and $\bar{v}_{ND} \leq v \leq \min\{\bar{v}_{\max}(i), \bar{v}_{\max}(i')\}$, we have $d_M(v; i) = d_M(v; i')$.[23] Since $Q(v)$ is continuous in v, and since the profit fitting-in function is continuous in its first argument, the previous equation also implies that the divestitures curve $d_M(v)$ is continuous in v. Recalling Figure 2, each conditional divestitures curve—and hence also the divestitures curve—therefore looks qualitatively like the red curve.

It is straightforward to show that with redundant assets the remedies exchange rate is given by

$$-d'_M(v) = \frac{\partial \pi(Q; \bar{c}_M(0))}{\partial Q} Q'(v) = s_M(Q(v)) \left[2 - s_M(Q(v))\sigma(Q(v))\right], \tag{12}$$

where $s_M(Q) \equiv r(Q; \bar{c}_M(0))/Q$ is the merged firm's market share. Consistent with Lemma 5, if the merged firm is active at $Q(v)$ (i.e., if $s_M(Q(v)) > 0$) then the remedies exchange rate is (locally) strictly positive, and otherwise it is equal to zero. To ensure that the remedies exchange rate is well behaved, we impose the following regularity condition:

Assumption 1. *For all* $Q > 0$ *such that* $P(Q) > 0$, *market demand satisfies*

$$\min\{3 - \sigma(Q), 2[1 - \sigma(Q)][2 - \sigma(Q)]\} + Q\sigma'(Q) \geq 0.$$

Assumption 1 holds provided that $\sigma'(Q)$ is not too negative, and therefore ensures that inverse demand does not become "too concave" as market-level output increases. The condition is trivially satisfied by any demand function with constant curvature, such as linear demand. It is also satisfied by demands that are derived from many common distributions. For example, if demand is proportional to $1-F(p)$, Condition 1 is satisfied when F is standard Normal, Logistic, and Type I Extreme Value; see the Online Appendix for further details.

[23] More formally, suppose that $\bar{v}_{\max}(i') \geq \bar{v}_{\max}(i)$ for all $i \in \mathcal{O}$. Then, the graph of any conditional divestitures curve is a subset of the graph of the conditional divestitures curve for i'.

Lemma 6. *Suppose that assets are redundant. The divestitures curve $d_M(v)$ is weakly convex (and strictly so if $d_M(v) > 0$).*

Lemma 6 shows that the divestitures curve is convex; equivalently, the remedies exchange improves as a higher level of consumer surplus is induced. To understand the result, first note that consumer surplus is convex in output. As such, as v increases, an additional dollar increase in v can be achieved through successively smaller increases in output Q. Second, note that in the special case where market demand is linear, a unit increase in Q reduces the market price by the same amount. At the same time, when Q is higher, the merged firm's market share is lower, and so it is hurt less by any given reduction in the market price. This explains why the divestitures curves is convex when demand is linear. In the more general case where demand is non-linear, a given increase in output Q does not reduce market price by the same amount. However, Assumption 1 ensures that demand does not become too concave—that is, price does not fall too quickly—as Q increases, and so the divestitures curve is still convex.

General case. We now turn to the general case, where assets are not necessarily redundant in that $\overline{c}_M(-k)$ is not constant. To this end, let $c^*_M(c; i)$ denote the minimized marginal cost of the merged firm when assets are divested to outsider i in such a way that the outsider's marginal cost equals $c \in [\overline{c}_i(K), \overline{c}_i(0)]$:

$$c^*_M(c; i) \equiv \min_{k \in [0,K]} \overline{c}_M(-k) \quad \text{s.t.} \quad \overline{c}_i(k) = c.$$

It is easy to see that $c^*_M(c; i)$ is weakly decreasing in c. Intuitively, as c increases fewer assets need to be divested, implying that the merged firm's marginal cost is lower.[24] For simplicity, we assume that $c^*_M(c; i)$ is also twice differentiable.[25]

Lemma 7. *At a point of differentiability, the slope of the conditional divestitures curve $d_M(v; i)$ is*

$$-d'_M(v; i) = s_M(Q(v); i) \left[2 - s_M(Q(v); i)\sigma(Q(v)) - 2\frac{(\overline{n}(Q(v); i) + 1 - \sigma(Q(v)))c^{*\prime}_M(c(Q(v)); i)}{1 + c^{*\prime}_M(c(Q(v)); i)} \right]$$

where $s_M(Q(v); i)$ and $\overline{n}(v; i)$ are the merged firm's market share and the number of active firms, respectively, when induced consumer surplus is v and the asset-receiving outsider is i, $c(v)$ is that outsider's marginal cost at consumer surplus v, and $c^{*\prime}_M(c(v); i) \equiv \partial c^*_M(c(v); i)/\partial c$.

[24] More formally, consider c' and c'' where $c' < c'' \leq \overline{c}_i(0)$, and suppose to the contrary that $c^*_M(c'; i) < c^*_M(c''; i)$. Notice that, if we let k' be an (efficient) divestiture associated with c', then by continuity there exists $\lambda \in [0, 1)$ such that $\overline{c}_i(\lambda k') = c''$. But then $\overline{c}_M(\lambda k') \leq c^*_M(c'; i) < c^*_M(c''; i)$, which is a contradiction.

[25] As an example, in the case of one-dimensional assets, this would hold if both $\overline{c}_M(-k)$ and $\overline{c}_i(k)$ are twice differentiable, with $\overline{c}'_i(k) < 0$ for all $k \in [0, K]$.

Comparing with the slope of the divestitures curve in the redundant assets from equation (12), there is a new term in the square brackets. This new term reflects the fact that now the merged firm's marginal cost may change as it moves down a conditional divestitures curve. (Note that if $c_M^{*\prime}(c(v);i) = 0$, such that the merged firm's cost does not change, then this new term is zero, and we are back in the redundant assets case.) This new term is positive if the merged firm is active (i.e., if $s_M(Q(v)) > 0$). To see this, note that for v to increase, the asset-receiving outsider has to be active, and divestitures must reduce its marginal cost by more than they increase the merged firm's marginal cost. Hence, we must have $c_M^{*\prime}(c(v);i) > -1$.

A natural case of interest is one in which assets are *complementary*:

Definition 3. *Assets are said to be complementary if, for any outsider* $i \in \mathcal{O}$, $c_M^*(c;i)$ *is strictly decreasing and concave in* $c \in [\overline{c}_i(K), \overline{c}_i(0)]$, *and* $\overline{c}_i(K) < \overline{c}_i(k)$ *for all* $k \neq K$.

A simple example of complementary assets is an environment where assets are one-dimensional and each firm has increasing returns from these assets. As the merged firm divests successively more of these assets, its marginal cost increases by less and less, whilst the cost of the asset-receiving outsider decreases by more and more, reaching its minimum when all K assets are divested.

In the proof of Lemma 8 below, we show that complementarity of assets implies that conditional divestitures curves are convex. Intuitively, this arises for two reasons. First, as in the redundant assets case, as we move down a given conditional divestitures curve, the merged firm's market share decreases, and so it is hurt less and less by the induced price decrease. Second, as explained above, as we move down a conditional divestitures curve, the merged firm's cost increases. Complementarity of assets ensures that this cost increase becomes smaller and smaller, thereby reinforcing the convexity of the conditional divestitures curve. In contrast to the redundant assets case, a conditional divestitures curve can have a downward jump, but only at $\overline{v}_{ND}$. Such a discontinuity may arise, as increasing consumer surplus slightly above $\overline{v}_{ND}$ may require divesting a discrete amount of assets—either because the asset-receiving outsider is initially inactive, or because initially its marginal cost decreases by less than the increase in the merged firm's marginal cost. See Figure 2 and the discussion of the blue and green conditional divestitures curves.

Convexity of the conditional divestitures curves is also inherited by the divestitures curve:

Lemma 8. *Suppose that assets are complementary. The divestitures curve* $d_M(v)$ *is weakly convex (and strictly so if* $d_M(v) > 0$*).*

Convexity of the conditional divestitures curves does *not* necessarily imply convexity of the divestitures curve, even though the latter is the upper envelope of the former. The reason is simply that the domains of the graphs of the conditional divestitures curves are, in general, not the same. In principle, this could lead to a downward jump in the divestitures curve at one or more $\overline{v}_{\max}(i)$; if so, this would imply that the divestitures curve were not convex.[26] Nevertheless, the proof of Lemma 8 shows that when assets are complementary, such downward jumps cannot occur.

3.2.4 Optimal Merger Remedies

We have established that the remedies exchange rate improves as consumer surplus increases, whenever assets are redundant (Lemma 6) or complementary (Lemma 8). Recall the merged firm's constrained optimization problem from equation (11):

$$\mathcal{L} = \max_{(v^h \in [\overline{v}^h_{ND}, \overline{v}^h_{\max}])^{h\in[0,1]}} \int_{[0,1]} \left[d^h_M(v^h) + \lambda(v^h - V^*) \right] dh.$$

Notice that the contribution of market h to the Lagrangian is strictly convex in v^h, and so is maximized at either $v^h = \overline{v}^h_{ND}$ or $v^h = \overline{v}^h_{\max}$. Hence, a solution to the above problem must be "bang bang". Intuitively, suppose to the contrary that there is a positive measure of markets where the proposed consumer surplus level is "interior". Then, because the remedies exchange rate is improving in each market, it would be more profitable for the merged firm to do fewer divestitures in some markets and more divestitures in others.

Given the bang bang property of the optimal merger proposal, we now investigate in which markets the merger partners should propose no divestitures (resulting in $\overline{v}^h_{ND}$) and in which they should propose "maximal" divestitures (resulting in $\overline{v}^h_{\max}$). To this end, it is useful to define the *average remedies exchange rate* in market h as:

$$a^h \equiv \frac{d^h_M(\overline{v}^h_{ND}) - d^h_M(\overline{v}^h_{\max})}{\overline{v}^h_{\max} - \overline{v}^h_{ND}}, \tag{13}$$

which equals the average slope of the divestitures curve in that market. In the following, we assume for simplicity that markets are heterogeneous, and that a^h has a continuous and strictly increasing distribution function. The following result is an immediate implication:

Proposition 6. *Suppose that assets in each market are either redundant or complementary. Then, the solution to the merger partners' maximization problem* (11) *is "bang bang": the merger partners propose in each market h consumer surplus $\overline{v}^h_{\max}$ if $a^h < \lambda$, and consumer*

[26] To see the potential issue, suppose that there are two asset-receiving outsiders, i and j. Suppose also that $d_M(v;i) > d_M(v;j)$ for all $\overline{v}_{ND} < v \leq \overline{v}_{\max}(i)$ and $\overline{v}_{\max}(i) < \overline{v}_{\max}(j)$. Then, $d_M(v) = d_M(v;i)$ for $\overline{v}_{ND} < v \leq \overline{v}_{\max}(i)$ and $d_M(v) = d_M(v;j)$ for $\overline{v}_{\max}(i) < v \leq \overline{v}_{\max}(j)$. That is, the divestitures curve $d_M(v)$ would have a downward jump at $v = \overline{v}_{\max}(i)$. However with complementary assets this issue does not arise.

surplus $\overline{v}^h_{ND}$ if the inequality is reversed. The Lagrange multiplier $\lambda > 0$ uniquely solves

$$\int_{[0,1]} \{\overline{v}^h_{ND}\mathbf{1}_{\{a^h>\lambda\}} + \overline{v}^h_{\max}\mathbf{1}_{\{a^h<\lambda\}}\}dh = V^*. \tag{14}$$

Given the bang bang property of the optimal solution, the merger partners do maximal divestitures in markets where the average exchange rate a^h is more favorable; divestitures are implemented where they give the biggest "bang for the buck". The Lagrange multiplier λ represents the loss in the merger partners' profit from a small increase in V^*, and is equal to the average remedies exchange rate in the "marginal" market where divestitures are carried out.

In our single-market analysis, Lemma 3 showed that the merger partners may optimally propose (more) divestitures in less competitive markets, as otherwise the merger may not be approved. We now show that the *opposite* result can obtain in our multi-market setting. To illustrate this, start with the case where assets are redundant:

Proposition 7. *Consider two markets, h and h', that are identical except that the latter is less competitive in that $\overline{c}^{h'}_j(k) - \overline{c}^h_j(k) = \Delta_j \geq 0$ for all k and all $j \in \mathcal{O}$. Suppose also that assets are redundant in these markets. Then, if it is optimal to propose no divestitures in market h, it is also optimal to propose no divestitures in the less competitive market h'.*

Note that the notion of a change in competitiveness used in Proposition 7 corresponds to part (ii) of Definition 1. In a multimarket setting with redundant assets, the merger partners optimally offer *fewer* divestitures in less competitive markets, because the average remedy exchange rate in these markets is worse. Intuitively, in less competitive markets the merged firm commands a larger market share, which implies that the merged firm is hurt more from any given increase in consumer surplus. Hence, it is more expensive for the merger partners to offer divestitures in such markets.[27] Another way to see this is to note that, when assets are redundant, for any given consumer surplus level v the merged firm's profit is independent of the marginal costs of outsiders—and hence is the same in markets h and h'. The solid blue curve in Figure 3 depicts this relationship between consumer surplus and merged firm profit. To understand Proposition 7, note that in the less competitive market $\overline{v}_{ND}$ and $\overline{v}_{\max}$ are both lower, and hence lie on a steeper part of the curve in the figure. This immediately implies that the average remedies exchange rate—which is the (absolute value of the) slope of the dashed line segments in the figure—is larger in the less competitive market.

When assets are complementary, the same result as in Proposition 7 obtains but under stronger conditions—as we demonstrate in Section C of the Online Appendix. Intuitively,

[27] As already noted, Proposition 7 uses the notion of competitiveness in part (ii) of Definition 1. The notion of competitiveness in part (i) would have no bite, since it only concerns a change in inverse demand at the pre-merger outcome, and does not constrain how inverse demand changes at $\overline{v}_{ND}$ and $\overline{v}_{\max}$.

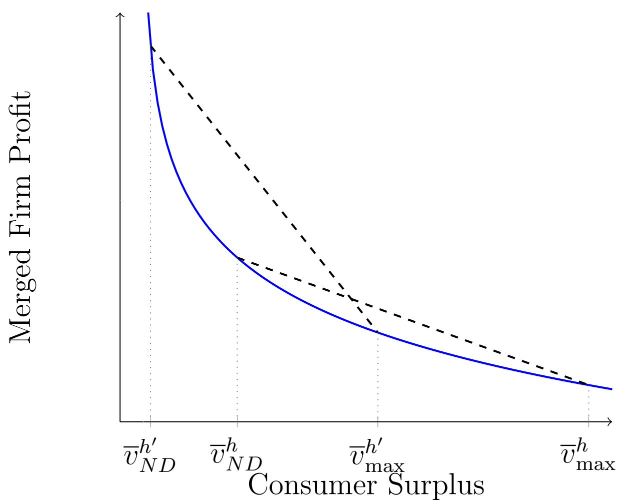

Figure 3: The remedies exchange rate in markets with different levels of competitiveness.

when assets are no longer redundant, a reduction in market competitiveness leads not only to a decrease in $\overline{v}_{ND}$ and $\overline{v}_{\max}$, but also to a (downward) shift in the relationship between consumer surplus and merged firm profit. Under some additional assumptions, this latter shift reinforces the effect due to a decrease in $\overline{v}_{ND}$ and $\overline{v}_{\max}$, ensuring that the merger partners still find it profitable to offer fewer divestitures in less competitive markets.

4 Extensions

We now show that our main insights are robust to allowing for bargaining between the merger partners and the antitrust authority, and to allowing the merger partners to earn revenues from asset divestitures. We relegate omitted proofs to the Online Appendix.

4.1 Bargaining

In our baseline model, we assumed that the merger partners propose divestitures and the authority can only accept or reject. However, in practice, in many jurisdictions there is some form of negotiation between the authorities and the merging parties. In Canada, for example, "a remedy is consensually negotiated between the agency and the parties to the transaction" (OECD, 2016). Meanwhile in the U.S., "the [merging] parties and the [FTC] staff negotiate a proposed settlement and finalize terms" (FTC, 2012).

In this section, we suppose that the merger partners and the antitrust authority engage in efficient bargaining over the merger and associated remedies. Our earlier analysis therefore corresponds to the special case where the merger partners have all the bargaining power.

Consider first our single-market analysis. As before, suppose that there exists some merger and associated remedy (k, i) which is both profitable and raises consumer surplus.

If the merger partners have all the bargaining power, our previous analysis has shown that the resulting consumer surplus level is $\max\{v(Q^*), \overline{v}_{ND}\}$. If instead the authority has all the bargaining power, the resulting consumer surplus level $\widehat{v}$ is the largest feasible consumer surplus such that the merger is profitable, i.e.,[28]

$$\widehat{v} \equiv \begin{cases} \overline{v}_{\max} & \text{if } d_M(\overline{v}_{\max}) \geq \sum_{j\in\mathcal{M}} \pi(Q^*; c_j), \\ d_M^{-1}\left(\sum_{j\in\mathcal{M}} \pi(Q^*; c_j)\right) & \text{otherwise.} \end{cases}$$

More generally, efficient bargaining implies that the resulting outcome lies on the divestitures curve, and induces a consumer surplus level between $\max\{v(Q^*), \overline{v}_{ND}\}$ and $\widehat{v}$. Note that Proposition 2 may no longer hold: even if a merger without remedies would not harm consumers, the antitrust authority may be able to negotiate a merger with divestitures, which is less profitable for the merger partners but which results in higher consumer surplus. However, a modified version of Proposition 3 does still hold: assuming that $\overline{c}_M(0) > \widehat{c}_M$, a merger will be proposed and approved if and only if $\underline{c}_M \leq \tilde{c}_M$. The reason is that the set of profitable and approvable mergers is non-empty if and only if this condition holds. Similarly, Proposition 4 also carries over: assuming again that $\overline{c}_M(0) > \widehat{c}_M$, as the market becomes less competitive, it is less likely that a merger is proposed and approved. The reason is that if the set of profitable and approvable mergers is initially empty, it remains empty when the market becomes less competitive.

We now turn to our multimarket analysis. Again, suppose that there exists some merger (and associated remedies $(k^h, i^h)^{h\in[0,1]}$) that is both profitable and raises aggregate consumer surplus. If the merger partners have all the bargaining power, our previous analysis implies that the resulting aggregate consumer surplus level is $\max\{V^*, \overline{V}_{ND}\}$, where V^* is pre-merger (aggregate) consumer surplus and $\overline{V}_{ND}$ is (aggregate) consumer surplus when there are no divestitures in any market. If instead the authority has some bargaining power, resulting in aggregate consumer surplus $\overline{V} > \max\{V^*, \overline{V}_{ND}\}$, then any efficient profile of remedies is the outcome of the following program:

$$\mathcal{L} = \max_{(v^h \in [\overline{v}^h_{ND}, \overline{v}^h_{\max}])^{h\in[0,1]}} \int_{[0,1]} \left[d^h_M(v^h) + \lambda(v^h - \overline{V})\right] dh.$$

This is identical to equation (11) (after replacing V^* with $\overline{V}$), and so it follows from earlier arguments that the solution is bang bang, as in Proposition 6. Let $\overline{\Pi}_M(\overline{V})$ be the merged firm's aggregate profit induced by this solution, and note that it is strictly decreasing in $\overline{V}$. If the authority has all the bargaining power, the resulting consumer surplus level $\widehat{V}$ is the largest feasible consumer surplus (aggregated over all markets) such that the merger is

[28]Note that for simplicity, here we assume that a merger is proposed provided it is weakly profitable (rather than strictly profitable, as was assumed earlier).

profitable, i.e.,

$$\widehat{V} \equiv \begin{cases} \int_{[0,1]} \overline{v}^h_{\max} dh & \text{if } \overline{\Pi}_M(\int_{[0,1]} \overline{v}^h_{\max} dh) \geq \Pi^*_M, \\ \overline{\Pi}^{-1}_M(\Pi^*_M) & \text{otherwise,} \end{cases}$$

where Π^*_M is the merger partners' pre-merger profit aggregated across all markets. More generally, efficient bargaining implies that the resulting aggregate consumer surplus level lies between $\max\{V^*, \overline{V}_{ND}\}$ and $\widehat{V}$. As the antitrust authority's bargaining power increases—and hence the resulting aggregate consumer surplus level $\overline{V}$ increases—the Lagrange multiplier λ increases, implying that the set of markets with maximal divestitures increases in the set order sense. Moreover, holding fixed bargaining power, under our earlier assumptions, maximal divestitures are "more likely" in more competitive markets, as in Proposition 7.

4.2 Asset Revenues

In our baseline model, we assumed that the merger partners do not obtain any revenue from divesting assets, even though any firm receiving assets benefits from this. While this is a strong assumption, it serves as a useful benchmark. In this section we relax this, and allow the merger partners to capture some of the profit gain that accrues to the asset-receiving outsider. Given the well-known problems with studying multi-player bargaining, we assume that in each market there is a single outsider capable of receiving assets.[29] In the following, we use a reduced-form approach, which nests standard two-player Nash bargaining as a special case.

4.2.1 Single-Market Analysis

We start with our single-market setting. Since the authority's problem is unchanged, our results from Section 2.3.1 on the acceptance set carry over. We now turn to the merger partners' problem. We assume that the merger partners' profit from merger (k, i) is given by

$$\beta \left[\pi(\overline{Q}^*(k,i); \overline{c}_M(-k)) + \alpha \pi(\overline{Q}^*(k,i); \overline{c}_i(k)) \right] + \overline{\pi}^o_M(\alpha, \beta), \tag{15}$$

where $\alpha \in [0,1]$, $\beta \in (0,1]$, and

$$\overline{\pi}^o_M(\alpha,\beta) \equiv \begin{cases} (1-\beta)\pi(\overline{Q}^*(0,i); \overline{c}_M(0)) - \alpha\beta\pi(\overline{Q}^*(0,i); \overline{c}_i(0)) & \text{if } \overline{c}_M(0) \leq \widehat{c}_M, \\ (1-\beta)\sum_{j\in\mathcal{M}} \pi(Q^*; c_j) - \alpha\beta\pi(Q^*; c_i) & \text{otherwise.} \end{cases}$$

This payoff function nests two bargaining processes. First, when $\beta = 1$, this represents the case where the merger partners obtain a share α of the increase in the outsider's profit due

[29] Alternatively, there could be multiple outsiders capable of receiving the assets, but before bargaining starts the merger partners have to commit to bargain with only one of them.

to the divested assets. (Note that our baseline model corresponds to the special case where $\alpha = 0$.) Second, when $\alpha = 1$, this represents standard Nash bargaining, with β denoting the bargaining power of the merger partners. Specifically, the merger partners receive their outside option plus a share β of the gain in joint profit due to the divested assets. In both cases, because we assume that only one outsider can receive assets, the outside option if bargaining breaks down is the no-divestitures outcome when $\overline{c}_M(0) \leq \widehat{c}_M$, and no merger otherwise.

Note that $\overline{\pi}_M^o(\alpha, \beta)$ does not depend on k. Hence, if the pre-merger consumer surplus level $v(Q^*)$ is strictly less than $\overline{v}_{\max}(i)$, so that there is a continuum of divestitures k that would not be blocked, the choice of *which* asset divestiture k will be proposed is *independent of the outside option*: the merger partners maximize a weighted sum of their profit and that of the asset-receiving outsider $\pi(\overline{Q}^*(k,i); \overline{c}_M(-k)) + \alpha\pi(\overline{Q}^*(k,i); \overline{c}_i(k))$ over the divestitures k that are not in the blocking set.

At each attainable consumer surplus level v, the value of the (conditional) divestitures curve $d_M(v; i)$ is equal to the payoff in equation (15), when maximized with respect to k and subject to the constraint $v(\overline{Q}^*(k,i); \overline{c}_i(k)) = v$. We now provide conditions to ensure that this curve remains downward sloping:

Lemma 9. *Assume that* $\max\{\overline{c}_M(-K), \overline{c}_i(K)\} < P(\overline{Q}^*(K,i))$*, such that both the merged firm and the asset-receiving outsider* i *are active when all assets are divested. Then, if* α *is sufficiently small, or if* $\overline{c}_i(\cdot)$ *is such that* $\overline{c}_i(K)$ *is sufficiently large, the (conditional) divestitures curve is strictly decreasing in* v *for all* $v \in [\overline{v}_{ND}, \overline{v}_{\max}(i)]$.

To understand the result, recall from Lemma 5, that any divestitures-induced increase in consumer surplus reduces the merged firm's profit. By the same token, that same increase in consumer surplus raises the profit of the asset-receiving outsider. Therefore, $d_M(v; i)$ is decreasing in v if either the weight α on the outsider's profit is sufficiently small, or else the outsider's marginal cost at maximal divestitures is sufficiently large (which ensures that its market share is sufficiently small, and hence the gain in its profit is small relative to the loss in the merged firm's profit). For example, in the Online Appendix we show that if post merger there is a duopoly, a sufficient condition for the divestitures curve to be downward sloping is that, for any divestiture, the market share of the asset-receiving outsider is less than $1 - \sqrt{1/2} \approx 0.29$.

Lemma 9 has two immediate implications. First, when a merger without divestitures is in the authority's acceptance set (which holds if and only if $\overline{c}_M(0) \leq \widehat{c}_M$), then this is what the merger partners will propose. That is, Proposition 2 carries over to this setting. Second, when such a merger is in the blocking set ($\overline{c}_M(0) > \widehat{c}_M$), then if a merger is proposed at all, it is a merger (k, i) that leaves the antitrust authority indifferent, as in Proposition 3. We now investigate how market competitiveness affects the likelihood of a merger being proposed.[30]

Proposition 8. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$ and that the divestitures curve is strictly decreasing in v. If no merger is proposed and approved in equilibrium, then the same is true after a decrease in the competitiveness of the market.*

Hence the insights of our earlier Proposition 4 carry over to the setting with endogenous asset revenues.

4.2.2 Multimarket Analysis

We now discuss the impact of (endogenous) asset revenues in our multimarket analysis. For simplicity assume that each asset-receiving outsider is present only in a zero measure of markets, such that the breakdown of negotiations with any such outsider cannot cause an otherwise profitable and approvable merger to become unprofitable or be blocked. Suppose there exists a profitable and approvable merger proposal $(k^h, i^h)^{h\in[0,1]}$, then the merged firm's payoff in market h given divestiture (k^h, i^h) in that market is

$$\tilde{\pi}^h_M(k^h, i^h) \equiv \beta^h \left[\pi^h(\overline{Q}^{h*}(k^h, i^h); \overline{c}^h_M(-k^h)) + \alpha^h \pi^h(\overline{Q}^{h*}(k^h, i^h); \overline{c}^h_{i^h}(k^h))\right] + \overline{\pi}^{h,o}_M(\alpha^h, \beta^h) \quad (16)$$

where

$$\overline{\pi}^{h,o}_M(\alpha^h, \beta^h) = (1-\beta^h)\pi^h(\overline{Q}^{h*}(0, i^h); \overline{c}^h_M(0)) - \alpha^h\beta^h\pi^h(\overline{Q}^{h*}(0, i^h); \overline{c}^h_{i^h}(0)).$$

The expression in equation (16) is essentially the same as the one in equation (15) from the single-market analysis, with one important difference: since the outsider in market h has zero measure, a merger will be proposed and approved irrespective of the outcome of bargaining in that market. Note that the bargaining parameters α and β are allowed to vary arbitrarily across markets.

The merger partners will choose divestitures $(k^h, i^h)^{h\in[0,1]}$ to maximize $\int_{[0,1]} \tilde{\pi}^h_M(k^h, i^h)dh$ subject to $\int_{[0,1]} v^h(\overline{Q}^{h*}(k^h, i^h))dh \geq V^*$. Using the above, we can define a divestitures curve $\tilde{d}^h_M(v^h)$ for each market h as before. To help ensure that the divestitures curve in each market is convex, we impose the following regularity condition:

Assumption 2. *For all $Q > 0$ such that $P(Q) > 0$, market demand satisfies $Q\sigma'(Q) < 2$.*

Assumption 2 holds provided that $\sigma'(Q)$ is not too positive. It is trivially satisfied by any demand function with constant curvature, such as linear demand. It is also satisfied by demands that are derived from many common distributions. For example, if demand is

[30] Note that since by assumption only one outsider is capable of receiving assets, we cannot distinguish between different "types" (active versus inactive) of outsider, so our earlier Proposition 5 has no analog.

proportional to $1 - F(p)$, as with our earlier Assumption 1, Assumption 2 is satisfied when F is standard Normal, Logistic, and Type I Extreme Value; see the Online Appendix for further details. When Assumptions 1 and 2 hold we can establish the following result:

Proposition 9. *The solution to the merger partners' problem is bang bang provided divestitures do not induce any firm to exit the market, and in a given market h either (i) assets are redundant, or (ii) assets are complementary, and the merged firm's market share is larger than α times the market share of the asset-receiving outsider at $v^h = \overline{v}^h_{\max}$.*

Under our regularity conditions on curvature $\sigma(Q)$ and the conditions in Proposition 9, the divestitures curve is convex. Therefore, following earlier arguments, the solution to the merger partners' problem is bang bang: in each market h, either $\overline{v}^h_{ND}$ or $\overline{v}^h_{\max}$ obtains. This is the analog of our earlier Proposition 6. Note that this proposition holds even if the conditions in Lemma 9 fail and the divestitures curve is not downward sloping everywhere.[31]

5 Conclusion

In this paper we provide a framework to study merger remedies—which are widely used in practice, but which have been largely overlooked by the existing literature. Our framework imposes minimal restrictions on how asset divestitures affect firms' marginal costs and product qualities. We study both the case where a merger affects only a single market, and the case where it affects multiple markets. We study the antitrust authority's optimal approval decision, and then solve for the merger partners' optimal decision of whether to propose a merger and, if so, which remedies (if any) to offer.

If the merger affects only a single market, we show that remedies cannot fully substitute for efficiencies in preventing consumer harm from a merger. We also demonstrate that as the market becomes less competitive, it is less likely that a merger is proposed and approved; but if a merger is proposed and approved, it is more likely to involve asset divestitures to a firm that was previously inactive.

If the merger instead affects multiple markets, and the antitrust authority is willing to balance gains and losses across these markets, the rate at which profit in any given market can be "converted" into consumer surplus plays a key role. We show that, in any given market, this remedies exchange rate is decreasing as assets are divested in such a way that the level of consumer surplus increases. Hence optimal merger remedies are "bang bang"—no divestitures in some markets, and large divestitures in others. Contrary to our single-market analysis, it can be optimal for the merger partners to concentrate divestitures in the more competitive markets, as the remedies exchange rate is often more favorable in such markets.

[31]Given that the divestitures curve is convex, if it has an upward-sloping portion it must be for sufficiently high levels of v. In principle, it could even be the case that the merged firm makes a higher profit at $\overline{v}^h_{\max}$ than at $\overline{v}^h_{ND}$.

In future research, our framework could be extended in various directions. First, it would be interesting to consider the case of price competition with differentiated products, e.g., as in Nocke and Schutz (forthcoming). Second, it would be worthwhile to investigate a setting in which merger opportunities and associated divestitures arise over time, e.g., as in Nocke and Whinston (2010). Third, while we have focused on merger remedies when the antitrust authority is concerned with unilateral effects, one could also consider remedies when there is the risk of coordinated effects, e.g., as in Compte, Jenny, and Rey (2002).

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A Omitted Proofs

Proof of Remark 1. For the first microfoundation, suppose the representative consumer has quasi-linear utility $U(Q) + H$, where $Q \equiv \sum_{i\in\mathcal{N}} w_i\tilde{q}_i$ is quality-adjusted total quantity, with $\tilde{q}_i$ being firm i's "raw" output and w_i a measure of its quality, and H is consumption of the Hicksian composite commodity. The representative consumer then wishes to

$$\max_{\{\tilde{q}_i\}_{i\in\mathcal{N}}} U\left(\sum_{i\in\mathcal{N}} w_i\tilde{q}_i\right) + y - \sum_{i\in\mathcal{N}} \tilde{p}_i\tilde{q}_i,$$

where $\tilde{p}_i$ is the price of a unit of output of firm i, and y is income. This implies that for any firm $i = 1$ we must have

$$\frac{\tilde{p}_i}{w_i} = U'(Q) \equiv P(Q).$$

Hence, firm i's program is

$$\max_{\tilde{q}_i} \tilde{q}_i\left[w_i U'\left(Q\right) - \tilde{c}_i\right],$$

where $\tilde{c}_i$ is its constant marginal cost. This can be rewritten as

$$\max_{q_i} q_i\left[P\left(Q\right) - c_i\right],$$

where $q_i \equiv w_i\tilde{q}_i$ is quality-adjusted output, and $c_i \equiv \tilde{c}_i/w_i$ is quality-adjusted marginal cost. This is the same optimization problem that a firm faces in a homogeneous goods Cournot model, as detailed in Section 2.2. That is, while each firm i is characterized by the tuple

$(w_i, \tilde{c}_i)$, the firm's payoff-relevant type is given by the scalar $c_i = \tilde{c}_i / w_i$. Consumer surplus is therefore

$$U(Q) - \sum_{i \in \mathcal{N}} \tilde{p}_i \tilde{q}_i = U(Q) - QP(Q) = \int_0^Q [P(z) - P(Q)] dz.$$

This coincides with consumer surplus in the homogeneous goods Cournot model, as detailed in Section 2.2.

For the second microfoundation, suppose the representative consumers has utility $U(Q) + \sum_{i \in \mathcal{N}} w_i q_i + H$, where $Q \equiv \sum_{i \in \mathcal{N}} q_i$ is total quantity, w_i is again firm i's quality, and H is consumption of the Hicksian composite commodity. By Roy's identity, inverse demand for firm i is given by

$$p_i \equiv P(Q) + w_i,$$

where $P \equiv U'$. Let $\tilde{c}_i$ denote firm i's marginal cost and assume $\tilde{c}_i > w_i$. Defining $c_i \equiv \tilde{c}_i - w_i$, firm i's program is thus identical to that in the homogeneous goods Cournot model:

$$\max_{q_i} q_i \left[P(Q) - c_i \right].$$

That is, while each firm i is characterized by the tuple $(w_i, \tilde{c}_i)$, the firm's payoff-relevant type is given by the scalar $c_i = \tilde{c}_i - w_i$. Consumer surplus is

$$U(Q) - \sum_{i \in \mathcal{N}} (p_i - w_i) q_i = U(Q) - QP(Q) = \int_0^Q [P(z) - P(Q)] dz,$$

which coincides with consumer surplus in the homogeneous goods Cournot model. □

Proof of Lemma 2. It follows from Lemma 1 that this cutoff $\widehat{c}_M$ exists. Moreover, it solves

$$r(Q^*; \widehat{c}_M) = \sum_{i \in \mathcal{M}} r(Q^*; c_i),$$

which immediately implies that $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$. □

Proof of Proposition 1. Merger (k, i) is in the acceptance set if and only if $\overline{Q}^*(k, i) \geq Q^*$. Using equations (3) and (4), and recalling that $r(Q; c)$ is decreasing in Q, this holds if and only if

$$r\left(Q^*; \overline{c}_M(-k)\right) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q^*; c_j) + r(Q^*; \overline{c}_i(k)) \geq \sum_{j \in \mathcal{N}} r(Q^*; c_j),$$

or equivalently,

$$r(Q^*;\overline{c}_M(-k)) + r(Q^*;\overline{c}_i(k)) \geq \sum_{j\in\mathcal{M}} r(Q^*;c_j) + r(Q^*;c_i).$$

We now prove that a necessary condition for this inequality to hold is that $r(Q^*;\overline{c}_i(k)) > 0$, i.e., the asset-receiving outsider is active after the merger if the merger-induced output is Q^*. On the way to a contradiction, suppose instead that $r(Q^*;\overline{c}_i(k)) = 0$. Then, as $r(Q;c)$ is weakly decreasing in its second argument, and since $\overline{c}_M(-k)$ is weakly decreasing, the left-hand side of the inequality is bounded above by $r(Q^*;\overline{c}_M(0))$. However, by assumption a merger without remedies decreases consumer surplus, and hence $r(Q^*;\overline{c}_M(0)) < \sum_{j\in\mathcal{M}} r(Q^*;c_j)$; as $r(Q^*;c_i) \geq 0$, we reach a contradiction. Since $r(Q^*;\overline{c}_i(k)) > 0$ then $P(Q^*) > \overline{c}_i(k)$. Using equation (1) to substitute out for $r(Q;c)$ we then obtain equation (6). □

Proof of Lemma 3. From the proof of Proposition 1, merger (k,i) strictly reduces total output (and is therefore in the blocking set) if and only if

$$r(Q^*;\overline{c}_M(-k)) + r(Q^*;\overline{c}_i(k)) < \sum_{j\in\mathcal{M}} r(Q^*;c_j) + r(Q^*;c_i).$$

Using equation (1) and the assumption that both merger partners are active pre merger, this condition can be rewritten as

$$2P(Q^*) - \sum_{j\in\mathcal{M}} c_j - \max\{P(Q^*) - \overline{c}_M(-k), 0\} - \max\{P(Q^*) - \overline{c}_i(k), 0\} + \max\{P(Q^*) - c_i, 0\} > 0.$$

As $P(Q^*)$ increases, the merger partners remain active, so this inequality remains valid. To complete the proof, note that the left-hand side is weakly increasing in $P(Q^*)$. □

Proof of Lemma 4. First, note that the inequality $\overline{c}_M(0) > \widehat{c}_M$ remains true as the market is made less competitive. Next, since $\overline{c}_M(0) > \widehat{c}_M$, Proposition 1 implies that merger (k,i) is in the acceptance set if and only if equation (6) holds. Since we assume that $\overline{c}_M(-k) < P(Q^*)$ and i is inactive pre merger, it is easy to see that $P(Q^*)$ cancels from the two sides of equation (6). □

Proof of Lemma 5. Towards a contradiction, suppose that there exist v_1 and $v_2 > v_1$ satisfying $d_M(v_1;i) \leq d_M(v_2;i)$ and $d_M(v_2;i) > 0$. Let k_j be a maximizer of the constrained optimization problem in equation (7) for $v = v_j$ where $j = 1, 2$. Let $Q_1 \equiv \overline{Q}^*(k_1,i)$ and

$Q_2 \equiv \overline{Q}^*(k_2, i)$. As $Q_2 > Q_1$ by assumption, and as the profit fitting-in function of any active firm is strictly decreasing in both arguments, it follows that $\overline{c}_M(-k_2) < \overline{c}_M(-k_1)$. Moreover, from equation (4), $Q_2 > Q_1$ also implies that

$$\begin{aligned}
& r(Q_1, \overline{c}_M(-k_2)) + r(Q_1, \overline{c}_i(k_2)) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q_1, c_j) \\
& \geq r(Q_2, \overline{c}_M(-k_2)) + r(Q_2, \overline{c}_i(k_2)) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q_2, c_j) = Q_2 \\
& > Q_1 = r(Q_1, \overline{c}_M(-k_1)) + r(Q_1, \overline{c}_i(k_1)) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q_1, c_j),
\end{aligned}$$

where the two inequalities follow because the output fitting-in function r is decreasing in its first argument, and because $Q_2 > Q_1$ by assumption, and both equalities follow from equilibrium condition (4). Hence,

$$r(Q_1, \overline{c}_M(-k_2)) + r(Q_1, \overline{c}_i(k_2)) > r(Q_1, \overline{c}_M(-k_1)) + r(Q_1, \overline{c}_i(k_1)).$$

At the same time, because $v_1 \geq \overline{v}_{ND}$ we must have $Q_1 \geq \overline{Q}^*(0, i)$, and hence by a reasoning similar to the above, we also have

$$r(Q_1, \overline{c}_M(0)) + r(Q_1, \overline{c}_i(0)) \leq r(Q_1, \overline{c}_M(-k_1)) + r(Q_1, \overline{c}_i(k_1)).$$

Therefore, by continuity, there must exist a $\lambda \in [0, 1)$ such that

$$r(Q_1, \overline{c}_M(-\lambda k_2)) + r(Q_1, \overline{c}_i(\lambda k_2)) = r(Q_1, \overline{c}_M(-k_1)) + r(Q_1, \overline{c}_i(k_1)).$$

That is, mergers $(\lambda k_2, i)$ and (k_1, i) both induce the same equilibrium output Q_1. Moreover, we have

$$\pi(Q_1; \overline{c}_M(-\lambda k_2)) \geq \pi(Q_1; \overline{c}_M(-k_2)) > \pi(Q_2; \overline{c}_M(-k_2)) \geq \pi(Q_1; \overline{c}_M(-k_1)),$$

where the first inequality follows from the assumption that $\overline{c}_M(\cdot)$ is weakly decreasing and the fact that the profit fitting-in function is weakly decreasing in its second argument; the second inequality follows from the fact that the profit fitting-in function of any active firm is strictly decreasing in its first argument (recall that $d_M(v_2; i) > 0$, so the merged firm is active at output Q_2 when its cost is $\overline{c}_M(-k_2)$); and the last inequality follows by assumption. But then k_1 is not a maximizer of the constrained optimization problem in equation (7) for $v = v_1$, a contradiction. □

Proof of Proposition 2. It follows from Lemma 2 that a merger without remedies is in $\mathcal{A}$ and so will be accepted.

First, we show that such a merger is strictly profitable. To see this, note that if $\overline{c}_M(0) = \widehat{c}_M$ the merger is strictly profitable because:

$$\pi(Q^*; \widehat{c}_M) = [P(Q^*) - \widehat{c}_M] r(Q^*; \widehat{c}_M) = [P(Q^*) - \widehat{c}_M] \sum_{j \in \mathcal{M}} r(Q^*; c_j) > \sum_{j \in \mathcal{M}} [P(Q^*) - c_j] r(Q^*; c_j),$$

where the second equality follows from the fact that a merger with post-merger marginal cost $\widehat{c}_M$ does not affect total output, and the inequality follows because $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$ holds by Lemma 2. Next, recall from Lemma 1 that any firm's equilibrium profit is decreasing in its marginal cost; hence the left-hand side of the above inequality is a lower bound on the profit of the merged firm for any $\overline{c}_M(0) \leq \widehat{c}_M$, implying that such a merger is strictly profitable.

Finally, note that a merger with divestitures must lead to weakly lower profit. This follows from Lemma 1 and the fact that divestitures would weakly increase the merged firm's marginal cost and weakly lower the marginal cost of the asset-receiving outsider. □

Proof of Proposition 3. As already argued in the main text, if a merger is proposed it must belong to $\mathcal{U}$ and the merged firm must be active. Because the profit fitting-in function of an active firm is strictly decreasing in its second argument, holding fixed total output (and hence also consumer surplus) at its pre-merger level, the merged firm's profit is maximized when its marginal cost equals $\underline{c}_M$. As the profit fitting-in function is strictly decreasing in its second argument for any active firm, such a merger is strictly profitable (and so is proposed and subsequently approved) if and only if $\underline{c}_M < \tilde{c}_M$. □

Proof of Proposition 4. First, note from equation (8) that $\tilde{c}_M$ is strictly decreasing in $P(Q^*)$. Second, we now show that $\underline{c}_M$ is weakly increasing in $P(Q^*)$. To this end, notice that

$$\underline{c}_M \equiv \min_{(k,i) \in \mathcal{U}} \overline{c}_M(-k) = \min_{(k,i) \in \mathcal{A}} \overline{c}_M(-k).$$

Suppose otherwise. Then there exists a $(k', i') \in \mathcal{A} \setminus \mathcal{U}$ such that $\overline{c}_M(-k') < \underline{c}_M$. As $\overline{c}_M(0) > \widehat{c}_M$ by assumption, continuity implies that there exists a $\lambda \in (0,1)$ such that merger $(\lambda k', i') \in \mathcal{U}$, and by monotonicity $\overline{c}_M(-\lambda k') \leq \overline{c}_M(-k')$—contradicting the assumption that $\underline{c}_M$ is the lowest cost in $\mathcal{U}$. To complete the proof, recall from Lemma 3 that $\mathcal{A}$ is weakly smaller in the less competitive market, implying that $\underline{c}_M$ is weakly larger in that market. □

Proof of Proposition 5. Note that in the more competitive market, it must hold that

$$\overline{c}_M(-k) = \underline{c}_M < \tilde{c}_M < P(Q^*),$$

where the equality and the first inequality follow from Proposition 3, and the second inequality follows from equation (8). By Lemma 4, as the market becomes less competitive, merger (k, i) remains in $\mathcal{A}$. Recall also from the proof of Proposition 4 that $\underline{c}_M$ is weakly larger in the less competitive market. Hence, even in the less competitive market, $\overline{c}_M(-k) = \underline{c}_M$, and so no acceptable merger is more profitable than merger (k, i). □

Proof of Lemma 6. It is straightforward to see from equation (12) that $d_M(v)$ is twice differentiable, except at the v' where $P(Q(v')) = \overline{c}_M(0)$ (if such a v' exists) because $s_M(Q(v))$ is not differentiable at $v = v'$.

Notice that $s_M(Q(v)) = 0$ for all $v \geq v'$ (if such a v' exists), and hence $d_M(v) = -d'_M(v) = 0$ for all $v \geq v'$. Now consider all remaining admissible $v < v'$, such that $s_M(Q(v)) > 0$. To simplify the exposition we henceforth omit the dependence of $Q(v)$ on v. As $s_M(Q) = r(Q; \overline{c}_M(0))/Q$, we obtain

$$\frac{ds_M(Q)}{dQ} = -\frac{1 + s_M(Q)[1 - \sigma(Q)]}{Q},$$

and, using equation (12),

$$-d''_M(v) = -Q'(v)\frac{s_M^2(Q)}{Q}\left\{2\left[\frac{1}{s_M(Q)} - \sigma(Q)\right]\left[\frac{1}{s_M(Q)} + 1 - \sigma(Q)\right] + Q\sigma'(Q)\right\}.$$

By Assumption 1, $-d''_M(v)$ is weakly negative, and strictly so if $s_M(Q) < 1$ (which necessarily holds if $v > \overline{v}_{ND}$). Hence, for any $\overline{v}_{ND} \leq v_1 < v'$ and any $v_2 > v_1$, we have $-d'_M(v_1) > -d'_M(v_2)$. □

Proof of Lemma 7. First, suppose $s_M(Q(v)) = 0$ and thus $d_M(v; i) = 0$. Then, by Lemma 5 the slope of the conditional divestitures curve is zero, consistent with the expression in Lemma 7. Second, suppose $s_M(Q(v)) > 0$. Note that we must have $1 + c^{*\prime}_M(c(Q); i) > 0$. Suppose to the contrary that $1 + c^{*\prime}_M(c(Q); i) \leq 0$. Then there exists a different vector of asset divestitures resulting in weakly higher Q and strictly higher profit for the merged firm. But from Lemma 5 the conditional divestitures curve must slope down, and so the initial point cannot have been on the conditional divestitures curve—a contradiction. For notational convenience, we henceforth omit the dependence of $Q(v)$ on v. Using equation (4) we have that

$$c'(Q) = \frac{P'(Q)[\overline{n}(Q;i) + 1 - \sigma(Q)]}{1 + c_M^{*\prime}(c(Q);i)}. \tag{17}$$

Next, differentiating the profit fitting-in function:

$$\frac{d\pi(Q; c_M^*(c(Q);i))}{dQ} = QP'(Q)s_M(Q)\left[2 - s_M(Q)\sigma(Q) - \frac{2c_M^{*\prime}(c(Q);i)}{P'(Q)}\frac{dc}{dQ}\right]. \tag{18}$$

Combining equations (17) and (18), and using the fact that $v'(Q) = -P'(Q)Q$, we obtain the expression for $d'_M(v;i)$ in the statement of the lemma. □

Proof of Lemma 8. We first show that any *conditional* divestitures curve $d_M(v;i)$ is weakly decreasing and convex, and strictly so when $v > \overline{v}_{ND}$ and $d_M(v;i) > 0$.

From the expression in Lemma 7, and using the fact that $0 > c_M^{*\prime}(c;i) > -1$ (from the proof of Lemma 7), it is straightforward to see that $d_M(v;i)$ is weakly decreasing, and strictly so when $d_M(v;i) > 0$ (or, equivalently, when $s_M(Q(v)) > 0$).

We now show that any conditional divestitures curve $d_M(v;i)$ is convex on $[\overline{v}_{ND}, \overline{v}_{\max}(i)]$, and strictly so when both $v > \overline{v}_{ND}$ and $d_M(v;i) > 0$. Let $v'(i) \equiv \sup\{v \in [\overline{v}_{ND}, \overline{v}_{\max}(i)] | s_M(Q(v)) > 0\}$. As $s_M(Q(v)) = 0$ for all $v \in (v'(i), \overline{v}_{\max}(i)]$, it is immediate from the equation in the statement of Lemma 7 that $d_M(v;i)$ is equal to zero (and thus weakly convex) on $[v'(i), \overline{v}_{\max}(i)]$.

Next, we show that $d_M(v;i)$ is strictly convex on $[\overline{v}_{ND}, v'(i))$. It is straightforward to see from Lemma 7 that $d_M(v;i)$ is twice differentiable almost everywhere on $[\overline{v}_{ND}, v'(i))$. Consider first any v in that interval at which $d''_M(v;i)$ exists. As $s_M(Q) = r(Q; c_M^*(c(Q);i))/Q$, we have:

$$s'_M(Q) = -\frac{1}{Q}\left[1 + s_M(Q)[1 - \sigma(Q)] - \frac{c_M^{*\prime}(c(Q);i)}{P'(Q)}\frac{dc}{dQ}\right] < 0,$$

where the inequality follows as each term inside the square brackets is strictly positive. The derivative of $d'_M(v;i)$ with respect to Q is

$$\begin{aligned}
&-2s'_M(Q)[1 - s_M(Q)\sigma(Q)] + [s_M(Q)]^2\sigma'(Q) \\
&+ 2\frac{c_M^{*\prime}(c(Q);i)}{1 + c_M^{*\prime}(c(Q);i)}\left[s'_M(Q)[\overline{n}(Q;i) + 1 - \sigma(Q)] - s_M(Q)\sigma'(Q)\right] \\
&+ \frac{2s_M(Q)[\overline{n}(Q;i) + 1 - \sigma(Q)]c_M^{*\prime\prime}(c(Q);i)c'(Q)}{[1 + c_M^{*\prime}(c(Q);i)]^2}.
\end{aligned} \tag{19}$$

We now prove that this is strictly positive. The first line of (19) is proportional to

$$2\left[\frac{1}{s_M(Q)} - \sigma(Q)\right]\left[\frac{1}{s_M(Q)} + 1 - \sigma(Q) - \frac{[\overline{n}(Q;i) + 1 - \sigma(Q)]c_M^{*\prime}(c(Q);i)}{(1 + c_M^{*\prime}(c(Q);i))s_M(Q)}\right] + Q\sigma'(Q).$$

Since $-1 < c_M^{*\prime}(c(Q);i) < 0$, Assumption 1 implies that this expression is weakly positive, and strictly positive for $s_M(Q) < 1$ (which necessarily holds for all $v > \overline{v}_{ND}$). The second line of (19) is proportional to

$$[\overline{n}(Q;i) + 1 - \sigma(Q)] \left[\frac{1}{s_M(Q)} + 1 - \sigma(Q) - \frac{[\overline{n}(Q;i) + 1 - \sigma(Q)] c_M^{*\prime}(c(Q);i)}{(1 + c_M^{*\prime}(c(Q);i)) s_M(Q)} \right] + Q\sigma'(Q).$$

Since $\overline{n}(Q;i) \geq 2$ the first square-bracketed term weakly exceeds $3 - \sigma(Q)$, and since $\sigma(Q) < 1$ by assumption the second square-bracketed term strictly exceeds 1, Assumption 1 ensures that the whole expression is strictly positive. By inspection, the third line of (19) is also strictly positive. It then follows that for any v where $d_M(v;i)$ is twice-differentiable, we have $d_M''(v;i) > 0$.

As $s_M(Q(v))$, $\sigma(Q(v))$ and $c_M^{*\prime}(c(Q(v));i)$ are all differentiable at any $v < v'(i)$, $d_M(v;i)$ is twice differentiable, except possibly at $v = \overline{v}_{ND}$ (where $d_M(v;i)$ may jump downwards) and at any v at which $\overline{n}(Q(v);i)$ jumps down (inducing an upward jump in the slope of $d_M(v;i)$). Note that any such non-differentiability preserves the convexity of $d_M(v;i)$.

We have thus shown that $d_M(v;i)$ is weakly convex on $[\overline{v}_{ND}, v'(i))$, and strictly so for $v > \overline{v}_{ND}$. Before, we had already shown that $d_M(v;i) = 0$ on $(v'(i), \overline{v}_{\max}(i)]$. As $d_M(v;i)$ is continuous everywhere, it follows that it is weakly convex everywhere, and strictly so in $(\overline{v}_{ND}, v'(i))$.

It remains to show that the divestitures curve $d_M(v)$, which is the upper envelope of the conditional divestitures curve, is weakly convex, and strictly so on $(\overline{v}_{ND}, \max_i v'(i))$. If all the conditional divestitures curves had the same support, then the result would follow trivially from the fact that the upper envelope of convex functions is convex. If, however, there are two asset-receiving outsiders i_1 and i_2 with $\overline{v}_{\max}(i_1) < \overline{v}_{\max}(i_2)$, this raises the possibility that the upper envelope has a downward jump at $\overline{v}_{\max}(i_1)$, namely if $d_M(\overline{v}_{\max}(i_1)) = d_M(\overline{v}_{\max}(i_1); i_1)$, implying that $d_M(v)$ is not convex. We now show that this possibility cannot arise. To see this, note that by the definition of complementary assets, $\overline{v}_{\max}(i) > \overline{v}_{ND}$ is uniquely obtained at $k = K$ for $i = i_1, i_2$, and this leads to a marginal cost $\overline{c}_M(-K)$ for the merged firm. Note that by continuity, we can divest strictly fewer than K assets to outsider i_2 and induce consumer surplus level $v = \overline{v}_{\max}(i_1)$; this results in a post-merger marginal cost for outsider i_2 strictly exceeding $\overline{c}_{i_2}(K)$, which by the definition of complementarity implies that the merged firm's marginal cost is strictly lower than $\overline{c}_M(-K)$. But this implies that $d_M(\overline{v}_{\max}(i_1); i_2) > d_M(\overline{v}_{\max}(i_1); i_1)$, a contradiction. □

Proof of Proposition 6. To prove the bang bang result, note that the contribution of market h to the Lagrangian in equation (11) is $d_M^h(v^h) + \lambda(v^h - V^*)$. By Lemmas 6 and 8 this is convex in v^h, and so is maximized by $v^h \in \{\overline{v}_{ND}^h, \overline{v}_{\max}^h\}$. The optimal solution has $v^h = \overline{v}_{\max}^h$

if and only if

$$d_M^h(\overline{v}_{\max}^h) + \lambda(\overline{v}_{\max}^h - V^*) > d_M^h(\overline{v}_{ND}^h) + \lambda(\overline{v}_{ND}^h - V^*),$$

which can be rewritten as $a^h < \lambda$. Equation (14) then follows immediately from the constraint $\int_{[0,1]} v^h(\overline{Q}^{h*}(k^h, i^h))dh = V^*$. The left-hand side of this equation is (by assumption) strictly less than V^* when $\lambda < \min_h a^h$, and strictly larger than V^* when $\lambda > \max_h a^h$; since the left-hand side is also continuous and strictly increasing in λ, there is a unique λ which solves equation (14). □

Proof of Proposition 7. Note first that since it is optimal to propose no divestitures in market h, we must have $a^h > \lambda > 0$ and hence $d_M^h(\overline{v}_{ND}^h) > 0$. By Lemma 1 it follows that $d_M^{h'}(\overline{v}_{ND}^{h'}) > 0$ as well. Lemma 1 also implies that $\overline{v}_{ND}^h \geq \overline{v}_{ND}^{h'}$ and $\overline{v}_{\max}^h \geq \overline{v}_{\max}^{h'}$. Note that since demand and the merged firm's cost are the same in both markets h and h', we have that $d_M^h(v) = d_M^{h\prime}(v)$ for any $v \in [\overline{v}_{ND}^h, \overline{v}_{\max}^h] \cap [\overline{v}_{ND}^{h\prime}, \overline{v}_{\max}^{h\prime}]$. Therefore dropping the market superscripts, the derivative of a in equation (13) with respect to $\overline{v}_{ND}$ is

$$\frac{d_M'(\overline{v}_{ND}) + a}{\overline{v}_{\max} - \overline{v}_{ND}} < 0,$$

where the inequality follows from the fact that $d_M(v)$ is weakly convex everywhere, and strictly so for all v such that $d_M(v) > 0$. Similarly, the derivative of a with respect to $\overline{v}_{\max}$ is

$$-\frac{d_M'(\overline{v}_{\max}) + a}{\overline{v}_{\max} - \overline{v}_{ND}} < 0,$$

where the inequality again follows from convexity of $d_M(v)$. Hence, $a^{h'} \geq a^h$. Applying Proposition 6 then gives the result. □

B Online Appendix

B.1 Omitted Proofs for Section 4

Proof of Lemma 9. To ease the exposition we drop the dependence of Q on v. Note that at any point on the divestitures curve, $1 + c_M^{*\prime}(c(Q); i) > 0$: since increasing consumer surplus through asset divestitures requires that the reduction in the outsider's marginal cost more than outweighs the increase in the merged firm's marginal cost. As the profit of asset-receiving outsider i equals $[P(Q) - c(Q)]^2/[-P'(Q)]$, its derivative with respect to Q is

$$s_i(Q; c(Q))P'(Q)Q \left\{ 2 \left[1 - \frac{\overline{n}(Q; i) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c(Q); i)} \right] - s_i(Q; c(Q))\sigma(Q) \right\},$$

where we have used the expression for $c'(Q)$ in equation (17), and where $\overline{n}(Q; i)$ denotes the number of active firms, and $s_i(Q; c(Q))$ denotes asset-receiving outsider i's market share. Using the fact that $v'(Q) = -P'(Q)Q$, and combining with the derivative of the merged firm's profit with respect to v in Lemma 7, the slope of the divestitures curve is β multiplied by

$$\begin{aligned} &- s_M(Q; i) \left[2 - s_M(Q; i)\sigma(Q) - 2\frac{[\overline{n}(Q; i) + 1 - \sigma(Q)]c_M^{*\prime}(c(Q); i)}{1 + c_M^{*\prime}(c(Q); i)} \right] \\ &+ \alpha s_i(Q; c(Q)) \left\{ -2 \left[1 - \frac{\overline{n}(Q; i) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c(Q); i)} \right] + s_i(Q; c(Q))\sigma(Q) \right\}, \end{aligned}$$

First, note that at $\alpha = 0$ the overall expression is strictly negative, so by continuity the same is true provided that α is sufficiently small. Second, consider the result on $\overline{c}_i(\cdot)$. Note that because $-1 < c_M^{*\prime}(c(Q); i) \leq 0$ at any point on the divestitures curve, the second line is positive; hence, if the overall expression is negative at $\alpha = 1$, it is also negative at any $\alpha \in [0, 1]$. It therefore suffices to establish that the following is strictly negative:

$$\begin{aligned} &- s_M(Q; i) \left[2 - s_M(Q; i)\sigma(Q) - 2\frac{[\overline{n}(Q; i) + 1 - \sigma(Q)]c_M^{*\prime}(c(Q); i)}{1 + c_M^{*\prime}(c(Q); i)} \right] \\ &+ s_i(Q; c(Q)) \left\{ -2 \left[1 - \frac{\overline{n}(Q; i) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c(Q); i)} \right] + s_i(Q; c(Q))\sigma(Q) \right\}. \end{aligned}$$

One can check that, holding all else fixed, the derivative of this with respect to $c_M^{*\prime}(c(Q); i)$ is

$$\frac{2[\overline{n}(Q; i) + 1 - \sigma(Q)]}{[1 + c_M^{*\prime}(c(Q); i)]^2} [s_M(Q; i) - s_i(Q; c(Q))].$$

Notice that by making $\overline{c}_i(\cdot)$ sufficiently large, we make $c(Q)$ sufficiently large for each Q, and hence we can ensure that $s_M(Q; i) > s_i(Q; c(Q))$, and so the above is positive. Thus, it is sufficient to prove that the expression of interest is strictly negative when evaluated at $c_M^{*\prime}(c(Q);i) = 0$, or equivalently:

$$-s_M(Q;i)\left[2 - s_M(Q;i)\sigma(Q)\right] + s_i(Q;c(Q))\left\{2\left[\overline{n}(Q;i) - \sigma(Q)\right] + s_i(Q;c(Q))\sigma(Q)\right\}.$$

Note that the first term is bounded from above by $-s_M(Q;i)\left[2 - s_M(Q;i)\right]$. Note also that if $\overline{c}_i(K)$ is sufficiently large, then $\overline{c}_i(k)$ is large for any $k \in [0,K]$, and so for any relevant Q, $c(Q)$ can also be made sufficiently large. Hence $s_M(Q;i)$ is bounded away from zero, while $s_i(Q;c(Q))$ can be made sufficiently small to ensure that the first term dominates the second. □

Proof of Proposition 8. Since $\overline{c}_M(0) > \widehat{c}_M$ a merger without divestitures is not approvable initially, and is also not approvable after the reduction in market competitiveness.

We will prove the result by contradiction. In particular, towards a contradiction, suppose that *initially* no merger is both approvable and profitable, but that *after* the market is made less competitive, there exists an approvable profitable merger. Since the divestitures curve is strictly decreasing, the best such merger in the latter case is one which leaves consumer surplus unchanged. Denote by k the associated divestiture. Using (15) the merger is profitable if and only if

$$\beta\left[\pi(\overline{Q}^*(k,i);\overline{c}_M(-k)) + \alpha\pi(\overline{Q}^*(k,i);\overline{c}_i(k))\right] + (1-\beta)\sum_{j\in\mathcal{M}}\pi(Q^*;c_j) - \alpha\beta\pi(Q^*;c_i) \geq \sum_{j\in\mathcal{M}}\pi(Q^*;c_j).$$

Given $\beta > 0$ and $\overline{Q}^*(k,i) = Q^*$ this is equivalent to

$$\pi(Q^*;\overline{c}_M(-k)) + \alpha\pi(Q^*;\overline{c}_i(k)) - \sum_{j\in\mathcal{M}}\pi(Q^*;c_j) - \alpha\pi(Q^*;c_i) \geq 0. \tag{20}$$

There are then two cases to consider. First, if asset-receiving outsider i is inactive pre merger, then (20) simplifies to

$$[P(Q^*) - \overline{c}_M(-k)]^2 + \alpha[P(Q^*) - \overline{c}_i(k)]^2 - \sum_{j\in\mathcal{M}}[P(Q^*) - c_j]^2 \geq 0,$$

where we use our earlier result that the asset-receiving outsider must be active post merger. The derivative of the above expression with respect to $P(Q^*)$ is

$$\begin{aligned} &2\left\{P(Q^*) - \overline{c}_M(-k) + \alpha[P(Q^*) - \overline{c}_i(k)] - \sum_{j\in\mathcal{M}}[P(Q^*) - c_j]\right\} \\ &= -2(1-\alpha)[P(Q^*) - \overline{c}_i(k)] \leq 0, \end{aligned}$$

where the equality uses the fact that a merger leaves consumer surplus unchanged if and only if $\overline{c}_M(-k) + \overline{c}_i(k) = \sum_{j\in\mathcal{M}} c_j$. Second, if asset-receiving outsider i is active pre merger, then (20) simplifies to

$$[P(Q^*) - \overline{c}_M(-k)]^2 + \alpha[P(Q^*) - \overline{c}_i(k)]^2 - \sum_{j\in\mathcal{M}} [P(Q^*) - c_j]^2 - \alpha[P(Q^*) - c_i]^2 \geq 0,$$

and its derivative with respect to $P(Q^*)$ is

$$\begin{aligned} &2\left\{P(Q^*) - \overline{c}_M(-k) + \alpha[P(Q^*) - \overline{c}_i(k)] - \sum_{j\in\mathcal{M}} [P(Q^*) - c_j] - \alpha[P(Q^*) - c_i]\right\} \\ &= -2(1-\alpha)[c_i - \overline{c}_i(k)] \leq 0, \end{aligned}$$

where the equality uses the fact from equation (1) that a merger leaves consumer surplus unchanged if and only if $P(Q^*) = \sum_{j\in\mathcal{M}} c_j - \overline{c}_M(-k) + c_i - \overline{c}_i(k)$. (Note that since the divestitures curve is strictly decreasing we must have $\overline{c}_M(-k) < P(Q^*)$, i.e., the merged firm is active.) In either case, we can conclude that *before* the market was made less competitive, there existed a profitable and approvable merger. To see this, notice that by assumption (20) holds when the market is less competitive. We have just argued that if we reduce $P(Q^*)$ the inequality in (20) still holds; moreover, as the market becomes more competitive, the original divestiture k remains approvable (by Lemma 3), and so since the divestitures curve is strictly decreasing, the merger partners can do (weakly) even better. But this contradicts the original supposition that before the market was made less competitive there was no profitable and approvable merger. □

Proof of Proposition 9. We prove that, under the conditions stated in the proposition, $\tilde{\pi}^h_M(k^h, i^h)$ is strictly convex in v^h. For simplicity, in what follows we drop the market index h.

First, totally differentiating asset-receiving outsider i's market share $s_i(Q) = -[P(Q) - c(Q)]/QP'(Q)$ with respect to Q gives

$$s_i'(Q) = \frac{1}{Q}\left[\frac{\overline{n}(Q) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c(Q))} - 1 - s_i(Q)[1 - \sigma(Q)]\right],$$

where we have used the expression for $c'(Q)$ in equation (17), and where $\overline{n}(Q; i)$ denotes the number of active firms, and $s_i(Q; c(Q))$ denotes asset-receiving outsider i's market share.

Taking the derivative of the objective $\tilde{\pi}_M$ with respect to v, and using $v'(Q) = -QP'(Q)$, yields

$$\frac{1}{\beta}\frac{d\tilde{\pi}_M}{dv} = -\, s_M(Q)\left[2 - s_M(Q)\sigma(Q) - 2\frac{c_M^{*\prime}(c_i(Q))[\overline{n}(Q) + 1 - \sigma(Q)]}{1 + c_M^{*\prime}(c_i(Q))}\right]$$

$$+ \alpha s_i(Q) \left\{ -2 \left[1 - \frac{\overline{n}(Q) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c_i(Q))} \right] + s_i(Q)\sigma(Q) \right\}.$$

As $v'(Q) > 0$, strict convexity of the divestitures curve is equivalent to $d^2\tilde{\pi}_M/dvdQ > 0$, which we now show to hold. Since there is no exit due to divestitures, changes in v (and hence Q) have no impact on $\overline{n}(Q)$. From equation (19) in the proof of Lemma 8, the derivative of the first line of the right-hand side is larger than

$$\frac{2s_M(Q)[\overline{n}(Q) + 1 - \sigma(Q)]c_M^{*\prime\prime}(c_i(Q))c_i'(Q)}{[1 + c_M^{*\prime}(c_i(Q))]^2}, \tag{21}$$

while the derivative of the second line equals $1/Q$ multiplied by

$$\begin{aligned} &2\alpha \left[\frac{\overline{n}(Q) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c_i(Q))} - 1 + s_i(Q)\sigma(Q) \right] \left[\frac{\overline{n}(Q) + 1 - \sigma(Q)}{1 + c_M^{*\prime}(c_i(Q))} - 1 - s_i(Q)[1 - \sigma(Q)] \right] \\ &- \alpha Q\sigma'(Q)s_i(Q) \left[\frac{2}{1 + c_M^{*\prime}(c_i(Q))} - s_i(Q) \right] - \frac{2\alpha s_i(Q)[\overline{n}(Q) + 1 - \sigma(Q)]c_M^{*\prime\prime}(c_i(Q))c_i'(Q)}{[1 + c_M^{*\prime}(c_i(Q))]^2}. \end{aligned} \tag{22}$$

Given that $c_M^{*\prime\prime}(c_i(Q)) \leq 0$ with complementary or redundant assets, and also the fact that $c_i'(Q) < 0$, the sum of (21) and the very last term in (22) is positive if $s_M(Q) \geq s_i(Q)$. Now, by assumption, we have $s_M(Q(\overline{v}_{\max})) \geq \alpha s_i Q(\overline{v}_{\max}))$. Since $\alpha \leq 1$ as well as the fact $s_M(Q)$ is decreasing in Q while $s_i(Q)$ is increasing in Q, we do indeed have $s_M(Q) \geq s_i(Q)$.

Hence it remains to prove that the first three terms in (22) are strictly positive. Notice that the two square-bracketed terms in the first line of (22) are positive and decreasing in $\sigma(Q)$. Therefore replacing them with $\sigma(Q) = 1$, and also using our assumption that $Q\sigma'(Q) < 2$, as well as $\overline{n}(Q) \geq 2$ the first three terms in (22) are strictly larger than

$$\begin{aligned} &2\alpha \left\{ \left[\frac{2}{1 + c_M^{*\prime}(c_i(Q))} - 1 + s_i(Q) \right] \left[\frac{2}{1 + c_M^{*\prime}(c_i(Q))} - 1 \right] - s_i(Q) \left[\frac{2}{1 + c_M^{*\prime}(c_i(Q))} - s_i(Q) \right] \right\} \\ &= 2\alpha \left\{ \left[\frac{2}{1 + c_M^{*\prime}(c_i(Q))} - 1 \right]^2 - s_i(Q) \left[1 - s_i(Q) \right] \right\}. \end{aligned}$$

This expression is strictly positive, as $c_M^{*\prime}(c_i(Q)) \in (-1, 0]$. □

B.2 Numerical Examples

We first give an example where, in the market-by-market approach, a reduction in the competitiveness of some markets renders profitable a merger that was previously unprofitable. Then we give an example where, in the balancing approach, a reduction in the competitiveness of some markets renders approvable a merger that would previously have been blocked.

B.2.1 Market-by-Market Approach

Start with a *single* market, with $n = 2$ firms and initial market demand curve $P(Q) = 1 - Q$. Suppose that pre-merger costs are $c_1 = c_2 = 1/2$. We can then compute the following pre-merger outcomes: $Q^* = 1/3$, $P(Q^*) = 2/3$, $v(Q^*) = 1/18$, and $\sum_{j=1}^{2} \pi(Q^*; c_j) = 1/18$. Now consider a merger between firms 1 and 2 (without divestitures) such that $\bar{c}_M(0) = 0$. We can then compute the following post-merger outcomes: $\overline{Q}^* = 1/2$, $P(\overline{Q}^*) = 1/2$, $v(\overline{Q}^*) = 1/8$, and $\pi(\overline{Q}^*; \bar{c}_M(0)) = 1/4$. Hence the merger increases consumer surplus in this market, and the increase in the merger partners' profit is

$$\pi(\overline{Q}^*; \bar{c}_M(0)) - \sum_{j=1}^{2} \pi(Q^*; c_j) = \frac{7}{36}. \tag{23}$$

Now suppose we make the market less competitive, by changing the demand curve such that now $P(Q) = 1 + \epsilon - Q$ where $\epsilon \in (0, 1)$. We can then compute the following pre-merger outcomes: $Q^* = (1 + 2\epsilon)/3$, $P(Q^*) = (2 + \epsilon)/3$, $v(Q^*) = (1 + 2\epsilon)^2/18$, and $\sum_{j=1}^{2} \pi(Q^*; c_j) = (1 + 2\epsilon)^2/18$. (Note that the pre-merger equilibrium price is indeed higher when the demand curve is modified in this way.) We can also compute the following post-merger outcomes: $\overline{Q}^* = (1 + \epsilon)/2$, $P(\overline{Q}^*) = (1 + \epsilon)/2$, $v(\overline{Q}^*) = (1 + \epsilon)^2/8$, and $\pi(\overline{Q}^*; \bar{c}_M(0)) = (1 + \epsilon)^2/4$. Hence the merger still increases consumer surplus in this market (given that $\epsilon < 1$), and the increase in the merger partners' profit is now

$$\pi(\overline{Q}^*; \bar{c}_M(0)) - \sum_{j=1}^{2} \pi(Q^*; c_j) = \frac{7 + 10\epsilon + \epsilon^2}{36}. \tag{24}$$

Notice that the expression in equation (24) is strictly larger than that in equation (23), i.e., in both cases, the merger increases market-level consumer surplus and is profitable, but the magnitude of the profit increase is larger in the less competitive market.

Now consider a *multimarket* context with two types of market. Type A markets are the same as those described above, while type B markets require divestitures to prevent market-level consumer surplus from falling, and these divestitures are such that in type B markets the merger reduces the merger partners' profits. Let α denote the fraction of type A markets. Note that there exist values of α such that initially the merger would be unprofitable for the merger partners, but would become profitable when the type A markets are made less competitive as above.

B.2.2 Balancing Approach

Start with a *single* market, with $n = 4$ firms and market demand curve $P(Q) = 1 - Q$. Suppose that initially pre-merger marginal costs are $c_1 = c_2 = c_3 = 1/2$ and $c_4 = 1$. We can then compute the following pre-merger outcomes: $Q^* = 3/8$, $P(Q^*) = 5/8$, and $v(Q^*) = 9/128$. (Note that before the merger, only firms $1, 2$, and 3 are active.) Now consider a merger between firms 1 and 2 and a divestiture $(k, 4)$ such that $\overline{c}_M(-k) = 1/2$ and $\overline{c}_4(k) = 1/2 + \epsilon$ where $\epsilon \in (0, 1/6)$. We can then compute the following post-merger outcomes: $\overline{Q}^* = (3 - 2\epsilon)/8$, $P(\overline{Q}^*) = (5 + 2\epsilon)/8$, and $v(\overline{Q}^*) = (3 - 2\epsilon)^2/128$. (Note that after the merger, the merged firm as well as firms 3 and 4 are active. In particular, firm 4 is active because $\epsilon < 1/6$ ensures that $P(\overline{Q}^*) > \overline{c}_4(k)$.) Hence the merger and divestitures reduce consumer surplus in this market by

$$v(Q^*) - v(\overline{Q}^*) = \frac{\epsilon(3 - \epsilon)}{32}. \tag{25}$$

Now suppose we make the market less competitive, by raising firm 3's pre-merger marginal cost such that $c_3 = 1/2 + \epsilon$. We can then compute the following pre-merger outcomes: $Q^* = (3 - 2\epsilon)/8$, $P(Q^*) = (5 + 2\epsilon)/8$, and $v(Q^*) = (3 - 2\epsilon)^2/128$. Again consider a merger between firms 1 and 2 and a divestiture $(k, 4)$ such that $\overline{c}_M(-k) = 1/2$ and $\overline{c}_4(k) = 1/2+\epsilon$. We can then compute the following post-merger outcomes: $\overline{Q}^* = (3-4\epsilon)/8$, $P(\overline{Q}^*) = (5+4\epsilon)/8$, and $v(\overline{Q}^*) = (3 - 4\epsilon)^2/128$. (Note that again, before the merger firms $1, 2$, and 3 are active, whereas after the merger the merged firm and also firms 3 and 4 are active.) Hence the merger and divestitures reduce consumer surplus by

$$v(Q^*) - v(\overline{Q}^*) = \frac{3\epsilon(1 - \epsilon)}{32}. \tag{26}$$

Notice that (25) is strictly greater than (26), i.e., although the merger reduces consumer surplus in both cases, the magnitude is lower in the less competitive market.

Now consider a *multimarket* context with two types of market. Type A markets are the same as those described above, while type B markets are such that a merger strictly raises consumer surplus. Let α denote the fraction of type A markets. Note that there exist values of α such that initially the merger would be blocked by the antitrust authority, but would be approved when the type A markets are made less competitive as above.

B.3 Examples of distributions satisfying Assumptions 1 and 2

Suppose that $Q = a[1 - F(P)]$ for some $a > 0$, where for simplicity we will henceforth omit the dependence of P on Q. Differentiating this equation, we find that

$$P'(Q) = -\frac{1}{af(P)} \quad \text{and} \quad P''(Q) = -\frac{f'(P)}{a^2 f(P)^3},$$

which then implies that

$$\sigma(Q) = -\frac{QP''(Q)}{P'(Q)} = -\frac{[1 - F(P)]f'(P)}{f(P)^2}.$$

We now use this to check Assumptions 1 and 2 for different distributions.

Generalized Pareto We have $F(P) = 1 - (1 + \eta P)^{-1/\eta}$ and $f(P) = (1 + \eta P)^{-(1+\eta)/\eta}$ for $\eta < 0$. Hence $\sigma(Q) = 1 + \eta$, and thus $\sigma'(Q) = 0$, so Assumptions 1 and 2 hold.

Logistic We have $F(P) = 1/(1 + e^{-P})$ and $f(P) = e^{-P}/(1 + e^{-P})^2$. Hence

$$\sigma(Q) = 1 - e^{-P} \quad \text{and} \quad \sigma'(Q) = -\frac{(1 + e^{-P})^2}{a}.$$

One can check that Assumption 1 holds if and only if

$$\min\{2 + e^{-P}, 2e^{-P}(1 + e^{-P})\} \geq e^{-P}(1 + e^{-P}),$$

which is clearly true for any $P > 0$. Assumption 2 also holds because $\sigma'(Q) < 0$.

Extreme Value Distribution We have $F(P) = e^{-e^{-P}}$ and $f(P) = e^{-e^{-P}} e^{-P}$. Hence

$$\sigma(Q) = -\frac{\left[1 - e^{-e^{-P}}\right]\left[e^{-P} - 1\right]}{e^{-e^{-P}} e^{-P}} \quad \text{and} \quad \sigma'(Q) = \frac{-\left[1 - e^{-e^{-P}}\right] - \left[e^{-P} - 1\right] e^{-P}}{a\left(e^{-e^{-P}} e^{-P}\right)^2}.$$

Letting $Z = e^{-P}$, one can check that

$$2[1 - \sigma(Q)][2 - \sigma(Q)] + Q\sigma'(Q) > 0 \iff m(Z) > 0,$$

where

$$m(Z) \equiv e^{2Z}[Z^2 + 1 - 3Z] + e^Z[3Z^2 + Z - 2] + 2Z + 1.$$

Simple calculations show that $m(0) = m'(0) = m''(0)$, that $m'''(0) = 3$, and that $m''''(Z) > 0$ for all $Z \in [0, 1]$. This implies that $m(Z) > 0$ for all $Z \in (0, 1)$ as required. One can also check that

$$3 - \sigma(Q) + Q\sigma'(Q) > 0 \iff \tilde{m}(Z) \equiv 3Z^2 - [e^Z - 1]^2[1 + [Z-1]Z] > 0.$$

Because $1 + (Z-1)Z < 1$ for all $Z \in (0,1)$, a sufficient condition for $\tilde{m}(Z) > 0$ is that

$$3Z^2 - [e^Z - 1]^2 = [Z\sqrt{3} - (e^Z - 1)][Z\sqrt{3} + (e^Z - 1)] > 0,$$

which holds because $Z\sqrt{3} - (e^Z - 1)$ is concave in Z, equals 0 at $Z = 0$, and is strictly positive at $Z = 1$, and thus is strictly positive for all $Z \in (0,1)$. Therefore Assumption 1 holds.

One can also check that the numerator of $\sigma'(Q)$ equals $-1 + e^{-Z} - Z^2 + Z$. This equals zero at $Z = 0$, it is strictly negative at $Z = 1$, its derivative with respect to Z evaluated at $Z = 0$ is zero, and it is concave in Z because its second derivative with respect to Z is $e^{-Z} - 2 < 0$. Hence the numerator of $\sigma'(Q)$ is strictly negative, and thus $\sigma'(Q) < 0$. Therefore Assumption 2 holds.

Generalized Extreme Value Distribution We have $F(P) = 1 - e^{-e^P}$ and $f(P) = e^{-e^P}e^P$. Hence

$$\sigma(Q) = 1 - e^{-P} \quad \text{and} \quad \sigma'(Q) = -\frac{e^{-P}}{ae^{-e^P}e^P}.$$

One can check that Assumption 1 holds if and only if

$$\min\{1 + 2e^P, 2(1 + e^{-P})\} \geq e^{-P},$$

which is clearly true for any $P > 0$. Assumption 2 also holds because $\sigma'(Q) < 0$.

Standard Normal Using the fact that $f'(P) = -Pf(P)$, we have

$$\sigma(Q) = \frac{P[1 - F(P)]}{f(P)} \quad \text{and} \quad \sigma'(Q) = -\left\{\frac{[1 + P^2][1 - F(P)]}{f(P)} - P\right\}\frac{1}{af(P)}.$$

Note that a simple lower bound on $f(P)/[1 - F(P)]$ for $P > 0$ is $(P + \sqrt{2 + P^2})/2$ (see, e.g., Duembgen, 2010); note that this bound itself strictly exceeds P. One can check that

$$2[1 - \sigma(Q)][2 - \sigma(Q)] + Q\sigma'(Q) > 0 \iff 4\left[\frac{f(P)}{1 - F(P)}\right]^2 - 5P\frac{f(P)}{1 - F(P)} + P^2 - 1 > 0.$$

Because $f(P)/[1-F(P)] > P$, the left-hand side of the second inequality is strictly increasing in $f(P)/[1 - F(P)]$ (for fixed P). Hence, to prove that the second inequality holds, it is sufficient to prove that it holds when replacing $f(P)/[1 - F(P)]$ with the lower bound $(P + \sqrt{2 + P^2})/2$. Doing this and simplifying, the second inequality reduces to $2 + P^2 > 0$

which is clearly true. One can also check that

$$3 - \sigma(Q) + Q\sigma'(Q) > 0 \iff \frac{f(P)}{1 - F(P)} > \sqrt{\frac{1 + P^2}{3}}.$$

It is straightforward to prove that the second inequality holds, given the lower bound on $f(P)/[1 - F(P)]$ provided above. Therefore Assumption 1 holds.

To show that Assumption 2 holds, note that from above we can write

$$Q\sigma'(Q) = -\left\{\frac{[1 + P^2][1 - F(P)]}{f(P)} - P\right\}\frac{1 - F(p)}{f(P)} < \frac{P[1 - F(P)]}{f(P)} = \sigma(Q) < 1,$$

where the final inequality follows because (from above) $f(P)/[1 - F(P)] > P$.

C Competitiveness and Complementary Assets

As discussed in the text, the result from Proposition 7 with redundant assets can be extended to the case with complementary assets, with some further assumptions.

Proposition 10. *Consider two markets, h and h', that are identical except that the latter is less competitive in that there exists some firm $j \in \mathcal{O}$ such that $\overline{c}_j^{h'}(k) - \overline{c}_j^{h}(k) = \Delta_j > 0$ for all k. Suppose also that assets are complementary in these markets. Then, if it is optimal to propose no divestitures in market h, it is also optimal to propose no divestitures in the less competitive market h', provided that (i) in market h' firm j is active at any (k, i), and (ii) in each of the two markets the number of active firms at $\overline{v}_{\max}$ is at least two and weakly smaller than at $\overline{v}_{ND}$.*[32]

Proof. Suppose that in the less competitive market h' it is optimal to divest (all K assets) to some outsider i (which may or may not be equal to j). We show that $a^{h\prime} \geq a^h$. To do this, we prove that if (all K) assets are divested to outsider i in market h, the average remedies exchange rate conditional on divesting to outsider i is weakly lower in market h than in market h'; since in market h it may be optimal to divest assets to a different outsider, it then follows that $a^{h\prime} \geq a^h$.

To simplify the exposition, we henceforth drop the market-level superscript. Note that if we let $Q(v)$ denote the market-level output associated with consumer surplus level v, we can rewrite a from equation (13) (conditional on divesting all assets to outsider i) as

$$\frac{\pi(Q(\overline{v}_{ND}); \overline{c}_M(-K)) - \pi(Q(\overline{v}_{\max}(i)); \overline{c}_M(-K))}{\overline{v}_{\max}(i) - \overline{v}_{ND}} + \frac{\pi(Q(\overline{v}_{ND}); \overline{c}_M(0)) - \pi(Q(\overline{v}_{ND}); \overline{c}_M(-K))}{\overline{v}_{\max}(i) - \overline{v}_{ND}}. \tag{27}$$

[32]Note that the assumption that the number of active firms is weakly smaller at $\overline{v}_{\max}$ than at $\overline{v}_{ND}$ would hold, for example, if the firm receiving all of the assets at $\overline{v}_{\max}$ is already active absent divestitures.

We now show that this expression is increasing in the marginal cost of outsider j. The first term can be shown to be increasing, using exactly the same steps as in the proof of Proposition 7. Next, we show that the numerator of the second term is increasing. To this end, we rewrite the numerator as

$$-\int_{\overline{c}_M(0)}^{\overline{c}_M(-K)} \frac{\partial \pi(Q(\overline{v}_{ND}); c)}{\partial c} dc = 2\int_{\overline{c}_M(0)}^{\overline{c}_M(-K)} r(Q(\overline{v}_{ND}); c) dc,$$

as $\partial\partial\pi(Q; c)/\partial c = -2r(Q; c)]$, using equations (1) and (2). This expression increases as the market is made less competitive, because $r(Q; c)$ is decreasing in Q while $\overline{v}_{ND}$ also decreases as the market becomes less competitive.

Finally, we show that the denominator of the second term, $\overline{v}_{\max}(i) - \overline{v}_{ND}$, decreases as firm j's marginal cost $\overline{c}_j(k)$ is shifted up. To do this, we make use of two of our assumptions. First, firm j is active in market h' at any level of divestitures (and therefore the same is true in market h because the only difference between the markets is that firm j has higher marginal cost in market h'). Second, in the case where $j = i$, the change in firm j's marginal cost is the same at $k = 0$ and $k = K$. Given these two assumptions, the derivative of $\overline{v}_{\max}(i) - \overline{v}_{ND}$ with respect to Δ_j at a point of differentiability is

$$-\frac{Q\left(\overline{v}_{\max}(i)\right)}{\overline{n}(Q(\overline{v}_{\max}(i)); i) + 1 - \sigma\left(Q\left(\overline{v}_{\max}(i)\right)\right)} + \frac{Q\left(\overline{v}_{ND}\right)}{\overline{n}(Q(\overline{v}_{ND}); i) + 1 - \sigma\left(Q\left(\overline{v}_{ND}\right)\right)},$$

where $\overline{n}(Q(v); i)$ denotes the number of active firms when consumer surplus is equal to v and the asset receiving outsider is firm i. This expression is negative because $2 \leq \overline{n}(Q(\overline{v}_{\max}(i)); i) \leq \overline{n}(Q(\overline{v}_{ND}); i)$ by assumption, and also because Assumption 1 ensures that $Q/(n + 1 - \sigma(Q))$ is weakly increasing in Q. $\square$

D Asset Revenues Duopoly Case

Recall from the proof of Lemma 9 that the slope of the divestitures curve is bounded from above by

$$-s_M(2 - s_M\sigma) + s_i[2(\overline{n} - \sigma) + s_i\sigma],$$

where to ease the exposition we have written s_M instead of $s_M(Q; i)$, s_i instead of $s_i(Q; c(Q))$, $\overline{n}$ instead of $\overline{n}(Q; i)$, and σ instead of $\sigma(Q)$. In this section, we provide a sufficient condition under which this expression is strictly negative in the case where $\overline{n} = 2$ (and thus $s_M = 1 - s_i$), i.e.,

$$-(1 - s_i)[2 - (1 - s_i)\sigma] + s_i[2(2 - \sigma) + s_i\sigma] \equiv x(s_i, \sigma) < 0. \quad (28)$$

Note that

$$\frac{\partial x(s_i,\sigma)}{\partial \sigma} = (1-s_i)^2 - 2s_i + (s_i)^2 = 1 + 2s_i^2 - 4s_i.$$

As this expression is decreasing in s_i, is positive at $s_i = 0$, and is negative at $s_i = 1$, $\partial x(s_i,\sigma)/\partial\sigma \geq 0$ if and only if $s_i \leq 1 - \sqrt{1/2}$. Hence, if $s_i \leq 1 - \sqrt{1/2}$, then $x(s_i,\sigma) \leq x(s_i,1) = -1 + 2s_i + 2s_i^2$, which is indeed strictly negative for $s_i \leq 1 - \sqrt{1/2}$.

That is, a sufficient condition for the divestitures curve to slope down is that, at any divestiture, the market share of the asset-receiving outsider be less than $1 - \sqrt{1/2} \approx 0.29$.