

# An Aggregative Games Approach to Merger Analysis in Multiproduct-Firm Oligopoly\*

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## Abstract

Using an aggregative games approach, we analyze horizontal mergers in a model of multiproduct-firm price competition with CES and logit demand, allowing for arbitrary firm and product heterogeneity. We provide conditions under which a merger raises consumer surplus, and establish the dynamic optimality of a myopic, consumer-surplus-based merger approval policy. We also study the aggregate surplus and external effects of a merger. Finally, we show that the market power effect of a merger, defined as the welfare effect in the absence of merger-specific synergies, can be approximated by the induced, naively computed change in the Herfindahl index.

## 1. Introduction

Using an aggregative games approach, we provide an analysis of horizontal mergers in a model of multiproduct-firm price competition with constant elasticity of substitution (CES) and multinomial logit (MNL) demand systems. The framework allows for arbitrary firm and product heterogeneity. The article makes three contributions. First, we provide conditions under which a merger raises consumer surplus, and establish the dynamic optimality of a myopic, consumer-surplus-based merger approval policy. Second, we provide conditions under

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which a merger raises aggregate surplus and has a positive external effect. Third, we show that the naively computed, merger-induced change in the Herfindahl index (HHI), which plays an important role in merger control, is a good approximation of the market power effect of a merger, defined as its welfare effect in the absence of synergies.

Almost all mergers involve multiproduct firms selling differentiated products. This is reflected in the literature on merger simulation (e.g., Hausman, Leonard, and Zona, 1994; Nevo, 2000a; Peters, 2006; Miller and Weinberg, 2017) and in the literature on upward pricing pressure and compensating marginal cost reductions (e.g., Werden, 1996; Froeb and Werden, 1998; Goppelsroeder, Schinkel, and Tuinstra, 2008; Farrell and Shapiro, 2010; Jaffe and Weyl, 2013), both of which have heavily influenced antitrust practice. Despite this, much of the theoretical literature on horizontal mergers and antitrust, including Farrell and Shapiro (1990), McAfee and Williams (1992), and Nocke and Whinston (2010, 2013), has focused on single-product firms in the homogeneous-goods Cournot setting. An open question is to what extent the insights derived in that earlier literature carry over to more realistic models of (price) competition with differentiated products and multiproduct firms.<sup>1</sup>

In this article, we develop a framework for horizontal merger analysis with multiproduct firms, allowing for arbitrary firm and product heterogeneity. The underlying oligopoly model is one of price competition with CES and MNL demand.<sup>2</sup> These demand systems give rise to an aggregative pricing game: each firm’s profit and optimal behavior depend on rival firms’ prices only through a single-dimensional aggregator. There exists a unique pricing equilibrium, with intuitive comparative statics (Nocke and Schutz, 2018). The resulting levels of consumer surplus and aggregate surplus can be expressed as functions of firms’ equilibrium market shares.

Important for merger analysis, our framework gives rise to *type aggregation*:<sup>3</sup> all relevant information about a firm’s product portfolio (the number of products as well as the qualities and marginal costs of those products) can be summarized in a uni-dimensional sufficient statistic—the firm’s “type.” Indeed, at the heart of the assessment of unilateral effects of horizontal mergers is the Williamson (1968) trade-off between the market power effect (which is due to the internalization of pricing externalities post merger) and the efficiency effect (which is due to potential merger-specific synergies). In our framework, merger-induced synergies can take many forms: some of the marginal costs of the merged firms’ products may go down (while those of others may go up); some of the products’ qualities may improve

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<sup>1</sup>For instance, Whinston (2007) notes: “[...] the Farrell and Shapiro analysis is based on the strong assumption that market competition takes a form that is described well by the Cournot model, both before and after the merger. [...] There has been no work that I am aware of extending the Farrell and Shapiro approach to other forms of market interaction. The papers that formally study the effect of horizontal mergers on price and welfare in other competitive settings [...] all assume that there are no efficiencies generated by the merger.”

<sup>2</sup>More flexible variants of this class of demand systems are ubiquitous in the empirical industrial organization literature (e.g., Berry, 1994; Berry, Levinsohn, and Pakes, 1995; Goldberg, 1995; Nevo, 2001; Björnerstedt and Verboven, 2016).

<sup>3</sup>In fact, Nocke and Schutz (2023) show that CES- or MNL-type demand is not only sufficient but also necessary for type aggregation to obtain.

(while others may degrade); and the merged entity may offer new products (while possibly withdrawing others).<sup>4</sup> The type aggregation property allows us to refrain from imposing any restrictions on the nature of the synergies, as all relevant information can be summarized in the merged firm’s post-merger type.

Using this framework, we first provide an analysis of the consumer surplus effects of mergers. We show that, for any merger, there exists a unique cutoff such that the merger increases consumer surplus if the post-merger type is above that cutoff, and decreases consumer surplus if it is below. As in the homogeneous-goods Cournot model (Farrell and Shapiro, 1990), for a merger to increase consumer surplus it must involve synergies. Moreover, the required synergies are larger the less competitive is the market pre-merger and the larger are the merging parties. This suggests that mergers inducing a larger (naively computed) increase in the Herfindahl index should indeed receive additional scrutiny, as put forward by Nocke and Whinston (2022).

Next, we embed the static pricing game in a dynamic model in which merger opportunities arise stochastically over time. In every period, firms involved in feasible but not-yet-approved mergers have to decide whether to propose their merger, and the antitrust authority has to decide which (if any) of the proposed mergers to approve. Under the key assumption that any given firm can be part of at most one (potential) merger, we show that a completely myopic merger approval policy is dynamically optimal. This extends the main insight of Nocke and Whinston (2010), derived in a homogeneous-goods Cournot setting, to the case of differentiated-products price competition with CES and MNL demand.

Turning to the aggregate surplus effects of mergers, we show that there also exists a post-merger cutoff type above which a merger increases aggregate surplus, and below which it decreases aggregate surplus.<sup>5</sup> That cutoff type is lower than the one for a consumer surplus standard: for a merger to increase aggregate surplus requires fewer synergies than for it to increase consumer surplus, and may not require any synergies at all.

Building on Farrell and Shapiro (1990)’s analysis of the homogeneous-goods Cournot setting, we also study the external effect of a merger, defined as the sum of the effect on consumer surplus and the non-merging firms’ profits. The aggregative properties of our oligopoly model allow us to decompose a merger into a sequence of *infinitesimal* mergers, where, along the sequence, the value of the aggregator changes continuously from its pre-merger to its post-merger equilibrium value. Using this insight, we show that a consumer-surplus-decreasing merger is more likely to have a positive external effect if the non-merging firms command larger pre-merger market shares and if these pre-merger market shares are more concentrated.<sup>6</sup> We also provide a simple and easily implementable test to check whether a consumer-surplus-decreasing merger has a positive external effect. That test only requires

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<sup>4</sup>This is in contrast to the literature on upward pricing pressure and compensating efficiencies (Werden, 1996; Froeb and Werden, 1998; Goppelsroeder, Schinkel, and Tuinstra, 2008; Farrell and Shapiro, 2010; Jaffe and Weyl, 2013), which assumes that synergies take the form of marginal cost reductions.

<sup>5</sup>An analogous result is unavailable in the homogeneous-goods Cournot model.

<sup>6</sup>The converse holds if the merger under consideration is consumer-surplus-increasing.

knowledge of the *pre-merger* market shares and of a demand elasticity parameter.

Many presumptions in merger control rely on the level and the merger-induced change in the Herfindahl index.<sup>7</sup> Defining the market power effect of a merger as its effect in the absence of synergies, we use Taylor approximations to show that the market power effect on consumer surplus and aggregate surplus is proportional to the naively computed change in the Herfindahl index. In providing additional support for the use of the change in the Herfindahl index as a screen for mergers, these approximation results on the welfare effects in the absence of synergies complement our earlier results on the size of the synergies required for consumers not to be harmed.

The analysis in the main text relies on CES and MNL demand systems, which are known to have the *Independence of Irrelevant Alternatives* (IIA) property—a very strong restriction on the implied substitution patterns.<sup>8</sup> In Appendix B, we extend our framework to allow for *nested* CES and *nested* MNL demand structures, thus allowing for substitution patterns that go beyond those implied by the IIA property. There, we show that if all firms are “broad” in that they own entire nests of products, then the mathematical structure underlying the oligopoly model is isomorphic to that in the framework without nests. Hence, all of our results carry over to this more general setting with broad firms. The restriction to broad firms, however, is substantial in that the IIA property still holds at the firm rather than the product level.<sup>9</sup> Further extending our framework, we study the coexistence of broad firms (that each own entire nests) and narrow firms (that each own a subset of products within a single nest), which allows for a slightly more substantial departure from the IIA property.<sup>10</sup> The results on the static and dynamic consumer surplus effects of mergers continue to hold in that setting. We also compare the synergy levels required for a merger among broad versus narrow firms not to harm consumers.

Our article is most closely related to the literature on the unilateral effects of horizontal mergers in industry equilibrium. In a diagrammatic analysis of a merger from perfect competition to monopoly, Williamson (1968) was the first to identify the welfare trade-off between the market power effect of a merger and its efficiency effect. Farrell and Shapiro (1990) provide a thorough analysis of this trade-off in a homogeneous-goods Cournot model. They give a necessary and sufficient condition for a merger to increase consumer surplus, and sufficient conditions for the external effect of a merger to be positive. In a dynamic setting with endogenous merger proposals (and approvals), Nocke and Whinston (2010) study

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<sup>7</sup>For instance, in the 2010 U.S. Horizontal Merger Guidelines, the “naively computed” post-merger Herfindahl index and its merger-induced change are proposed as indicators of the “likely competitive effects of a merger.” The current (2023) Guidelines strengthen these presumptions: “A merger that creates or further consolidates a highly concentrated market that involves an increase in the HHI of more than 100 points is presumed to substantially lessen competition or tend to create a monopoly.”

<sup>8</sup>A demand system is said to have the IIA property if the ratio of demands for any two products is independent of the price of any third product.

<sup>9</sup>That is, the ratio of market shares of any two firms is still independent of the prices charged by any third firm.

<sup>10</sup>Specifically, the ratio of market shares of two narrow firms in different nests does depend on the prices of a third narrow firm, provided the latter offers products in one of the two nests.

the dynamic optimality of a myopic, consumer-surplus-based merger approval policy in a homogeneous-goods Cournot model. We extend Farrell and Shapiro (1990) and Nocke and Whinston (2010)’s analyses to the case of differentiated-goods price competition with multi-product firms.<sup>11,12,13</sup> We also complement Nocke and Whinston (2022) in providing support for using the merger-induced change in the Herfindahl index as a screen for horizontal mergers. However, whereas they focus on the level of synergies required for a merger not to harm consumers, we also derive the loss in consumer and aggregate surplus from a merger without synergies and relate that loss to the Herfindahl index.

The remainder of the article is organized as follows. In Section 2, we introduce the oligopoly model and solve it using aggregative games techniques. There, we also show that the type aggregation property permits a tractable analysis of mergers in multiproduct-firm oligopoly. We study the consumer surplus effects of mergers, in both static and dynamic settings, in Section 3. We turn to the aggregate surplus and external effects of mergers in Section 4. In Section 5, we show that the merger-induced, naively computed change in the Herfindahl index approximates the market power effect of a merger. We discuss various extensions in Section 6.

## 2. Mergers in Multiproduct-Firm Oligopoly

In this section, we present the baseline oligopoly model with CES and MNL demand, solve the model using aggregative games techniques, and use the type aggregation property to simplify the treatment of mergers among multiproduct firms.<sup>14</sup>

□ **The Oligopoly Model.** Consider an industry with a set  $\mathcal{N}$  of imperfectly substitutable products. The representative consumer’s quasilinear indirect utility function is given by

$$y + V(p) = y + V_0 \log \left[ H^0 + \sum_{j \in \mathcal{N}} h_j(p_j) \right], \quad (1)$$

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<sup>11</sup>A separate, less-related strand of literature studies the profitability of mergers in the absence of merger-specific synergies (Salant, Switzer, and Reynolds, 1983; Perry and Porter, 1985; Deneckere and Davidson, 1985). In recent work, Johnson and Rhodes (2021) study the profitability and consumer-surplus effects of mergers in a Cournot setting with pure vertical product differentiation, where each firm may provide one or two (exogenously given) quality levels. Another literature, pioneered by Kamien and Zang (1990), studies the limits of monopolization through mergers in the absence of antitrust policy.

<sup>12</sup>A recent literature focuses on the effects of mergers and merger policy on investment and innovation (e.g., Gowrisankaran, 1999; Federico, Langus, and Valletti, 2018; Mermelstein et al., 2020; Jullien, Bourreau, and Lefouili, 2021; Motta and Tarantino, 2021).

<sup>13</sup>Anderson, Erkal, and Piccinin (2020) use an aggregative games approach to study an oligopoly model with single-product firms under price or quantity competition. They show that a merger without synergies lowers consumer surplus in the short run. In the long-run free-entry equilibrium, ignoring integer constraints, such a merger does not affect consumer surplus.

<sup>14</sup>We refer the reader to Appendix B for extensions with nested demand structures.

where  $y > 0$  is the consumer's income,  $V_0 > 0$  is a market size parameter,  $H^0 \geq 0$  is a baseline-utility parameter, and

$$h_j(p_j) = \begin{cases} \exp\left(\frac{a_j - p_j}{\lambda}\right) & \text{in the case of MNL,} \\ a_j p_j^{1-\sigma} & \text{in the case of CES.} \end{cases}$$

The parameter  $a_j > 0$  summarizes vertical product characteristics and will be referred to as the quality of product  $j$ ;  $\sigma > 1$  and  $\lambda > 0$  measure the substitutability of products. Defining the industry-level aggregator as

$$H(p) \equiv H^0 + \sum_{j \in \mathcal{N}} h_j(p_j),$$

indirect utility can be rewritten as  $V(p) = V_0 \log H(p)$ .

Applying Roy's identity, we obtain the demand for product  $i$ :

$$D_i(p) = V_0 \frac{-h'_i(p_i)}{H(p)}. \quad (2)$$

It is well known that demand system (2) can also be derived from discrete/continuous choice (see Anderson, de Palma, and Thisse, 1987, 1992; Nocke and Schutz, 2018). With such a micro-foundation,  $V_0$  is the total number of consumers,  $h_i/H$  is the probability that a consumer picks product  $i$ , and  $-h'_i/h_i$  is the number of units of product  $i$  a consumer purchases conditional on having chosen that product.<sup>15</sup> Moreover,  $\log H^0$  is the mean utility of the outside option. In the remainder of the article, we normalize  $V_0$  to 1.

Each product  $i$  has constant marginal cost of production  $c_i > 0$ . The set of firms,  $\mathcal{F}$ , is a partition of  $\mathcal{N}$ . That is, each firm has exclusive property rights over the production of a subset of products. The profit of firm  $f \in \mathcal{F}$  is given by

$$\Pi^f = \sum_{j \in f} (p_j - c_j) D_j(p) = \sum_{j \in f} (p_j - c_j) \frac{-h'_j(p_j)}{H(p)}. \quad (3)$$

Firms compete by simultaneously setting prices. We seek the Nash equilibrium of this multiproduct-firm pricing game.

Firms' market shares will play an important role in our analysis. We define the market share of firm  $f$  as

$$s^f = \sum_{j \in f} \frac{h_j}{H}.$$

In the discrete/continuous choice micro-foundation mentioned above,  $s^f$  corresponds to the probability that any given consumer chooses one of firm  $f$ 's products. Moreover,  $s^f$  is equal to

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<sup>15</sup>Under MNL demand,  $-h'_i/h_i$ , the conditional demand for product  $i$ , is constant and equal to  $1/\lambda$ ; under CES demand, it is equal to  $(\sigma - 1)/p_i$ .

firm  $f$ 's output share under MNL demand and to firm  $f$ 's revenue share under CES demand. In both cases, the firms' market shares add up to  $1 - H^0/H$ , where  $H^0/H$  is the market share of the outside option.

□ **Equilibrium Analysis.** We now turn to the equilibrium analysis of our multiproduct-firm pricing game, adopting the aggregative games approach of Nocke and Schutz (2018). The key steps of this approach are as follows. As each firm's profit and marginal profit depend on rivals' prices only through the uni-dimensional aggregator  $H$ , one can take first-order conditions firm by firm and solve for each firm's optimal price vector as a function of the aggregator—the first dimensionality reduction in our analysis. We find that each firm optimally sets the same (normalized) markup for each one of its products—the second dimensionality reduction. We also show that all relevant information about each firm's product portfolio can be summarized by a uni-dimensional sufficient statistic—the third dimensionality reduction. Having written optimal prices as a function of the aggregator, the final step in the equilibrium analysis involves finding the aggregator level that is consistent with firms' pricing decisions.

Differentiating equation (3) yields firm  $f$ 's first-order condition for product  $i \in f$ :

$$\frac{1}{H} \left( -h'_i - (p_i - c_i)h''_i - \frac{\partial H}{\partial p_i} \sum_{j \in f} (p_j - c_j) \frac{-h'_j}{H} \right) = 0.$$

Simplifying, we obtain

$$\frac{p_i - c_i}{p_i} \frac{p_i h''_i}{-h'_i} = 1 + \Pi^f. \quad (4)$$

The left-hand side of equation (4) is the product of two terms: the Lerner index of product  $i$  and the price elasticity of demand that would prevail if the aggregator were held fixed (i.e., under monopolistic competition). Following Nocke and Schutz (2018), we call the left-hand side the  $\iota$ -markup on product  $i$ . As the right-hand side is the same for every  $i \in f$ , firm  $f$  charges the same  $\iota$ -markup,  $\mu^f > 1$ , for each of its products. Under MNL demand, the elasticity term  $-p_i h''_i/h'_i$  is equal to  $p_i/\lambda$ , and so the  $\iota$ -markup on product  $i$  is proportional to the absolute markup,  $p_i - c_i$ , on that product. Under CES demand, the elasticity term is constant and equal to  $\sigma$ , and the  $\iota$ -markup is thus proportional to the Lerner index.

Let  $\alpha$  be equal to  $(\sigma - 1)/\sigma$  under CES demand and equal to 1 under MNL demand. Using the common  $\iota$ -markup property and the fact that  $(h'_j)^2/h''_j = \alpha h_j$ , we can rewrite  $\Pi^f$  as

$$\Pi^f = \sum_{j \in f} (p_j - c_j) \frac{-h'_j}{H} = \sum_{j \in f} \frac{p_j - c_j}{p_j} \frac{p_j h''_j}{-h'_j} \frac{(h'_j)^2}{h''_j} \frac{1}{H} = \sum_{j \in f} \mu^f \alpha h_j \frac{1}{H} = \alpha \mu^f s^f.$$

Inserting this expression into equation (4) and rearranging yields

$$\mu^f = \frac{1}{1 - \alpha s^f}. \quad (5)$$

Intuitively, firms with larger market shares have more market power, and therefore set higher  $\iota$ -markups. In the limit as the firm's market share goes to zero, its optimal markup converges to one. This limiting markup corresponds to the  $\iota$ -markup under monopolistic competition, where firms—regardless of their size—perceive the aggregator  $H$  as fixed when setting their prices.

The above derivations imply that firm  $f$ 's equilibrium profit can be written as

$$\Pi^f = \alpha \mu^f s^f = \mu^f - 1. \quad (6)$$

Turning to the demand side, we now express firm  $f$ 's market share as a function of the aggregator  $H$  and firm  $f$ 's  $\iota$ -markup  $\mu^f$ . Under CES demand, the common  $\iota$ -markup property implies that  $p_j = c_j \sigma / (\sigma - \mu^f)$  for every  $j \in f$ , and so

$$s^f = \frac{1}{H} \sum_{j \in f} a_j \left( \frac{\sigma}{\sigma - \mu^f} c_j \right)^{1-\sigma} = \frac{1}{H} \underbrace{\sum_{j \in f} a_j c_j^{1-\sigma}}_{\equiv T^f} (1 - (1 - \alpha) \mu^f)^{\frac{\alpha}{1-\alpha}}. \quad (7)$$

Under MNL demand, we have  $p_j = c_j + \lambda \mu^f$  for every  $j \in f$ , so that

$$s^f = \frac{1}{H} \sum_{j \in f} \exp \left( \frac{a_j - (c_j + \lambda \mu^f)}{\lambda} \right) = \frac{1}{H} \underbrace{\sum_{j \in f} \exp \left( \frac{a_j - c_j}{\lambda} \right)}_{\equiv T^f} \exp(-\mu^f). \quad (8)$$

Equations (7) and (8) can be summarized as follows:

$$s^f = \begin{cases} \frac{T^f}{H} (1 - (1 - \alpha) \mu^f)^{\frac{\alpha}{1-\alpha}} & \text{under CES demand,} \\ \frac{T^f}{H} e^{-\mu^f} & \text{under MNL demand.} \end{cases} \quad (9)$$

This equation can be viewed as expressing firm-level demand as a decreasing function of the firm's markup, with  $T^f/H$  acting as a demand shifter.

In the following, we refer to  $T^f$  as firm  $f$ 's type. As we shall see below, that uni-dimensional sufficient statistic aggregates all the relevant information about firm  $f$ 's product portfolio—the *type aggregation property*.<sup>16</sup> If firm  $f$  were the only firm and priced all of its products at marginal cost, and if there were no outside option, then  $\log T^f$  would be equal to consumer surplus. That is,  $T^f$  is a measure of firm  $f$ 's ability to provide utility to consumers.

The above analysis implies that if  $H$  is an equilibrium aggregator level, then firm  $f$ 's markup and market share  $\mu^f$  and  $s^f$  jointly solve the system of equations (5) and (9). It is straightforward to show that this system has a unique solution,  $(m(T^f/H), S(T^f/H))$ . We call  $m(T^f/H)$  and  $S(T^f/H)$  the firm's *markup fitting-in function* and *market-share fitting-in*

<sup>16</sup>Nevo and Rossi (2008) were the first to obtain the type aggregation property in the case of MNL demand. They dubbed  $\log T^f$  the adjusted inclusive value of firm  $f$ .



function, respectively. Both fitting-in functions are increasing,  $m' > 0$  and  $S' > 0$ , i.e., a firm that has a higher type and operates in a less competitive environment (lower  $H$ ) sets a higher markup and commands a higher market share. Moreover, the range of  $S$  is the entire interval  $(0, 1)$ . Using equation (6), we obtain the *profit fitting-in function*  $\pi(T^f/H) = m(T^f/H) - 1$ .

The final step of the analysis consists in finding the aggregator level that is consistent with the firms' pricing decisions. That level is pinned down by the equilibrium condition

$$\frac{H^0}{H} + \sum_{f \in \mathcal{F}} S\left(\frac{T^f}{H}\right) = 1, \quad (10)$$

which says that market shares add up to one. The continuity and monotonicity properties of  $S$ , along with the fact that  $S$  has full range, imply that equation (10) has a unique solution. This establishes equilibrium existence and uniqueness.

The following proposition summarizes these insights and provides intuitive comparative statics:<sup>17</sup>

**Proposition 1.** *The multiproduct-firm pricing game has a unique equilibrium. The equilibrium aggregator level  $H^*$  is the unique solution of equation (10); firm  $f$  sets a markup of  $m(T^f/H^*)$ , commands a market share of  $S(T^f/H^*)$ , and earns a profit of  $\pi(T^f/H^*)$ . An increase in  $T^f$  raises firm  $f$ 's equilibrium markup, market share, and profit, lowers the markup, market share, and profit of any rival, and raises consumer surplus and aggregate surplus.*

*Proof.* See Section 5 in Nocke and Schutz (2018). □

□ **Modeling Mergers.** Consider a merger between the firms  $\mathcal{M} \subsetneq \mathcal{F}$ , and let  $\mathcal{O} \equiv \mathcal{F} \setminus \mathcal{M}$  be the set of non-merging firms—the outsiders. We assume throughout that the merger does not directly affect the outsiders. That is, post merger each firm  $f \in \mathcal{O}$  continues to offer the same products, with the same qualities and costs, implying that  $T^f$  remains unchanged.

By contrast, the merger is allowed to affect the merging firms' product portfolios. In particular, the marginal costs and qualities of the merging firms' pre-existing products may change, and the merged firm may offer fewer or more products than before. Our aggregative games tools and the type aggregation property allow us to account for such changes in a parsimonious way, as all that matters is the type of the merged firm,  $T^M$ .

A special case of interest arises when the merger does not involve any synergies, in the sense that the merged firm supplies the same products, with the same qualities and costs, as the merger partners did pre-merger. In that case, the type of the merged firm is given by  $T^M = \sum_{f \in \mathcal{M}} T^f$ . We say that the merger involves (positive) synergies if  $T^M > \sum_{f \in \mathcal{M}} T^f$ .

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<sup>17</sup>For comparative statics in aggregative oligopoly games with single-product firms, see also Anderson, Erkal, and Piccinin (2020).

### 3. Consumer Surplus Effects of Mergers

In this section, we analyze the consumer surplus effects of mergers, which are at the heart of merger control. We begin by studying a static setting and then turn to a dynamic setting with endogenous mergers.

□ **Static Analysis.** Consider a merger  $M$  between the firms in  $\mathcal{M}$ . Let  $H^*$  (resp.,  $\bar{H}^*$ ) denote the equilibrium value of the aggregator before (resp., after) the merger. As consumer surplus is increasing in the value of that aggregator, we say that the merger is *CS-increasing* (resp., *CS-decreasing*) if  $\bar{H}^* > H^*$  (resp.,  $\bar{H}^* < H^*$ ); it is *CS-neutral* if  $\bar{H}^* = H^*$ .

Suppose the merger is CS-neutral. This implies that the market share of each outsider  $g \in \mathcal{O}$ ,  $S(T^g/H^*)$ , and the market share of the outside option,  $H^0/H^*$ , are unaffected by the merger. As market shares add up to one (equation (10)), this implies that the post-merger market share of the merged firm is equal to the sum of the pre-merger market shares of the merger partners:

$$S\left(\frac{T^M}{H^*}\right) = \sum_{f \in \mathcal{M}} S\left(\frac{T^f}{H^*}\right),$$

where we have used the fact that  $\bar{H}^* = H^*$  by CS-neutrality.

As  $S$  is strictly increasing and has full range, there exists a unique cutoff type  $\hat{T}^M$  such that the merger is CS-neutral if and only if  $T^M = \hat{T}^M$ . By Proposition 1,  $\bar{H}^*$  is strictly increasing in  $T^M$ , implying that the merger is CS-increasing if  $T^M > \hat{T}^M$ , and CS-decreasing if the inequality is reversed.

As the market-share fitting-in function  $S$  is strictly concave (see Lemma 5 in Online Appendix C) and satisfies  $S(0) = 0$ , that function is sub-additive. This implies that the cutoff type satisfies  $\hat{T}^M > \sum_{f \in \mathcal{M}} T^f$ . That is, for the merger to be CS-nondecreasing it has to involve positive synergies.<sup>18</sup>

Next, we argue that a merger that does not involve synergies is profitable for the merger partners.<sup>19</sup> To see this, note first that such a merger lowers the equilibrium aggregator level, as it is CS-decreasing, as shown above. This, in turn, implies that the outsiders raise their prices, as the markup fitting-in function  $m$  is increasing. As a result, the merger partners face less competition, and therefore make strictly higher profits after the merger. By Proposition 1, a merger involving positive synergies must be even more profitable than one that does not.

We summarize these insights in the following proposition:

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<sup>18</sup>Farrell and Shapiro (1990) obtain the same conclusion in the homogeneous-goods Cournot model. By contrast, Johnson and Rhodes (2021) show that a merger without synergies may benefit consumers in a Cournot setting with pure vertical product differentiation.

<sup>19</sup>This is usually the case in models of price competition with differentiated products (see, e.g., Deneckere and Davidson, 1985).

**Proposition 2.** *For a merger among the firms in  $\mathcal{M}$ , there exists a unique  $\hat{T}^M > \sum_{f \in \mathcal{M}} T^f$  such that the merger is CS-neutral if the post-merger type satisfies  $T^M = \hat{T}^M$ , CS-decreasing if  $T^M < \hat{T}^M$ , and CS-increasing if  $T^M > \hat{T}^M$ . That is, for the merger to be CS-nondecreasing requires strictly positive synergies. If the merger involves weakly positive synergies,  $T^M \geq \sum_{f \in \mathcal{M}} T^f$  (and in particular if it is CS-nondecreasing), then it is privately profitable in that it strictly increases the joint profit of the merger partners.*

We now turn to the comparative statics of the post-merger cutoff-type  $\hat{T}^M$ . First, we consider the thought experiment of changing the pre-merger aggregator level  $H^*$  while holding fixed the characteristics of the merger. Second, we compare two alternative mergers in a given industry, thus holding fixed the pre-merger aggregator level  $H^*$ .

The first comparative statics result shows that the synergies required for a merger to be CS-nondecreasing are smaller the more competitive is the market before the merger:

**Proposition 3.** *For a merger among the firms in  $\mathcal{M}$ , the post-merger cutoff type  $\hat{T}^M$  is strictly decreasing in the pre-merger level of the aggregator,  $H^*$ .*

*Proof.* See Online Appendix D. □

To see the intuition, consider a merger between two symmetric single-product firms, producing products  $i$  and  $j$  at pre-merger marginal cost  $c$ , and charging the pre-merger price  $p^*$ . Suppose the merger-induced synergies materialize only through a symmetric marginal cost reduction. As shown by Werden (1996), for the merger to be CS-neutral, the common post-merger marginal cost  $\hat{c}$  must be such that

$$c - \hat{c} = \frac{d}{1-d}(p^* - c), \quad (11)$$

where  $d \equiv -(\partial D_j / \partial p_i) / (\partial D_i / \partial p_i)$  is the diversion ratio between goods  $i$  and  $j$ .

The left-hand side of equation (11) gives the required change in marginal cost whereas the right-hand side represents the increase in market power due to the post-merger internalization of competitive externalities. An increase in the pre-merger aggregator level  $H^*$  reduces the right-hand side through two channels: it lowers both the pre-merger equilibrium price  $p^*$  and the diversion ratio  $d$ .<sup>20</sup> Proposition 3 generalizes this intuition to mergers between arbitrary sets of firms with arbitrary forms of synergies.

We now turn to our second comparative statics result. It shows that the synergies required for a merger to be CS-nondecreasing are larger for mergers involving larger firms, holding fixed the pre-merger aggregator level  $H^*$ :

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<sup>20</sup>In our model, the diversion ratio between two symmetric single-product firms can be shown to be equal to  $\alpha s^* / (1 - \alpha s^*)$ , which is increasing in the equilibrium market share  $s^*$ , and thus decreasing in  $H^*$ .

**Proposition 4.** Consider a merger between the firms in  $\mathcal{M} = \{f, g\}$ , resp.,  $\mathcal{M}' = \{f', g'\}$ , where  $T^f \geq T^{f'}$  and  $T^g > T^{g'}$ . Then, the “larger” merger  $\mathcal{M}$  requires larger synergies than  $\mathcal{M}'$ , in the sense of a larger fractional increase in type.<sup>21</sup>

$$\frac{\hat{T}^{\mathcal{M}}}{T^f + T^g} > \frac{\hat{T}^{\mathcal{M}'}}{T^{f'} + T^{g'}}.$$

*Proof.* See Online Appendix D. □

To see the intuition, suppose demand takes the MNL form, each merger involves symmetric single-product firms, and merger-induced synergies materialize only through a symmetric reduction in the common marginal cost. The right-hand side of equation (11) is larger for merger  $\mathcal{M}$  than  $\mathcal{M}'$ , as each merger partner in  $\mathcal{M}$  has a higher pre-merger market share, implying that both its pre-merger diversion ratio  $d$  and its margin  $p^* - c$  are larger. Hence, the (absolute and percentage) cost reduction necessary for the merger to be CS-neutral is larger for the larger merger.<sup>22</sup>

□ **Dynamic Analysis.** In industries in which merger opportunities are not isolated events, a static analysis of the consumer surplus effect of a given proposed merger may be inappropriate: the approval decision on a merger may affect both the consumer surplus effects of future mergers, and therefore the set of mergers that will be approved in the future, and the profitability of future mergers, and therefore the set of mergers that will be proposed in the future.

In the following, we show that a completely myopic merger approval policy, whereby, in every period, the antitrust authority approves only those mergers that raise consumer surplus given current market conditions, is dynamically optimal. This extends the main insight of Nocke and Whinston (2010), derived in the context of a homogeneous-goods Cournot model, to the case of differentiated-goods price competition with MNL and CES demand.

**Framework.** Following Nocke and Whinston (2010), we assume that there is a collection of potential mergers,  $M_1, \dots, M_K$ , corresponding to sets of merger partners  $\mathcal{M}_1, \dots, \mathcal{M}_K$ , and that all of these mergers are disjoint, i.e.,  $\mathcal{M}_k \cap \mathcal{M}_l = \emptyset$  for  $k \neq l$ . Disjointness means that each firm has a distinct set of natural merger partners that have the potential to create sizable synergies by merging.

There are  $\tau < \infty$  periods in which mergers may become feasible, and be proposed to the antitrust authority for approval. Any merger  $M_k$  may become feasible at the beginning of period  $1 \leq t \leq \tau$  with probability  $p_t^{M_k}$ , where  $\sum_t p_t^{M_k} \leq 1$ . Once merger  $M_k$  has become

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<sup>21</sup>If the merger partners were the only firms and were pricing all of their products at marginal cost both pre- and post-merger, and if there were no outside option, then the logarithm of this fractional increase would give the merger-induced increase in consumer surplus. The proposition implies that the larger merger also requires a larger absolute increase in type  $\hat{T}^{\mathcal{M}} - (T^f + T^g) > \hat{T}^{\mathcal{M}'} - (T^{f'} + T^{g'})$ .

<sup>22</sup>As the firms involved in the larger merger have a lower marginal cost pre merger, a larger absolute cost reduction implies a larger percentage cost reduction.

feasible, the merger partners learn the realization of their post-merger type  $T^{M_k}$ , drawn from a continuous probability distribution  $G_t^{M_k}$ .

If merger  $M_k$  has become feasible in period  $t$ , or became feasible earlier but has not yet been approved, the merger partners decide whether to propose it to the antitrust authority. We assume that the merger is proposed if and only if it is in the merger partners' joint interest to do so. When doing so, they observe the type not only of their own merger but also that of any other feasible but not yet approved merger (as well as the type of every firm).

If a feasible merger is proposed, the antitrust authority observes its efficiency (i.e., the post-merger type); the authority also observes the types of all firms. Market structure (as summarized by the vector of firm types) changes according to the authority's approval decisions. Importantly, although a blocked merger cannot be consummated, it can be proposed again in the future.

At the end of period  $t$ , firms compete in prices under complete information, as described in Section 2. Payoffs in each period therefore depend only on the market structure at the end of that period. Firms and the authority discount future payoffs with factor  $\delta \leq 1$ .

**Results.** The main result of this subsection is that the myopically CS-maximizing merger policy is dynamically optimal in that it maximizes the discounted sum of consumer surplus. The myopically CS-maximizing merger policy is the merger approval rule that, in each period  $t$ , maximizes consumer surplus in that period, given current market structure and the set of proposed mergers.<sup>23</sup> In the remainder of this subsection, we provide the intuition and the formal statement for the main result, and refer the reader to Appendix A for details.

Our result on the dynamic optimality of a myopically CS-maximizing merger policy comes in two parts. First, we ignore the incentive constraints for proposing mergers and show that the myopically CS-maximizing merger policy maximizes discounted consumer surplus *if all feasible but not yet approved mergers are proposed in each period*. Second, we show that there exists a subgame-perfect equilibrium in which all feasible but not yet approved mergers are indeed proposed in each period. Moreover, any subgame-perfect equilibrium induces the same optimal sequence of period-by-period consumer surpluses.

The first part follows from a fundamental sign-preserving complementarity in the consumer surplus effects of mergers, formally stated in Lemma 1 in Appendix A. Consider two disjoint mergers  $M_k$  and  $M_l$ , and suppose first that each is CS-nondecreasing given current market structure, i.e.,  $T^{M_k} \geq \hat{T}^{M_k}$  and  $T^{M_l} \geq \hat{T}^{M_l}$ . If merger  $M_k$  is implemented first, then  $H^*$  weakly increases as the merger is CS-nondecreasing. By Proposition 3, this implies that  $\hat{T}^{M_l}$  weakly decreases so that the condition for merger  $M_l$  to be CS-nondecreasing,  $T^{M_l} \geq \hat{T}^{M_l}$ , continues to hold. By the same argument, if both mergers are CS-decreasing

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<sup>23</sup>There may be more than one set of merger approvals that maximizes consumer surplus in a given period but, if so, these sets differ only by mergers that are CS-neutral given the other mergers in those sets. Proposition 1 and the fact that post-merger types are drawn from continuous distributions imply that any given merger is CS-neutral with probability zero. For ease of exposition, we assume that the antitrust authority approves the largest myopically CS-maximizing set of mergers (which can be shown to be unique).

given current market structure, then implementing merger  $M_k$  increases the cutoff type for the other merger  $M_l$ , implying that  $M_l$  remains CS-decreasing. Proposition 3 implies that a CS-increasing merger  $M_k$  can induce an otherwise CS-decreasing merger  $M_l$  to become CS-nondecreasing. If so, merger  $M_k$  remains CS-increasing conditional on merger  $M_l$  taking place, as shown in Lemma 2 in Appendix A.

These insights imply that if the antitrust authority approves only mergers that are CS-nondecreasing at the time of approval, then it will not have ex post regret about previously approved mergers (as these remain CS-nondecreasing) nor about previously rejected mergers (as these remain feasible and therefore can be implemented once they become CS-nondecreasing). Hence, as shown in Lemma 3 in Appendix A, if all feasible but not yet approved mergers are proposed in each period, then the myopically CS-maximizing merger policy maximizes discounted consumer surplus, no matter what the realization of feasible mergers is.

The second part in establishing the dynamic optimality of a myopic merger approval policy consists in showing that there exists a subgame-perfect equilibrium in which, in each period, every feasible but not yet approved merger is proposed (and that any other equilibrium is outcome-equivalent). We already established in Proposition 2 that a CS-nondecreasing merger  $M_k$  is privately profitable, holding fixed the market structure among outsiders. Lemma 4 in Appendix A establishes that a CS-nondecreasing merger is still privately profitable even if it induces (directly or indirectly) other mergers to become CS-nondecreasing, resulting in their approval. Hence, any merger that a CS-maximizing antitrust authority would ever want to approve will be proposed in equilibrium.

We thus obtain:

**Proposition 5.** *Suppose that the antitrust authority adopts the myopically CS-maximizing merger policy. Then, all feasible mergers being proposed in each period after any history is a subgame-perfect equilibrium. The resulting outcome maximizes discounted consumer surplus, no matter what the realized sequence of feasible mergers. Moreover, every subgame-perfect equilibrium results in the same optimal level of consumer surplus in each period.*

*Proof.* See Appendix A. □

As in Nocke and Whinston (2010)'s homogeneous-goods Cournot model, a myopically CS-maximizing merger policy is dynamically optimal in a strong sense: the antitrust authority could not improve upon the resulting outcome even if it had perfect foresight about future realizations of feasible mergers (which it does not) nor if it had the power to undo previously approved mergers (which we assume it does not).

## 4. Aggregate Surplus and External Effects of Mergers

Although most antitrust authorities have adopted a consumer surplus standard, or something close to it, it is also important to study the impact of mergers on aggregate surplus, which

we undertake next.

□ **Aggregate Surplus Effects.** Consider a merger  $M$  among the firms in  $\mathcal{M}$ , and let  $T^M$  be the merged firm's type. If  $T^M = \hat{T}^M$ , where  $\hat{T}^M$  is the cutoff type defined in Proposition 2, then the merger is CS-neutral. Moreover, as the merger does not affect the equilibrium value of the aggregator, it has no impact on the outsiders' equilibrium profits. As the merger is profitable by Proposition 2, it is therefore aggregate-surplus-increasing (AS-increasing). Furthermore, it is straightforward to show that the merger is AS-decreasing if  $T^M$  is small.<sup>24</sup> The continuity of aggregate surplus in types implies the existence of a cutoff type  $\tilde{T}^M$  that makes the merger AS-neutral. By monotonicity of aggregate surplus (Proposition 1), that cutoff type is unique, and the merger is AS-increasing if  $T^M > \tilde{T}^M$ , and AS-decreasing if  $T^M < \tilde{T}^M$ .

We summarize these insights in the following proposition:

**Proposition 6.** *For a merger among the firms in  $\mathcal{M}$ , there exists a unique  $\tilde{T}^M < \hat{T}^M$  such that the merger is AS-neutral if the post-merger type satisfies  $T^M = \tilde{T}^M$ , AS-decreasing if  $T^M < \tilde{T}^M$ , and AS-increasing if  $T^M > \tilde{T}^M$ .*

Note that there is no counterpart to Proposition 6 in Farrell and Shapiro (1990)'s classical analysis. The reason is that, in the homogeneous-goods Cournot model, equilibrium aggregate surplus is not a monotonic function of firms' marginal costs (Lahiri and Ono, 1988; Zhao, 2001). By contrast, we are able to leverage the monotonicity of aggregate surplus in firms' types to obtain Proposition 6.

That  $\tilde{T}^M < \hat{T}^M$  follows immediately from the fact that a CS-neutral merger is AS-increasing. Whether an AS-neutral merger must involve positive synergies (i.e.,  $\tilde{T}^M > \sum_{f \in \mathcal{I}} T^f$ ) is unclear. On the one hand, a merger without synergies lowers the equilibrium aggregator. On the other, it reallocates market shares toward the outsiders, which can raise aggregate surplus if those firms are initially producing too little relative to the merger partners.

An example where a merger without synergies is AS-increasing can easily be constructed in the case of MNL demand without an outside option ( $H^0 = 0$ ). Suppose there are three firms, 1, 2, and 3, with pre-merger types  $T^1 = 1$  and  $T^2 = T^3 = 1/2$ . In the aggregate-surplus-maximizing pre-merger allocation, which can be obtained by setting all markups equal to zero, firm 1 commands a market share of  $1/2$ , whereas firms 2 and 3 each receive a market share of  $1/4$ . The equilibrium is efficient if and only if it induces that allocation, which arises if and only if all firms charge the same markup.<sup>25</sup> As firm 1 has a higher type,

<sup>24</sup>As  $T^M$  tends to zero, post-merger aggregate surplus converges to equilibrium aggregate surplus when firm  $M$  does not exist. This limiting value is also equal to equilibrium aggregate surplus before the merger if the firms in  $\mathcal{M}$  do not exist. As aggregate surplus is strictly increasing in types, that value is strictly lower than the actual pre-merger aggregate surplus.

<sup>25</sup>Under MNL demand without outside option, the total quantity consumed is fixed, and so higher markups are merely a transfer from consumers to firms. Aggregate surplus is therefore maximized at equal markups, no matter what their level.

it sets a higher markup than its rivals in equilibrium, resulting in an inefficient allocation. Consider now a merger  $M$  between firms 2 and 3, and, assuming no synergies, let  $T^M = 1$ . As firm 1 and the merged firm have the same type, they charge the same equilibrium markups, implying that the post-merger equilibrium allocation is efficient. The merger is therefore AS-increasing.<sup>26</sup>

□ **External Effects.** We now extend Farrell and Shapiro (1990)’s analysis of the external effects of a merger, defined as the sum of its impact on consumer surplus and outsiders’ profits. To the extent that a merger is proposed by the merger partners only if it is in their joint interest to do so, a positive external effect is a sufficient (“safe harbor”) condition for the merger to raise aggregate surplus. The idea behind focusing on the external effect is that the profitability of a merger depends on the magnitude of internal cost savings, and that these are hard to assess for an antitrust authority. As we shall see below, the sign of the external effects of a merger can be related to pre-merger market shares.

Consider a merger  $M$  among the firms in  $\mathcal{M}$ , and let  $\mathcal{O}$  be the set of outsiders. Let  $H^*$  and  $\bar{H}^*$  denote the pre- and post-merger equilibrium values of the aggregator, respectively. Recalling that  $\pi(T/H) = m(T/H) - 1$ , the external effect of the merger can be written as

$$\mathcal{E}^M = \log \bar{H}^* - \log H^* + \sum_{f \in \mathcal{O}} \left( m \left( \frac{T^f}{\bar{H}^*} \right) - m \left( \frac{T^f}{H^*} \right) \right) = - \int_{H^*}^{\bar{H}^*} \frac{\eta(H)}{H} dH,$$

where

$$\eta(H) \equiv -1 + \sum_{f \in \mathcal{O}} \frac{T^f}{H} m' \left( \frac{T^f}{H} \right).$$

Hence, as in Farrell and Shapiro (1990), the merger can be thought of as a sequence of infinitesimal mergers  $dH$ , where, along the sequence, the value of the aggregator changes progressively from  $H^*$  to  $\bar{H}^*$ . The sign of the external effect of an infinitesimal CS-decreasing (resp. CS-increasing) merger is thus given by  $\eta(H)$  (resp.  $-\eta(H)$ ). In Online Appendix E, we show that  $\eta(H)$  can be rewritten as

$$\eta(H) = -1 + \sum_{f \in \mathcal{O}} \frac{\alpha s^f (1 - s^f)}{(1 - \alpha s^f)(1 - s^f + \alpha (s^f)^2)}, \quad (12)$$

where  $s^f = S(T^f/H)$ . The first term on the right-hand side corresponds to the merger’s consumer surplus effect. The second term corresponds to the effect on outsider’s profits. Note that it has the opposite sign to the first term, as outsiders’ profits are decreasing in  $H$ . The magnitude of the second term depends both on the level of outsiders’ market shares and their concentration.

We now formalize the notions of larger and more concentrated market shares of outsiders.

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<sup>26</sup>By the same token, with MNL demand, no outside option, and three firms 1, 2, and 3 such that  $T^1 = T^2 = T^3$ , a merger between firms 2 and 3 is AS-decreasing if it does not give rise to synergies.



A pre-merger industry structure among outsiders is a vector  $(s^f)_{f \in \mathcal{O}}$ , where  $s^f \in (0, 1)$  for every  $f \in \mathcal{O}$ , and  $\sum_{f \in \mathcal{O}} s^f < 1$ . Let  $s = (s^f)_{f \in \mathcal{O}}$  and  $s' = (s'^f)_{f \in \mathcal{O}'}$  be two pre-merger industry structures. Outsiders have *larger market shares* under  $s$  than under  $s'$ , denoted  $s \geq_1 s'$ , if there exists a one-to-one mapping  $\kappa : \mathcal{O}' \rightarrow \mathcal{O}$  such that  $s^{\kappa(f)} \geq s'^f$  for every  $f \in \mathcal{O}'$ . Outsiders' market shares are *more concentrated* under outsider industry structure  $s$  than under  $s'$ , denoted  $s \geq_2 s'$ , if  $s$  and  $s'$  have the same length and the distribution of  $s$  is a mean-preserving spread of the distribution of  $s'$  (Rothschild and Stiglitz, 1970).

In the remainder of this subsection, we focus on CS-decreasing mergers to fix ideas. The following proposition shows that a CS-decreasing merger has a negative external effect when products are poor substitutes. When, instead, products are close substitutes, a merger is more likely to have a positive external effects if the market shares of the outsider are larger or more concentrated.

**Proposition 7.** *Let  $\bar{\alpha} = \frac{3}{2}(\sqrt{57} - 7) \simeq 0.82$ .*

1. *If  $\alpha \leq \bar{\alpha}$ , then any CS-decreasing merger has a negative external effect. If, instead,  $\alpha > \bar{\alpha}$ , then there exist CS-decreasing mergers that have a positive external effect, and CS-decreasing mergers that have a negative external effect.*

2. *Consider two infinitesimal CS-decreasing mergers,  $M$  and  $M'$ , with pre-merger outsider industry structures  $s = (s^f)_{f \in \mathcal{O}}$  and  $s' = (s'^f)_{f \in \mathcal{O}'}$ . Suppose one (or both) of the following holds:*

(i)  *$s \geq_1 s'$  and  $s^f \leq s^* \simeq 0.68$  for every  $f \in \mathcal{O}$ .*

(ii)  *$s \geq_2 s'$ ,  $s^f \leq \hat{s} \simeq 0.29$  for every  $f \in \mathcal{O}$ , and  $s'^f \leq \hat{s}$  for every  $f \in \mathcal{O}'$ .*

*If merger  $M'$  has a positive external effect, then so does merger  $M$ .*

*Proof.* See Online Appendix E. □

To understand the intuition, recall that the profit of an outsider  $f$  can be written as

$$\Pi^f = \alpha \mu^f s^f = \begin{cases} \alpha \frac{T^f}{H} \mu^f (1 - (1 - \alpha)\mu^f)^{\frac{\alpha}{1-\alpha}} & \text{under CES,} \\ \frac{T^f}{H} \mu^f e^{-\mu^f} & \text{under MNL.} \end{cases} \quad (13)$$

The positive impact of an infinitesimal CS-decreasing merger on  $\Pi^f$  can be decomposed into two effects. First, holding  $\mu^f$  fixed, the merger increases  $\Pi^f$  by  $\Pi^f \times |dH/H|$ . Hence, the “direct” effect on outsiders’ joint profit is positive and proportional to their joint profit. Second, each outsider responds by increasing its markup  $\mu^f$ . This “indirect” effect is negative because oligopolistic markups are always above those of monopolistically competitive firms (which perceive  $H$  as fixed), and so any further increase must reduce profit for a fixed  $H$ . Indeed, as can be seen from equation (13),  $\Pi^f$  is decreasing in  $\mu^f$  for  $\mu^f > 1$ .

Consider now the first part of the proposition. The condition  $\alpha \leq \bar{\alpha}$  can be satisfied only in the CES case, where it translates to  $\sigma \leq \bar{\sigma} \simeq 5.7$ . To see why a CS-decreasing must

have a negative external effect for small  $\sigma$ , note that total expenditure is at most  $\sigma - 1$  (and equal to  $\sigma - 1$  if there is no outside option). Hence, the direct effect on outsiders' profits is bounded above by  $(\sigma - 1)|dH/H|$ , whereas the indirect effect is always negative and the effect on consumer surplus is  $-|dH/H|$ . This implies that  $\eta$  is negative for  $\sigma \leq 2$ . The proposition shows that the cutoff value of  $\sigma$  below which the external effect is always negative is, in fact, much larger.

Consider now the second part of the proposition, which (by the first part) has bite only if  $\alpha > \bar{\alpha}$ . Recall that the direct effect on outsider  $f$ 's profits is  $\Pi^f |dH/H|$ . The indirect effect can be shown to equal

$$-\Pi^f \frac{\alpha(s^f)^2}{1 - s^f + \alpha(s^f)^2} \left| \frac{dH}{H} \right|.$$

If  $s^f$  is small, the indirect effect is thus dwarfed by the direct effect. As the sum of outsiders' profits is increasing and convex in their market shares, the overall effect on outsiders' profits tends to be larger when those market shares are larger or more concentrated, provided the outsiders are not too large.

As  $s^f$  increases, the relative size of the indirect effect on outsider  $f$ 's profit increases. In the limit as  $s^f$  goes to one (and therefore the other outsiders' market shares vanish), the indirect effect fully offsets the direct effect, so that the external effect must be negative. This explains the presence of the cutoffs  $s^*$  and  $\hat{s}$  in the statement of the proposition.

We close this section by discussing the external effect of a non-infinitesimal CS-decreasing merger. From Proposition 7, such a merger always has a negative external effect if  $\alpha \leq \bar{\alpha}$ . Suppose now that  $\alpha > \bar{\alpha}$ . By continuity, the result in Proposition 7—that a merger is more likely to have a positive external effect, the larger or more concentrated are the outsiders—continues to hold as long as the merger's impact on the equilibrium aggregator level is not too large.

Moreover, regardless of the magnitude of the merger-induced decrease in  $H$ , a sufficient condition for the merger to have a positive external effect is that  $\eta(H^*) > 0$  (i.e., at the pre-merger aggregator level, an infinitesimal CS-decreasing merger has a positive external effect). The reason is the following. The external effect of the merger is the integral of the external effects of the infinitesimal mergers along the path from  $H^*$  to  $\bar{H}^* < H^*$ . As the merger is CS-decreasing by assumption, outsiders' market shares increase along that sequence. Hence, if  $\eta(H^*) > 0$ , then, by part 2(i) of Proposition 7,  $\eta(H)$  remains positive along the sequence (provided no outsider reaches a market share larger than  $s^*$ ), and so the external effect of the merger is positive. Importantly, checking whether  $\eta(H^*) > 0$  involves using only the outsiders' *pre-merger* market shares (see equation (12)).

## 5. The Herfindahl Index and the Welfare Effects of Mergers

Industry-level concentration measures feature prominently in merger analysis and—based on the revised (2023) U.S. Merger Guidelines—will play an even larger role in the years to come. The common presumption is that the market power effect of a merger tends to be larger when (i) the (naively computed) post-merger Herfindahl index (HHI) is larger and (ii) the (naively computed) merger-induced increase in HHI is larger. Here, “naively computed” means that market shares are computed assuming that the market shares of the non-merging outsiders are not affected by the merger.

Propositions 3 and 4 in Section 3 above provide some theoretical support for the use of the merger-induced variation in the HHI to screen mergers. Defining the HHI as  $\text{HHI} \equiv \sum_{f \in \mathcal{F}} (s^f)^2$ , the (naively computed) increase in the index induced by merger  $\mathcal{M} = \{f, g\}$  is given by  $\Delta\text{HHI} = 2s^f s^g$ , and is larger for a merger involving larger firms.

Proposition 3 shows that, holding fixed the types of the merger partners, a decrease in the pre-merger aggregator level  $H^*$ , resulting in a higher  $\Delta\text{HHI}$ , raises the required level of synergies for the merger to be CS-nondecreasing. Proposition 4 shows that, holding fixed  $H^*$ , a merger involving firms with higher types, and thus resulting in a higher  $\Delta\text{HHI}$ , also raises that required level of synergies. Along the same lines, Nocke and Whinston (2022) show, theoretically and empirically, that the synergy level required for consumers not to be harmed is positively related to  $\Delta\text{HHI}$  but unrelated to the (naively computed) post-merger level of HHI when controlling for  $\Delta\text{HHI}$ .

In the remainder of this section, we take a different approach by relating the welfare effects of a merger *in the absence of synergies* to concentration measures. By considering a merger without synergies, we therefore isolate its pure market power effect. Using approximation techniques, we show that  $\Delta\text{HHI}$  is an appropriate measure of the market power effect of the merger. To this end, we first relate measures of industry performance to the equilibrium market share vector.

Let  $s = (s^f)_{f \in \mathcal{F}}$  be the profile of equilibrium market shares. Assume that consumers have access to an outside option ( $H^0 > 0$ ), so that  $\sum_{f \in \mathcal{F}} s^f < 1$ .<sup>27</sup> Equation (10) implies that the equilibrium aggregator level  $H^*$  is equal to  $H^0 / (1 - \sum_{f \in \mathcal{F}} s^f)$ . As shown in Anderson and Nocke (2014), this implies that consumer surplus can be written as a function of market

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<sup>27</sup>In the presence of an outside option ( $H^0 > 0$ ), computing market shares in practice is well known to be non-trivial, as the potential market size may be hard to determine. This issue is ubiquitous in the literature on demand estimation in differentiated-products industries (see, e.g., Berry, 1994; Berry, Levinsohn, and Pakes, 1995; Nevo, 2001). Nevo (2000b) provides guidance on how to proceed. If the outside option represents consuming an imported good and importers have no market power, in that they form a perfectly or monopolistically competitive fringe, then computing the market share of the outside option is particularly simple: all that is required is knowledge of those importers’ sales (in value under CES demand, and in volume under MNL demand).

shares:<sup>28</sup>

$$CS(s) = \log H^0 - \log \left( 1 - \sum_{f \in \mathcal{F}} s^f \right). \quad (14)$$

Thus, consumer surplus depends only on the sum of the firms' market shares. The intuition is that all the products are equally good substitutes for the outside option, as the elasticity of  $D_i$  with respect to  $H^0$  is independent of  $i \in \mathcal{N}$ .

As firm  $f$ 's equilibrium profit is  $\mu^f - 1$  and  $\mu^f = 1/(1 - \alpha s^f)$ , aggregate surplus can also be written as a function of market shares:

$$AS(s) = \log H^0 - \log \left( 1 - \sum_{f \in \mathcal{F}} s^f \right) + \sum_{f \in \mathcal{F}} \frac{\alpha s^f}{1 - \alpha s^f}.$$

Note that aggregate surplus is increasing in the vector of market shares. Moreover, by convexity of  $s^f/(1 - \alpha s^f)$ , a mean-preserving spread of market shares raises industry profit and therefore aggregate surplus.<sup>29</sup>

We now show that the merger's  $\Delta HHI$  is an adequate measure of its market power effect. We proceed as follows. We fix the pre-merger vector of market shares  $s$ , and use this vector to compute the pre-merger market performance measures  $CS(s)$  and  $AS(s)$  and recover the underlying pre-merger type vector  $(T^f(s))_{f \in \mathcal{F}}$ . Assuming no synergies, the merged firm's type is  $T^M(s) = \sum_{f \in \mathcal{M}} T^f(s)$ . We then use the post-merger type vector to obtain the post-merger equilibrium vector of market shares  $\bar{s}(s)$ . The post-merger welfare measures are  $CS(\bar{s}(s))$  and  $AS(\bar{s}(s))$ . Hence, the market power effect of the merger is  $CS(\bar{s}(s)) - CS(s)$  or  $AS(\bar{s}(s)) - AS(s)$ .

Applying Taylor's theorem, we obtain the following second-order approximation results on the welfare effects of mergers in the absence of synergies:

**Proposition 8.** *In the neighborhood of  $s = 0$ , the market power effect of the merger is.*<sup>30,31</sup>

$$\begin{aligned} CS(\bar{s}(s)) - CS(s) &= -\alpha \Delta HHI(s) + o(\|s\|^2), \\ AS(\bar{s}(s)) - AS(s) &= -\alpha \Delta HHI(s) + o(\|s\|^2). \end{aligned}$$

*Proof.* See Online Appendix F. □

Hence, the market power effect of a merger is proportional to  $\Delta HHI$ , where the proportionality coefficient is the elasticity measure  $\alpha$ . Perhaps surprisingly, this holds regardless

<sup>28</sup>See Armstrong and Vickers (2018) on the related concept of consumer surplus as a function of quantities.

<sup>29</sup>This is akin to the homogeneous-goods Cournot model, where consumer surplus depends only on aggregate output, and aggregate profit is proportional to the Herfindahl index, for a fixed aggregate output.

<sup>30</sup> $o(\cdot)$  is Landau's little-o notation:  $f(x) = o(g(x))$  in the neighborhood of  $x = x^0$  if  $f(x)/g(x) \xrightarrow{x \rightarrow x^0} 0$ .

<sup>31</sup>While the HHI is unit-free, consumer surplus is usually measured in dollars. Here, consumer surplus is measured in units of the Hicksian composite commodity, the price of which was normalized to one in equation (1).

of whether the market power effect is measured in terms of consumer surplus or aggregate surplus.<sup>32</sup>

In Appendix A, we provide approximation results around monopolistic competition conduct.<sup>33</sup> There, we show that, around monopolistic competition conduct and absent synergies, the effect of a merger on consumer surplus (and on aggregate surplus when holding fixed the market share of the outside option) is proportional to the change in HHI.

## 6. Discussion and Conclusion

We provide a merger analysis in a multiproduct-firm oligopoly model with CES and MNL demand. The model allows for arbitrary product heterogeneity in terms of marginal costs and qualities, and allows firms to differ in their product portfolios. The demand system gives rise to an aggregative pricing game; the equilibrium is unique and has intuitive comparative statics. Moreover, the type aggregation property permits rich forms of merger-specific synergies through marginal cost reductions, quality improvements, or new products. Finally, consumer surplus and aggregate surplus can be expressed as functions of firm-level equilibrium market shares.

We derive three sets of results. First, we study the consumer surplus effects of mergers in both static and dynamic settings. For a merger to be CS-increasing requires that the merger generates efficiencies. These efficiencies need to be larger when the industry is less competitive before the merger, or when the merger partners are larger. In a dynamic context, in which merger opportunities arise stochastically over time and merger proposals are endogenous, a completely myopic consumer-surplus-based merger approval policy is dynamically optimal.

Second, we study the aggregate surplus and external effects of mergers. For a merger to be AS-increasing requires fewer efficiencies than for it to be CS-increasing and may, in fact, not require any efficiencies at all. The external effect of a CS-decreasing merger is always negative when products are poor substitutes. When instead products are close substitutes, the external effect is positive if the outsiders' pre-merger market shares are sufficiently large or sufficiently concentrated.

Third, we relate the magnitude of the market power effect of a merger to concentration measures. In particular, we show that—absent synergies—the effect of a merger on consumer surplus and aggregate surplus is approximately proportional to the naively computed, merger-induced variation in the Herfindahl index.

We close the article by briefly discussing various extensions of the baseline model:

**Nested demand systems.** In the main text, we confine attention to CES and MNL demand. These demand systems are known to have the IIA property, thus exhibiting unreal-

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<sup>32</sup>The effects on consumer surplus and aggregate surplus do differ at the third order.

<sup>33</sup>Specifically, we introduce a conduct parameter,  $\theta \in [0, 1]$ , and assume that each firm believes the impact of  $p_i$  on the aggregator is  $\theta \partial H / \partial p_i$ . Under this parameterization, oligopolistic competition conduct corresponds to  $\theta = 1$  and monopolistic competition conduct to  $\theta = 0$ .

istically simple substitution patterns. In Appendix B, we extend our analysis by considering *nested* CES and MNL demand systems. As products within the same nest are closer substitutes to each other than products in different nests, this extension allows for substitution patterns that go beyond those implied by the IIA property. To ensure that the game retains aggregative properties, we impose the restriction that each firm is broad in that it owns one or several entire nests.<sup>34</sup> We show that firm types can be defined in such a way that fitting-in functions continue to satisfy equations (5)–(9) while the equilibrium aggregator level is still pinned down by the adding-up condition (10). It follows that all of the results in the main text carry over to this richer setting.

**Broad and narrow firms.** The restriction to broad firms in the above extension means that the IIA property still holds, albeit at the firm rather than the product level. Relaxing this restriction, we study in the second part of Appendix B the coexistence of broad firms (which own entire nests of products) and narrow firms (which own only a subset of the products within a single nest). As pointed out in the introduction, the IIA property does not apply to two narrow firms offering products in different nests. We show that an aggregative games approach can still be applied in such a setting and that there exists a unique equilibrium, with intuitive comparative statics.<sup>35</sup> This insight allows us to extend most of our results on the static and dynamic consumer surplus effects of mergers to both broad mergers (which involve only broad firms) and narrow mergers (which involve only narrow firms operating in the same nest). One notable exception is that a narrow merger giving rise to a larger naively computed variation in the Herfindahl index may in fact require *fewer* synergies to benefit consumers, showing that presumptions of anti-competitive effects based solely on market shares can be misguided.

We also compare the synergy levels required to make a broad merger and an “equivalent” narrow merger not harm consumers. A broad and a narrow merger are deemed equivalent if both sets of merger partners command the same industry-level market shares before the merger, so that both sets of firms appear to have the same degree of market power. Although one might expect a narrow merger to raise more competitive concerns than an equivalent broad merger, as the products of the merger partners are closer substitutes, we show that the narrow merger requires in fact fewer synergies if the nest in which the narrow firms are present has a sufficiently high market share.

**Merger analysis with IIA demand.** Anderson, Erkal, and Piccinin (2020) show that any IIA demand system can be obtained from utility maximization of a representative consumer

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<sup>34</sup>For a treatment of oligopoly with symmetric broad firms, see Anderson and de Palma (1992, 2006).

<sup>35</sup>In recent work, Garrido (forthcoming) proves existence of equilibrium with nested CES/MNL demand without imposing a restriction on the relationship between the firm and nest partitions. However, this comes at the cost of losing both the type aggregation property and the uni-dimensionality of the industry-level aggregator, which are essential for a tractable merger analysis.

with an indirect utility of the form

$$V(p) = \Psi \left( H^0 + \sum_{j \in \mathcal{N}} h_j(p_j) \right).$$

In the special case of CES (resp. MNL) demand,  $\Psi = \log$  and  $h_j$  takes the CES (resp. MNL) form for all  $j$ . In Nocke and Schutz (2023), we extend the analysis in the present article by allowing for general  $\Psi$ , while maintaining the assumption necessary for type aggregation to obtain, namely that  $h_j$  takes the CES form for every  $j$  or the MNL form for every  $j$ . There, we show that prices, locally, can be strategic complements or substitutes, depending on the local behavior of the curvature of  $\Psi$ . We find that all of the insights of the present article are robust to the more general demand specification as long as prices remain strategic complements.

## Appendix A: Deferred Results and Proofs

□ **Dynamic Optimality of Myopic CS-Based Merger Approval Policy.** In the following, we gather intermediate results on the dynamic optimality of a myopic CS-based merger approval policy.

**Lemma 1.** *If merger  $M_l$  is CS-nondecreasing in isolation, it remains CS-nondecreasing if another merger  $M_k$ ,  $k \neq l$ , that is CS-nondecreasing in isolation takes place. If merger  $M_l$  is CS-decreasing in isolation, it remains CS-decreasing if another merger  $M_k$ ,  $k \neq l$ , that is CS-decreasing in isolation takes place.*

*Proof.* This follows immediately from Proposition 3. □

**Lemma 2.** *Suppose that merger  $M_k$  is CS-nondecreasing in isolation whereas merger  $M_l$  is CS-decreasing in isolation but CS-nondecreasing once merger  $M_k$  has taken place. Then, merger  $M_k$  is CS-increasing conditional on merger  $M_l$  taking place.*

*Proof.* This follows immediately from the assumption that consumer surplus is higher after both mergers are implemented, but lower after only  $M_l$  is implemented. □

**Lemma 3.** *Suppose that all feasible but not yet approved mergers are proposed in each period. Then, the myopically CS-maximizing merger policy maximizes discounted consumer surplus, no matter what the realization of feasible mergers is.*

*Proof.* The lemma is the analogue of Lemma 4 in Nocke and Whinston (2010), and its proof is identical. It suffices to make the following two observations. First, Lemma 4 in Nocke and Whinston (2010) states the result for the “most lenient” myopically CS-maximizing merger policy. However, the result and proof also hold for the “least lenient” such policy. As noted in footnote 23, these two policies are generically identical in our model, as every merger is,

generically, either CS-increasing or CS-decreasing, but not CS-neutral. Second, the proof of Lemma 4 uses the monotonicity property of Lemma 2 in Nocke and Whinston (2010). It is straightforward to see that Lemmas 5 and 6 in Nocke and Whinston (2010) hold in our setup, implying that the monotonicity property of Lemma 2 carries over as well.  $\square$

**Lemma 4.** *Suppose that merger  $M_k$  is CS-nondecreasing given current market structure whereas merger  $M_l$  is CS-decreasing but becomes CS-nondecreasing once  $M_k$  has been implemented. Then, the joint profit of the firms in  $M_k$  is strictly higher if both mergers take place than if none does.*

*Proof.* Think of implementing merger  $M_l$  at step one. As that merger is CS-decreasing by assumption, the equilibrium level of the aggregator,  $H^*$ , must decrease, which strictly raises the profit of each firm in  $\mathcal{M}_k$ . Next, implement merger  $M_k$  at step two: as that merger remains, by Lemma 2, CS-nondecreasing after  $M_l$  has taken place, it is profitable by the second part of Proposition 2. Thus, the joint profit of the firms in  $\mathcal{M}_k$  strictly increases at each step.  $\square$

*Proof of Proposition 5.* Proposition 5 in the main text is the analogue of Proposition 3 in Nocke and Whinston (2010), and its proof is identical. (Note that, in the present model, the most and least lenient myopically CS-maximizing merger policies generically coincide.) The proof in Nocke and Whinston (2010) makes explicit use of the statement about the private profitability of CS-nondecreasing mergers in Corollary 1 as well as of Lemmas 2, 4 and 5 in that article. The profitability statement of Corollary 1 in Nocke and Whinston (2010) corresponds to the second part of Proposition 2 in this article whereas Lemma 4 in Nocke and Whinston (2010) corresponds to our Lemma 3. As noted in the proof of our Lemma 3, Lemmas 5 and 6 in Nocke and Whinston (2010) hold in our setup, implying that Lemma 2 in Nocke and Whinston (2010) carries over as well.  $\square$

$\square$  **HHI and the Welfare Effects of Mergers: Approximations around Monopolistic Competition Conduct.** Under monopolistic competition, firms believe to have no impact on the aggregator, i.e., firm  $i$  believes that  $\partial H/\partial p_i$  is equal to zero. We now bridge the gap between monopolistic competition conduct and fully-fledged “Bertrand-Nash” conduct. Specifically, let  $\theta \in [0, 1]$  be a conduct parameter, and assume that each firm believes that the impact of  $p_i$  on the aggregator is  $\theta \partial H/\partial p_i$  instead of  $\partial H/\partial p_i$ , that is, firms internalize their impact on the aggregator only to extent  $\theta$ .<sup>36</sup>

The analysis proceeds along the same lines as in Section 2 above (see Online Appendix G for details). There exists a unique equilibrium aggregator level  $H^*(\theta)$ . It is easy to see that  $H^*(\theta)$  and the fitting-in functions  $m(\cdot, \theta)$ ,  $S(\cdot, \theta)$ , and  $\pi(\cdot, \theta)$  all tend to their value under monopolistic competition as  $\theta$  tends to 0, and to their value under fully-fledged oligopoly as  $\theta$  tends to 1.

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<sup>36</sup>This treatment of firm conduct is closely related to the classical approach under quantity competition with homogeneous products surveyed by Bresnahan (1989).



We now turn to the approximation of the welfare effects of mergers around monopolistic competition conduct. Let  $(T^f)_{f \in \mathcal{F}}$  be the pre-merger type vector and  $T^M = \sum_{f \in \mathcal{M}} T^f$  the merged firm's type, under the assumption of no synergies. Let  $CS(\theta)$  and  $AS(\theta)$  denote pre-merger consumer surplus and aggregate surplus, respectively. Similarly,  $\overline{CS}(\theta)$  and  $\overline{AS}(\theta)$  denote post-merger consumer surplus and aggregate surplus, respectively. The naively computed, merger-induced change in the Herfindahl index is denoted  $\Delta HHI(\theta)$ . We find:

**Proposition 9.** *In the neighborhood of  $\theta = 0$ , the market power effect of the merger is:*

$$\begin{aligned} \overline{CS}(\theta) - CS(\theta) &= -\alpha \Delta HHI(\theta)\theta + o(\theta), \\ \overline{AS}(\theta) - AS(\theta) &= -\alpha \Delta HHI(\theta) \left( 1 - \alpha \sum_{f \in \mathcal{F}} s^f(\theta) \right) \theta + o(\theta). \end{aligned}$$

*Proof.* See Online Appendix G. □

Note that the merger's market power effect on consumer surplus is independent of whether the approximation is taken around small market shares or monopolistic competition conduct. By contrast, the market power effect on aggregate surplus when approximated around monopolistic competition conduct differs slightly from that when approximated around small market shares; that difference, however, vanishes as market shares become small.

## Appendix B: Nested Demand Structures

In this appendix, we extend the multiproduct-firm pricing game in the main text to allow for *nested* CES (NCES) and *nested* MNL (NMNL) demand. In the first part of the appendix, we assume that all firms are “broad” in that they own entire nests of products. We show that the resulting fitting-in functions solve the same system of equations as under the non-nested demand structures in the main text, implying that all of the results in the article carry over to this more general setting. In the second part of the appendix, we introduce, in addition to broad firms, “narrow” firms that own only a strict subset of products within a single nest. We show that all our results on the consumer surplus effects of mergers continue to hold.

□ **Nested Demand Structures with Broad Firms.** Each product now belongs to a nest of products; the set of nests is denoted  $\mathcal{L}$ , a partition of  $\mathcal{N}$ . Products within the same nest are viewed by consumers as closer substitutes with each other than products in different nests. Specifically, the representative consumer's quasilinear indirect utility function is given by

$$y + V(p) = y + V_0 \log \left[ H^0 + \sum_{l \in \mathcal{L}} \left( \sum_{j \in l} h_j(p_j) \right)^\beta \right], \quad (15)$$

where  $0 < \beta \leq 1$  is a parameter measuring the substitutability of products across nests relative to that within nests,<sup>37</sup> and the  $h_j$  functions take again either the CES or MNL form.

Defining the nest- and industry-level aggregators as

$$H_l(p_l) = \sum_{j \in l} h_j(p_j), \text{ where } p_l \equiv (p_j)_{j \in l} \forall l \in \mathcal{L},$$

$$\text{and } H(p) = H^0 + \sum_{l \in \mathcal{L}} (H_l(p_l))^\beta,$$

the indirect utility takes again the form  $V(p) = V_0 \log H(p)$ .

Applying Roy's identity, we obtain the demand for product  $i$  in nest  $l$ :

$$D_i(p) = V_0 \beta \frac{-h'_i(p_i)}{h_i(p_i)} \frac{h_i(p_i)}{H_l(p_l)} \frac{H_l(p_l)^\beta}{H(p)} = V_0 \beta \frac{-h'_i(p_i) H_l(p_l)^{\beta-1}}{H(p)}. \quad (16)$$

As shown in Nocke and Schutz (2018), demand system (16) can alternatively be derived from discrete/continuous choice. With such a micro-foundation,  $V_0 \beta$  is the total number of consumers,  $H_l^\beta / H$  is the probability that a given consumer chooses nest  $l$ ,  $h_i / H_l$  is the probability that a consumer picks product  $i$  conditional on having chosen nest  $l$ , and  $-h'_i / h_i$  is the number of units of product  $i$  a consumer purchases conditional on having chosen product  $i$ . Moreover,  $(\log H^0) / \beta$  is the value of the outside option. In the remainder, we normalize  $V_0$  to 1.

Each product  $i \in \mathcal{N}$  has again constant marginal cost of production  $c_i > 0$ . The set of firms,  $\mathcal{F}$ , is now a partition of  $\mathcal{L}$ . That is, each firm is broad in that it has property rights over the production of all products within one or more nests.<sup>38</sup> (We relax this assumption in the second part of this appendix.) The profit of firm  $f \in \mathcal{F}$  is given by

$$\Pi^f = \sum_{l \in f} \sum_{i \in l} (p_i - c_i) D_i(p).$$

We seek the Nash equilibrium of the resulting multiproduct-firm pricing game.

The market share of firm  $f$  is

$$s^f = \sum_{l \in f} \frac{(H_l)^\beta}{H}.$$

In the discrete/continuous choice micro-foundation mentioned above,  $s^f$  corresponds to the probability that any given consumer chooses one of firm  $f$ 's products. Moreover,  $s^f$  is equal to firm  $f$ 's market share in volume under NMNL demand, and to firm  $f$ 's market share in value under NCES demand. In both cases, the firms' market shares add up to  $1 - H^0 / H$ , where  $H^0 / H$  is again the market share of the outside option.

<sup>37</sup>If  $\beta = 1$ , the nest structure is irrelevant.

<sup>38</sup>One way to interpret this restriction is that each firm owns one or several brands (nests), with products being closer substitutes within a brand than across brands. Another interpretation is that each firm owns multiple products, with each product being available in different varieties.

**The Monopolistic Competition Benchmark.** Before analyzing the above oligopolistic pricing game, it is instructive to consider first the monopolistic competition benchmark. Under monopolistic competition, firms do not internalize the impact of their behavior on the industry aggregator  $H$ , i.e., they behave as if  $\partial H/\partial p_i = 0$ .

Under this behavioral assumption, the first-order condition of profit maximization for product  $i \in n \in f$  is given by

$$\frac{H_n^{\beta-1}}{H} \left( -h'_i - (p_i - c_i)h''_i + (1 - \beta) \frac{\partial H_n}{\partial p_i} \frac{\sum_{j \in n} (p_j - c_j)h'_j}{H_n} \right) = 0,$$

which can be rewritten as

$$\frac{p_i - c_i}{p_i} \frac{p_i h''_i}{-h'_i} = 1 + (1 - \beta) \frac{\sum_{j \in n} (p_j - c_j)(-h'_j)}{H_n}. \quad (17)$$

If  $\beta = 1$  (i.e., in the absence of nests), we immediately obtain that firm  $f$  sets the Lerner index of product  $i$  equal to the reciprocal of the perceived price elasticity of demand. If  $\beta < 1$ , firm  $f$  internalizes self-cannibalization effects within its own nests, and it optimally sets a Lerner index that exceeds that in the absence of nests.

As in the main text, the left-hand side of equation (17) is the  $\iota$ -markup on product  $i$ . As the right-hand side is the same for every  $i \in n$ , firm  $f$  charges the same  $\iota$ -markup,  $\tilde{\mu}_n > 1$ , for each product  $i$  in nest  $n$ . Under NCES demand, this implies that the Lerner index of product  $i$  is equal to  $\tilde{\mu}_n/\sigma$ , whereas under NMNL demand, the absolute markup  $p_i - c_i$  is equal to  $\tilde{\mu}_n \lambda$ .

Using the common  $\iota$ -markup property within nest  $n$ , the sum on the right-hand side of equation (17) can be written as:

$$\sum_{j \in n} (p_j - c_j)(-h'_j) = \sum_{j \in n} \frac{p_j - c_j}{p_j} \frac{h''_j}{-h'_j} \frac{(h'_j)^2}{h''_j} = \tilde{\mu}_n \sum_{j \in n} \frac{(h'_j)^2}{h''_j} = \tilde{\alpha} \tilde{\mu}_n \sum_{j \in n} h_j = \tilde{\alpha} \tilde{\mu}_n H_n, \quad (18)$$

where  $\tilde{\alpha} = (\sigma - 1)/\sigma < 1$  under NCES demand and  $\tilde{\alpha} = 1$  under NMNL demand. Equation (17) therefore boils down to

$$\tilde{\mu}_n = \frac{1}{1 - \tilde{\alpha}(1 - \beta)} \equiv \mu^{\text{mc}}. \quad (19)$$

As  $\mu^{\text{mc}}$  does not depend on the identity of nest  $n$  nor on the identity of firm  $f$ , the monopolistically competitive  $\iota$ -markup  $\mu^{\text{mc}}$  is the same for each product  $i \in \mathcal{N}$ .

**Equilibrium Analysis.** We now turn to the equilibrium analysis of our multiproduct-firm pricing game. This requires adapting the aggregative games approach taken in Section 5 of Nocke and Schutz (2018), where each firm is restricted to own only a single nest.

The first-order condition for product  $i$  in nest  $n$  owned by firm  $f$  is given by

$$\frac{H_n^{\beta-1}}{H} \left( -h'_i - (p_i - c_i)h''_i + (1 - \beta) \frac{\partial H_n}{\partial p_i} \frac{\sum_{j \in n} (p_j - c_j)h'_j}{H_n} + \frac{H_n^{1-\beta}}{H} \frac{\partial H}{\partial p_i} \sum_{l \in f} H_l^{\beta-1} \sum_{j \in l} (p_j - c_j)h'_j \right) = 0.$$

The last term on the left-hand side, which is absent under monopolistic competition, captures the impact of the price change through the aggregator  $H$ . Simplifying, we obtain

$$\frac{p_i - c_i}{p_i} \frac{p_i h''_i}{-h'_i} = 1 + (1 - \beta) \frac{\sum_{j \in n} (p_j - c_j)(-h'_j)}{H_n} + \beta \frac{1}{H} \sum_{l \in f} H_l^{\beta-1} \sum_{j \in l} (p_j - c_j)(-h'_j). \quad (20)$$

Hence, despite the additional term on the right-hand side, firm  $f$  continues to charge the same  $\iota$ -markup on every product  $i$  in nest  $n$ . That is, there exists  $\tilde{\mu}_n > 1$  such that

$$\frac{p_i - c_i}{p_i} \frac{p_i h''_i}{-h'_i} = \tilde{\mu}_n$$

for every  $i \in n$ .

Using the common  $\iota$ -markup property within each nest  $l$  and equation (18), equation (20) can be rewritten as

$$\tilde{\mu}_n (1 - \tilde{\alpha}(1 - \beta)) = 1 + \tilde{\alpha}\beta \frac{1}{H} \sum_{l \in f} \tilde{\mu}^l H_l^\beta, \quad (21)$$

which immediately implies that  $\tilde{\mu}_n = \tilde{\mu}_{n'} \equiv \tilde{\mu}^f$  for every  $n, n' \in f$ . Firm  $f$  therefore applies the same  $\iota$ -markup  $\tilde{\mu}^f$  to all the products in its portfolio. Using this common  $\iota$ -markup property, both within and across nests, equation (21) simplifies to

$$\tilde{\mu}^f (1 - \tilde{\alpha}(1 - \beta)) = 1 + \tilde{\alpha}\beta \tilde{\mu}^f \frac{\sum_{l \in f} H_l^\beta}{H} = 1 + \tilde{\alpha}\beta \tilde{\mu}^f s^f. \quad (22)$$

Define the elasticity measure  $\alpha \equiv \tilde{\alpha}\beta/(1 - \tilde{\alpha}(1 - \beta))$ , and note that  $\alpha < 1$  under NCES demand and  $\alpha = 1$  under NMNL demand. Using equation (22), we can decompose firm  $f$ 's  $\iota$ -markup as follows:

$$\tilde{\mu}^f = \underbrace{\frac{1}{1 - \tilde{\alpha}(1 - \beta)}}_{\equiv \mu^{\text{mc}}} \underbrace{\frac{1}{1 - \alpha s^f}}_{\equiv \mu^f}.$$

That is, under oligopoly, firm  $f$ 's  $\iota$ -markup  $\tilde{\mu}^f$  is the product of the monopolistically competitive  $\iota$ -markup  $\mu^{\text{mc}}$  and a market power factor, the normalized markup  $\mu^f > 1$ . As  $\mu^f$  is increasing in  $s^f$ , this decomposition reveals that firms with larger market shares have more market power, and therefore set higher  $\iota$ -markups.

Equations (18) and (22) yield a simple formula for firm  $f$ 's equilibrium profit:

$$\Pi^f = \tilde{\alpha}\beta \tilde{\mu}^f s^f = \mu^f - 1. \quad (23)$$

Next, we express firm  $f$ 's market share as a function of the industry-level aggregator  $H$  and firm  $f$ 's normalized markup  $\mu^f$ . Under NCES demand,

$$\begin{aligned} s^f &= \frac{1}{H} \sum_{l \in f} \left( \sum_{j \in l} a_j \left( \frac{\sigma}{\sigma - \tilde{\mu}^f c_j} \right)^{1-\sigma} \right)^\beta \\ &= \frac{1}{H} \underbrace{\sum_{l \in f} \left( \sum_{j \in l} a_j c_j^{1-\sigma} \right)^\beta}_{\equiv T^f} (1 - (1 - \tilde{\alpha}) \tilde{\mu}^f)^{\frac{\tilde{\alpha}\beta}{1-\tilde{\alpha}}} = \frac{T^f}{H} (1 - (1 - \alpha) \mu^f)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Under NMNL demand,

$$s^f = \frac{1}{H} \sum_{l \in f} \left( \sum_{j \in l} \exp \left( \frac{a_j - c_j}{\lambda} - \tilde{\mu}^f \right) \right)^\beta = \frac{1}{H} \underbrace{\sum_{l \in f} \left( \sum_{j \in l} \exp \left( \frac{a_j - c_j}{\lambda} \right) \right)^\beta}_{\equiv T^f} \exp(-\mu^f).$$

Firm  $f$ 's type,  $T^f$ , summarizes again all the relevant information about firm  $f$ 's product portfolio.

Hence, if  $H$  is an equilibrium aggregator level, then firm  $f$ 's markup and market share  $\mu^f$  and  $s^f$  jointly solve the following system of equations:

$$\mu^f = \frac{1}{1 - \alpha s^f}, \tag{24}$$

$$s^f = \begin{cases} \frac{T^f}{H} (1 - (1 - \alpha) \mu^f)^{\frac{\alpha}{1-\alpha}} & \text{under NCES demand,} \\ \frac{T^f}{H} e^{-\mu^f} & \text{under NMNL demand.} \end{cases} \tag{25}$$

Importantly, this system of equations is identical to the one in the main text, consisting of equations (5) and (9) in Section 2. Hence, the markup and market-share fitting-in functions are exactly as in the main text, and given by  $m(T^f/H)$  and  $S(T^f/H)$ . From equation (23), we see that the profit fitting-in function continues to satisfy  $\pi(T^f/H) = m(T^f/H) - 1$ . The equilibrium aggregator level is still pinned down by the condition that market shares add up to one (equation (10)).

We summarize these insights in the following proposition:

**Proposition 10.** *The multiproduct-firm pricing game with broad firms has a unique equilibrium. The equilibrium aggregator level  $H^*$  is the unique solution of equation (10). In equilibrium, firm  $f \in \mathcal{F}$  sets a markup of  $m(T^f/H^*)$ , commands a market share of  $S(T^f/H^*)$ , and earns a profit of  $\pi(T^f/H^*)$ .*

*Proof.* The only thing left to prove is that first-order conditions are necessary and sufficient for global optimality. This is done in Online Appendix H.  $\square$

As the results on mergers in the main text depend only on the properties of the fitting-in

functions, and as merger-induced synergies can be defined exactly as before, it follows that all of these results carry over to this more general setting with broad firms.

□ **Merger Analysis with Broad and Narrow Firms.** In the first part of this appendix, we assumed that each firm owns all of the products in one or more nests, implying that competition takes place only across nests but not within nests. We now relax this restriction by introducing, in addition to broad firms, “narrow” firms that own only a strict subset of products within a single nest. The demand system continues to be of the NCES or NMNL type.

We partition the set of nests  $\mathcal{L}$  into two subsets:  $\mathcal{L}^b$  and  $\mathcal{L}^n$ . The set  $\mathcal{L}^b$  is further partitioned into a set of broad firms  $\mathcal{F}^b$ . The novelty is that each nest  $l \in \mathcal{L}^n$  is partitioned into a set of narrow firms  $\mathcal{F}_l$ . By assumption, the products of a given narrow firm  $f \in \mathcal{F}_l$  are all contained in nest  $l$ . We assume that the partition  $\mathcal{F}_l$  contains at least two elements for every  $l \in \mathcal{L}^n$ . (If  $\mathcal{F}_l$  were a singleton, we would classify the corresponding firm as a broad firm.) In the following, when studying a narrow firm  $f \in \mathcal{F}_l$ , we will often write  $f \in l$  with a slight abuse of notation. The set of narrow firms is denoted  $\mathcal{F}^n \equiv \bigcup_{l \in \mathcal{L}^n} \mathcal{F}_l$ , and the set of firms is  $\mathcal{F} \equiv \mathcal{F}^b \cup \mathcal{F}^n$ .

The restriction that each firm is either narrow or broad ensures that the oligopoly game retains some aggregative properties in the following sense: the behavior of broad firm  $f$  depends only on the value of the industry aggregator  $H$ , whereas the behavior of narrow firm  $f \in l$  depends solely on the values of the industry aggregator  $H$  and the nest-level aggregator  $H_l$ . When studying mergers, we need to ensure that the post-merger oligopoly model continues to satisfy the restriction that each firm is either narrow or broad. This means that we need to confine attention to the following types of mergers: mergers between broad firms such that the merged entity is also a broad firm; mergers between narrow firms operating in the same nest  $l$  such that all of the merged firms’ products are in nest  $l$ .

In what follows, we provide an informal overview of a merger analysis with broad and narrow firms, referring the reader to Online Appendix I for a formal treatment. We study the pricing game with broad and narrow firms, using an aggregative games approach. We then extend our analysis of the static and dynamic consumer surplus effects of mergers to this more general setting. Finally, we show that, to be CS-nondecreasing, a merger between narrow firms in the same nest may require fewer or more synergies than an “equivalent” merger between broad firms.

**Oligopoly with broad and narrow firms.** The fitting-in functions of broad firm  $f$  are unaffected by the presence of narrow firms and are therefore as characterized in the first part of this appendix. Consider now narrow firm  $f$  in nest  $l$ . Its industry-level market share  $s^f$  can be decomposed into its nest-level market share  $\tilde{s}^f$  and the market share of its nest  $s_l$ :  $s^f \equiv \tilde{s}^f s_l$ , where

$$\tilde{s}^f = \frac{\sum_{j \in f} h_j(p_j)}{H_l} \quad \text{and} \quad s_l = \frac{H_l^\beta}{H}.$$

From the first-order condition, we find that narrow firm  $f$  charges the same  $\iota$ -markup

$$\tilde{\mu}^f = \frac{1}{1 - \tilde{\alpha}\tilde{s}^f(1 - \beta + \beta s_l)}. \quad (26)$$

for all its products. Intuitively, the firm sets a high markup if it has a high market share in its nest or if its nest commands a high market share at the industry level. Using the definition of market shares and the common  $\iota$ -markup property, we obtain

$$\tilde{s}^f = \frac{(T^f)^{\frac{1}{\beta}}}{H_l} \times \begin{cases} (1 - (1 - \tilde{\alpha})\tilde{\mu}^f)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} & \text{under NCES demand,} \\ e^{-\tilde{\mu}^f} & \text{under NMNL demand,} \end{cases} \quad (27)$$

where the firm's type  $T^f$  is given by  $T^f = \left(\sum_{j \in f} h_j(c_j)\right)^\beta$ . Note that  $\log T^f$  corresponds again to the consumer surplus firm  $f$  would deliver if it were to price all of its products at marginal costs and no other products were offered.

There exists a unique pair of markup and nest-level market share,

$$\tilde{m} \left( \frac{(T^f)^{\frac{1}{\beta}}}{H_l}, s_l \right) \quad \text{and} \quad \tilde{S} \left( \frac{(T^f)^{\frac{1}{\beta}}}{H_l}, s_l \right),$$

that solves equations (26)–(27). Given industry-level aggregator  $H$ , the nest-level aggregator  $H_l(H)$  is pinned down by nest-level market shares having to add up to one:<sup>39</sup>

$$\sum_{f \in l} \tilde{S} \left( \frac{(T^f)^{\frac{1}{\beta}}}{H_l}, \frac{H_l^\beta}{H} \right) = 1.$$

The equilibrium industry-level aggregator  $H^*$  is the unique solution to industry-level market shares adding up to one:

$$\frac{H^0}{H} + \sum_{f \in \mathcal{F}^b} S \left( \frac{T^f}{H} \right) + \sum_{l \in \mathcal{L}^n} \frac{H_l(H)^\beta}{H} = 1.$$

We show in Online Appendix I (Propositions 12 and 13) that there exists a unique equilibrium with intuitive comparative statics, extending the results in Section 5 of Nocke and Schutz (2018).

**Consumer surplus effects of mergers.** We now revisit the results of Section 3 when broad and narrow firms coexist, making again use of the type aggregation property, which applies not only to broad but also to narrow firms. First, note that all those results extend

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<sup>39</sup>Anderson, Erkal, and Piccinin (2016) were the first to use the concept of nest-level aggregator, which they refer to as “sub-aggregator.”

immediately when confining attention to “broad mergers,” i.e., to mergers between broad firms (see the first part of this appendix). In the following, we consider the consumer surplus effects of a merger between narrow firms that operate in the same nest, assuming throughout that all of the products of the merged firm are still within that same nest. We refer to those mergers as “narrow mergers.” As mentioned above, the reason for the restriction is that after the merger, it must still be possible to partition the set of firms into broad and narrow firms for our aggregative games approach to apply.

Consider a narrow merger  $M$  between the firms in  $\mathcal{M}$  operating in the same nest  $l$ . The merger is CS-neutral if the post-merger nest-level market share of the merged firm is equal to the combined pre-merger market shares of the merger partners:

$$\sum_{f \in \mathcal{M}} \tilde{S} \left( \frac{(T^f)^{\frac{1}{\beta}}}{H_l^*}, \frac{(H_l^*)^\beta}{H^*} \right) = \tilde{S} \left( \frac{(T^M)^{\frac{1}{\beta}}}{H_l^*}, \frac{(H_l^*)^\beta}{H^*} \right),$$

where  $H_l^*$  and  $H^*$  are the pre-merger nest-level and industry-level aggregators and  $T^M$  is the post-merger type. This equation uniquely pins down a cutoff type  $\hat{T}^M(H_l^*, H^*)$  below which a merger is CS-decreasing and above which it is CS-increasing (see Proposition 14 in Online Appendix I). As  $\tilde{S}$  is strictly sub-additive in its first argument, a CS-nondecreasing narrow merger must involve strictly positive synergies:

$$\hat{T}^M(H_l^*, H^*) > \left( \sum_{f \in \mathcal{M}} (T^f)^{\frac{1}{\beta}} \right)^\beta.$$

Note that the right-hand side of the inequality does indeed give the post-merger type in the absence of synergies.<sup>40</sup>

Turning our attention to the comparative statics of the cutoff type, we find that  $\hat{T}^M(H_l^*, H^*)$  is decreasing in both of its arguments, implying that a merger requires fewer synergies to be CS-nondecreasing if the merging firms face more competition within their nest (higher  $H_l^*$ ) or from other nests (higher  $H^*$ )—the counterpart of Proposition 3 (see Propositions 15–17 in Online Appendix I). That earlier proposition implied that, holding fixed the types of the merger partners, changes in market structure give rise to a positive correlation between the naively computed, merged-induced change in the Herfindahl index, and the synergy level required for the merger to be CS-nondecreasing. This is no longer true for a narrow merger: holding fixed pre-merger types and  $H^*$ , an increase in  $H_l^*$  may result in larger pre-merger industry-level market shares for the merger partners, despite decreasing the required syner-

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<sup>40</sup>Under no synergies, the post-merger type of the merged firm is

$$T^M = \left( \sum_{f \in \mathcal{M}} \sum_{j \in f} h_j(c_j) \right)^\beta = \left( \sum_{f \in \mathcal{M}} (T^f)^{1/\beta} \right)^\beta.$$



gies.<sup>41</sup> On the other hand, a change in  $H^*$ , holding fixed pre-merger types and  $H_l^*$ , results in a positive correlation between pre-merger market shares and the required synergy level.<sup>42</sup>

**Interactions between mergers and dynamic merger policy.** The sign-preserving complementarity in the consumer-surplus effect of disjoint broad mergers (Lemma 1 in Appendix A) carries over to any admissible pair of mergers  $M_1$  and  $M_2$ , where each  $M_i$  can be a narrow or a broad merger. In case both  $M_1$  and  $M_2$  are narrow mergers, we do not impose any restriction on whether they are in the same nest or not. The result follows as  $\widehat{T}^{M_i}(H_l^*, H^*)$ , the cutoff type for a narrow merger  $M_i$  in nest  $l$ , is decreasing in both of its arguments and any CS-increasing merger raises the industry aggregator and all nest-level aggregators. Moreover, it is still the case that a CS-increasing merger remains CS-increasing even if it induces an otherwise CS-decreasing merger to become CS-increasing—the counterpart of Lemma 2 in Appendix A.

A CS-neutral merger, whether narrow or broad, is privately profitable, as it must involve positive synergies and affects none of the aggregators. As comparative statics are well-behaved, this implies that a CS-nondecreasing merger is profitable as well. Moreover, firms involved in a CS-nondecreasing merger are better off even if their merger induces otherwise CS-decreasing mergers to become CS-nondecreasing, resulting in their approval.

In the dynamic framework sketched in Section 3, the results outlined above imply the dynamic optimality of a myopic CS-based merger policy—the counterpart of Proposition 5 (see Proposition 19 in Online Appendix I).

**Broad vs. narrow mergers.** We close this appendix by comparing the synergy levels required for a broad merger  $M_b$  and an “equivalent” narrow merger  $M_n$  to be CS-nondecreasing. Intuitively, one would expect the required synergy level to be higher for the narrow merger, as the merger partners compete more fiercely within the same nest than across nests. As we show below, this intuition is incomplete: to be CS-nondecreasing, a narrow merger may in fact require fewer synergies than an equivalent broad merger.

Formally, a broad merger is equivalent to a narrow one if the pre-merger vector of industry-level market shares of the merger partners is  $(s^f)_{f \in \mathcal{M}}$  for both mergers. Pre-merger types for the broad and narrow mergers can be recovered (up to a multiplicative constant) by solving

$$s^f = S \left( \frac{T_b^f}{H^*} \right) \quad \text{and} \quad \frac{s^f}{s_l} = \widetilde{S} \left( \frac{(T_n^f)^{\frac{1}{\beta}}}{H_l^*}, s_l \right).$$

The cutoff types  $\widehat{T}_b^M$  and  $\widehat{T}_n^M$ , which make the broad and the narrow merger CS-neutral, can

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<sup>41</sup>In the NMNL case, this arises when  $\beta$  and  $s_l$  are high enough and the merger partners have sufficiently high nest-level market shares.

<sup>42</sup>We also establish the counterpart of Proposition 4. That is, we show that holding fixed  $H_l^*$  and  $H^*$ , a narrow merger in nest  $l$  requires larger synergies to be CS-nondecreasing if pre-merger types are larger. See Proposition 18 in Online Appendix I.

be backed out by solving similar equations, recalling that a merger is CS-neutral if and only if the industry-level market share of the merged firm is equal to the combined pre-merger market shares of the merger partners. Our goal is to compare

$$E_b = \frac{\widehat{T}_b^M}{\sum_{f \in \mathcal{M}} T_b^f} \quad \text{and} \quad E_n = \frac{\widehat{T}_n^M}{\left(\sum_{f \in \mathcal{M}} (T_n^f)^{\frac{1}{\beta}}\right)^\beta},$$

the required synergy levels for the broad and the narrow merger.

For simplicity, we confine attention to mergers between symmetric firms.<sup>43</sup> We find:

**Proposition 11.** *Consider two equivalent broad and narrow mergers between  $N$  symmetric firms. Let  $\bar{s}$  be the combined pre-merger industry-level market shares of the merger partners and  $s_l$  the pre-merger market share of the narrow merger's nest. There exists a threshold  $\widehat{s}_l \in (\bar{s}, 1)$  such that the broad merger requires fewer synergies than the narrow one,  $E_b < E_n$ , if  $s_l < \widehat{s}_l$ , whereas the opposite is true if  $s_l > \widehat{s}_l$ .*

*Proof.* See Online Appendix I. □

Intuitively, two opposing effects are at work. On the one hand, narrow firms face more intense competition, and therefore charge lower markups, compared to equivalent broad firms. A narrow merger eliminating that competition therefore requires stronger synergies than an equivalent broad merger. On the other hand, if there are non-merging rivals within the same nest, the merged narrow firm still faces more intense competition than an equivalent merged broad firm, implying that the narrow merger requires fewer synergies. The magnitude of the latter effect is increasing in the market share of the narrow merger's nest, holding fixed the industry-level market shares of the merger partners.

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<sup>43</sup>In the case of NMNL demand, Proposition 11 extends to mergers involving asymmetric firms provided the nest-level market share of each narrow merger partner does not exceed 3/4. (See Lemma 27 in Online Appendix I.)

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