CHICAGO JOURNALS

Dynamic Merger Review
Author(s): Volker Nocke and Michael D. Whinston
Source: *The Journal of Political Economy*, Vol. 118, No. 6 (December 2010), pp. 1201-1251
Published by: The University of Chicago Press
Stable URL: http://www.jstor.org/stable/10.1086/658161
Accessed: 22/02/2011 18:21

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Dynamic Merger Review

Volker Nocke

University of Mannheim and Centre for Economic Policy Research

Michael D. Whinston

Northwestern University and National Bureau of Economic Research

We analyze the optimal dynamic policy of an antitrust authority toward horizontal mergers when merger proposals are endogenous and occur over time. Approving a currently proposed merger may affect the profitability and welfare effects of potential future mergers, whose characteristics may not yet be known. We identify conditions under which discounted expected consumer surplus is maximized by using a completely myopic merger review policy that approves a merger if and only if it does not lower consumer surplus given the current market structure. We also discuss a number of extensions as well as factors that undermine the optimality of myopic merger review policies.

I. Introduction

The traditional approach to the review of a horizontal merger stresses the trade-off between market power and efficiencies. Mergers, which

We thank two referees and the editor, Sam Kortum, as well as Pat DeGraba, Glenn Ellison, Peter Eso, Joe Farrell, Ramon Fauli-Oller, Chiara Fumagalli, Tracy Lewis, Tore Nilsson, Patrick Rey, Carl Shapiro, Ron Siegel, Lucy White, members of the Toulouse Network for Information Technology, and audiences at numerous conferences and research seminars for their comments. Nocke gratefully acknowledges financial support from the National Science Foundation and the University of Pennsylvania Research Foundation, as well as the hospitality of Northwestern University's Center for the Study of Industrial Organization. Whinston thanks the National Science Foundation, the Toulouse Network for Information Technology, and the Leverhulme Trust for financial support, as well as Nuffield College and the Oxford University Department of Economics for their hospitality. We thank Kei Kawai and Ben Mermelstein for research assistance.

[*Journal of Political Economy,* 2010, vol. 118, no. 6]
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cause firms to internalize pricing externalities among former rivals, increase the exercise of market power and therefore tend to reduce social welfare. However, since they can create efficiencies, horizontal mergers may instead increase welfare. This trade-off was first articulated by Williamson (1968) for the case of an antitrust authority who wants to maximize aggregate surplus, using a diagram like figure 1. In the diagram, a competitive industry merges to become a monopolist that charges the price p' but lowers its marginal cost of production from c to c'. Whether aggregate surplus increases or not depends on whether the dark grey deadweight loss triangle exceeds the light grey efficiency gain. A similar, though even more straightforward, trade-off arises when an antitrust authority instead applies a consumer surplus standard to merger approval decisions, as is (roughly) the case in both the U.S. and EU legal regimes.[1] In that case, the marginal cost reduction must be large enough that the price does not increase for the merger to be approved.

More recently, Farrell and Shapiro (1990) (see also McAfee and Williams 1992) have provided a more complete and formal analysis of this trade-off for settings with Cournot competition. Farrell and Shapiro provide a necessary and sufficient condition for a merger to increase consumer surplus as well as a sufficient condition for a merger to increase aggregate surplus.

With few exceptions, however, the literature on merger review has focused on the approval decision for a single merger. In reality, though, mergers are usually not one-time events. That is, one proposed merger in an industry may be followed by others. In that case, approval of a merger today based on current conditions, as in the Farrell and Shapiro test, appears inappropriate. Rather, an antitrust authority in general needs to determine the welfare effect of the current proposed merger given the potential for future merger approvals and given the fact that today's merger approval decision may alter the set of mergers that are later proposed.

In this paper, we identify conditions under which this apparently difficult problem has a simple resolution, in which an antitrust authority that wants to maximize consumer surplus can accomplish this objective by using a completely myopic merger review policy that approves a merger today if and only if it does not lower consumer surplus given the current market structure. These conditions provide, we think, a

[1] The U.S. Horizontal Merger Guidelines, issued by the U.S. Department of Justice and the Federal Trade Commission on August 19, 2010, e.g., state that "the Agency will not challenge a merger if cognizable efficiencies . . . likely would be sufficient to reverse the merger's potential harm to consumers in the relevant market." The European Commission Guidelines on the Assessment of Horizontal Mergers under the Council Regulation on the Control of Concentrations between Undertakings, issued on February 5, 2004, state that "the relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger."

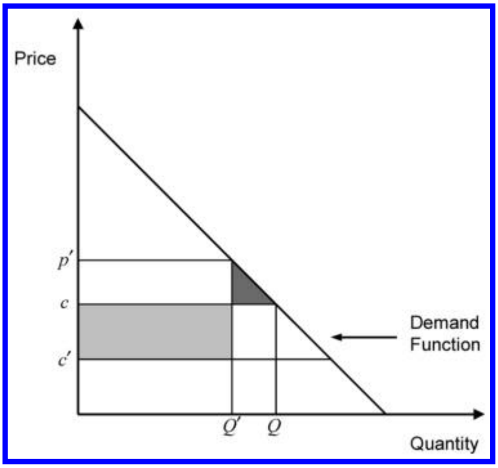

FIG. 1.—The Williamson trade-off in merger review: deadweight loss of market power (dark-shaded triangle) versus efficiency gain (light-shaded rectangle).

useful benchmark for thinking about the dynamic merger review problem, both identifying cases in which a myopic policy is in fact optimal and clarifying what factors can undermine the optimality of a myopic policy, necessitating a more complex consideration of dynamic trade-offs.[2]

We begin in Section II by establishing some preliminary characterizations of consumer surplus enhancing mergers and their interactions. Our central results focus on a model of Cournot competition with constant returns to scale. Most important, we show in Section II that there is a form of complementarity between mergers in that setting. In particular, mergers that enhance consumer surplus continue to be consumer surplus enhancing if other mergers that enhance consumer surplus take place. Similarly, mergers that reduce consumer surplus continue to be consumer surplus reducing if other mergers that reduce consumer surplus take place. That is, the sign of a merger's consumer surplus effect is unchanged if another merger whose consumer surplus effect has the same sign takes place. This result, which is of independent interest, sets the stage for our main result, which is contained in Section III.

[2] Nilssen and Sorgard (1998), Matsushima (2001), and Motta and Vasconcelos (2005) study mergers and antitrust review in a dynamic context. In these papers, two mergers between two nonoverlapping pairs of firms can take place sequentially. We discuss these papers further in Sec. IV. Kamien and Zang (1990), Gowrisankaran (1999), Fauli-Oller (2000), and Pesendorfer (2005) are among a much larger set of articles that study equilibrium merger decisions in dynamic models but without considering merger policy (and sometimes without allowing for efficiencies).

In Section III we embed our Cournot competition framework into a dynamic model in which merger opportunities arise, and may be proposed, over time. We show that if the set of possible mergers is disjoint, in the sense that each firm is involved in at most one potential merger, and if mergers that are not approved in a given period are still feasible at any later date, then a completely myopic consumer surplus–based approval policy maximizes discounted consumer surplus for every possible realization of the set of feasible mergers.

In Section IV, we discuss both extensions of this result and factors that may undermine it, considering more limited information possessed by firms about rivals' merger possibilities, other models of competition (homogeneous and differentiated product price competition), demand shifts, entry, continuing efficiency improvements, the presence of fixed costs and exit, merger proposal and implementation costs, merger blocking costs, the nondisjointness of mergers, and the use of an aggregate surplus criterion.

Section V presents conclusions. There, we note how our model naturally gives rise to the emergence of endogenous merger waves and also discuss implications for the analysis of breakups.

II. Mergers in the Cournot Model

A. *Cournot Oligopoly*

Consider an industry with N firms producing a homogeneous good and competing in quantities. Let $\mathcal{N} \equiv \{1, 2, \ldots, N\}$ denote the set of firms. Firm i's cost of producing q_i units of output is given by $C_i(q_i) = c_i q_i$, where $c_i > 0$ is firm i's marginal cost. Thus, for now, we restrict attention to firms producing under constant returns to scale. The inverse market demand is given by the twice-differentiable function $P(Q)$, where $Q \equiv \sum_{i \in \mathcal{N}} q_i \geq 0$ is industry output. We make the following (standard) assumption on demand.

ASSUMPTION 1. For any $Q > 0$ such that $P(Q) > 0$, (i) $P'(Q) < 0$ and (ii) $P'(Q) + QP''(Q) < 0$. Moreover, (iii) $\lim_{Q \to \infty} P(Q) = 0$.

Part i of the assumption says that demand is downward sloping, part ii implies that quantities are strategic substitutes and that each firm's profit maximization problem is strictly concave, and part iii in conjunction with $c_i > 0$ for all i implies that the equilibrium aggregate output is bounded.

Let $Q_{-i} \equiv \sum_{j \neq i} q_j$ denote the aggregate output of all firms other than i. Firm i's best response is

$$b(Q_{-i} | c_i) = \arg\max_{q_i \geq 0} \, [P(Q_{-i} + q_i) - c_i] q_i. \tag{1}$$

As is well known (see, e.g., Farrell and Shapiro 1990), assumption 1 implies that each firm's best-response function $b(Q_{-i}|c_i)$ satisfies $\partial b(Q_{-i}|c_i)/\partial Q_{-i} \in (-1, 0)$ at all Q_{-i} such that $b(Q_{-i}|c_i) > 0$.

Under assumption 1, there is a unique Nash equilibrium. Let Q^* and q_i^* denote, respectively, the industry output and firm i's output in equilibrium. The first-order condition for problem (1) is

$$P(Q^*) - c_i + P'(Q^*)q_i^* \leq 0, \quad = 0 \text{ if } q_i^* > 0, \tag{2}$$

so output levels in this equilibrium satisfy

$$q_i^* = -\frac{P(Q^*) - c_i}{P'(Q^*)} \tag{3}$$

if $c_i < P(Q^*)$, and $q_i^* = 0$ otherwise. Assumption 1 also implies that the equilibrium is "stable," so that comparative statics are "well behaved." For example, we will make use of two comparative statics properties: First, a reduction in an active firm's marginal cost increases its equilibrium output and profit, reduces the output and profit of each of its active rivals, and increases aggregate output and consumer surplus. Second, following any change in the incentives of a subset of firms, the equilibrium aggregate output increases if and only if the equilibrium output of that set of firms increases (see Farrell and Shapiro 1990, 111, lemma).

B. The Consumer Surplus Effect of Mergers

Consider a merger between a subset $M \subseteq \mathcal{N}$ of firms that will result in a postmerger marginal cost of $\bar{c}_M$. Denoting aggregate output before the merger by Q^* and after by $\overline{Q}^*$, the change in consumer surplus is $CS(\overline{Q}^*) - CS(Q^*)$, where

$$CS(Q) = \int_0^Q [P(s) - P(Q)]ds.$$

Since $CS'(Q) = -QP'(Q) > 0$, a merger increases consumer surplus if and only if it induces an increase in industry output. We will say that a merger is *CS-neutral* if the merger does not affect consumer surplus. Similarly, we will say that a merger is *CS-increasing* if consumer surplus following the merger is larger than before and *CS-decreasing* if it is smaller. Finally, a merger is *CS-nondecreasing* if it is not CS-decreasing and is *CS-nonincreasing* if it is not CS-increasing.

We will say that a merger involves active firms if at least one of the merging firms is producing a positive quantity before the merger (and hence has $c_i < P(Q^*)$). Observe that a merger involving only inactive firms is always CS-nondecreasing and weakly profitable. The following result catalogs some useful properties of CS-neutral mergers involving active firms.

LEMMA 1. If a merger M involving active firms is CS-neutral, then (1) it causes no changes in the output of any nonmerging firm or in the total output of the merging firms, (2) the merged firm's postmerger margin equals the sum of the active merging firms' premerger margins:

$$P(Q^*) - \bar{c}_M = \sum_{i \in M} \max\{0, P(Q^*) - c_i\}, \quad (4)$$

and (3) the merger is profitable (it weakly increases the joint profit of the merging firms) and is strictly profitable if it involves at least two active firms.

Proof. To see property 1, observe that by first-order condition (2) there is a unique output level for each nonmerging firm i that is compatible with any given level of aggregate output Q. Since aggregate output is unchanged by a CS-neutral merger, all nonmerging firms' outputs are unchanged. In turn, this implies that the total output of the merging firms must be unchanged as well. For property 2, note that the merged firm's postmerger first-order condition (using property 1) is

$$P(Q^*) - \bar{c}_M + \Big(\sum_{i \in M} q_i^*\Big) P'(Q^*) = 0. \quad (5)$$

Summing the premerger first-order conditions of the active merger partners yields

$$\sum_{i \in M_+} [P(Q^*) - c_i + q_i^* P'(Q^*)] = 0, \quad (6)$$

where $M_+ = \{i \in M : q_i^* > 0\}$. Since for all $i \in M \backslash M_+$ we have $P(Q^*) \leq c_i$ and $q_i^* = 0$, it follows that[3]

$$\sum_{i \in M} \max\{0, P(Q^*) - c_i\} + \Big(\sum_{i \in M} q_i^*\Big) P'(Q^*) = 0. \quad (7)$$

Combining equations (5) and (7) yields condition (4). Property 3 holds since the merging firms' joint output has not changed (property 1), but their margin has weakly increased and has strictly increased if the merger involves at least two active firms (property 2). QED

As emphasized by Farrell and Shapiro (1990), it follows from property 2 of lemma 1 that a CS-neutral merger among two or more active firms must reduce marginal cost below the marginal cost of the most efficient merger partner ($\bar{c}_M < \min_{i \in M}\{c_i\}$). The following corollary (which follows from properties 2 and 3 of lemma 1 and the fact that the postmerger aggregate output, $\overline{Q}^*$, and the profit of the merged firm are both decreasing in the merged firm's marginal cost, $\bar{c}_M$) identifies the threshold level of postmerger marginal cost that makes a merger CS-nondecreasing and records that any CS-nondecreasing merger is profitable for the merging firms.

[3] Throughout the paper, $A \backslash B$ denotes the elements of set A that are not in set B.

Corollary 1. A merger involving active firms is CS-neutral if

$$\bar{c}_M = \hat{c}_M(Q^*) \equiv P(Q^*) - \sum_{i \in M} \max\{0, P(Q^*) - c_i\},$$

CS-increasing if $\bar{c}_M < \hat{c}_M(Q^*)$, and CS-decreasing if $\bar{c}_M > \hat{c}_M(Q^*)$. Moreover, any CS-nondecreasing merger is profitable for the merging firms and is strictly profitable if it is CS-increasing or involves at least two active firms.

Corollary 1 implies that an antitrust authority concerned with maximizing consumer surplus and confronted with a single merger involving active firms in set M would strictly prefer to approve the merger if $\bar{c}_M < \hat{c}_M(Q^*)$, would be willing to approve it if $\bar{c}_M \leq \hat{c}_M(Q^*)$, and would strictly prefer to reject it if $\bar{c}_M > \hat{c}_M(Q^*)$. Moreover, any merger among active firms that the antitrust authority would be willing to approve is strictly profitable for the merging parties.

Observe also that the threshold $\hat{c}_M(Q^*)$ is nondecreasing in Q^* and is strictly increasing in Q^* if the merger involves at least two active firms. Thus, the larger Q^* is (and the lower the premerger price), the more likely it is that a merger is CS-nondecreasing. This fact will play a central role in our analysis, where one merger may lead to a change in industry output prior to the proposal of another merger. To see the intuition for this result, consider a proposed merger between two symmetric firms, each of which has a premerger marginal cost c in a market with a linear inverse demand function with slope $b < 0$. An increase in the premerger aggregate output Q^* (perhaps due to a cost-reducing merger among these firms' rivals) leads these two firms to reduce their premerger outputs q^*. However, the market power effect from the merger (involving internalization of competitive externalities) depends solely on the merging firms' outputs in such a market: the merger is CS-neutral if its marginal cost reduction $\Delta c = c - \bar{c}_M$ exactly equals the market power effect, $-bq^*$. Since the market power effect is reduced when the premerger outputs of the merging firms decrease whereas the efficiency gain remains unchanged, the threshold for a CS-nondecreasing merger is relaxed when the premerger aggregate output Q^* increases.[4]

Figure 2 illustrates the cases of CS-neutral, CS-increasing, and CS-decreasing mergers. The figure considers a merger involving two active firms, $M_1 = \{1, 2\}$, in a four-firm industry. The complementary set of firms is $M_2 = \{3, 4\}$. The axes in the figure measure the joint outputs of the two sets of firms, q_{M_1} and q_{M_2}. The curves labeled $r_{M_1}(q_{M_2})$ and $r_{M_2}(q_{M_1})$ depict the "premerger group-response functions" of each set of firms prior to the merger. Specifically, M_1's premerger group-

[4] More generally, with a nonlinear inverse demand function, a lower premerger margin $P(Q^*) - c$ due to a larger premerger aggregate output implies a smaller premerger absolute value of $P'(Q^*)q^*$ (see eq. [2]), so the same idea applies.

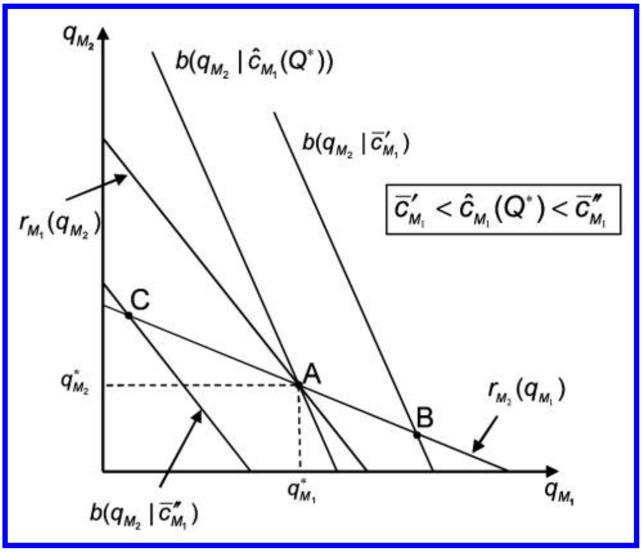

FIG. 2.—A merger involving the firms in M_1. Depending on the merged firm's marginal cost, the merger may be CS-neutral (point A), CS-increasing (point B), or CS-decreasing (point C).

response function gives the joint premerger Nash equilibrium output of the firms in M_1, $r_{M_1}(q_{M_2})$, conditional on the firms in M_2 producing q_{M_2} in total,

$$r_{M_1}(q_{M_2}) \equiv \{q_1 + q_2 : q_1 = b(q_2 + q_{M_2}|c_1) \text{ and } q_2 = b(q_1 + q_{M_2}|c_2)\},$$

and similarly for $r_{M_2}(q_{M_1})$. It is routine to verify that these group-response functions satisfy $-1 < r'_{M_i}(q_{M_j}) < 0$.

The equilibrium before the merger is point A in the figure, the intersection of the two premerger group-response curves $r_{M_1}(q_{M_2})$ and $r_{M_2}(q_{M_1})$; at that point, the outputs of groups M_1 and M_2 are $q^*_{M_1}$ and $q^*_{M_2}$, respectively, and all four firms are playing best responses given their premerger costs. A merger M_1 that results in a postmerger marginal cost $\bar{c}_{M_1}$ causes the merged firm to have best-response curve $b(q_{M_2}|\bar{c}_{M_1})$.[5]

[5] Any postmerger best-response curve $b(q_{M_2}|\bar{c}_{M_1})$ must cross the premerger group-response curve $r_{M_1}(q_{M_2})$ from above at any interior intersection of these two curves, $(\bar{q}_{M_1}, \bar{q}_{M_2}) \gg 0$. To see this fact, observe that at any $q_{M_2} > \bar{q}_{M_2}$ such that $r_{M_1}(q_{M_2}) > 0$, we have

$$\begin{aligned} b(q_{M_2}|\bar{c}_{M_1}) &= b(q_{M_2}|\hat{c}_{M_1}(\bar{q}_{M_2} + r_{M_1}(\bar{q}_{M_2}))) \\ &> b(q_{M_2}|\hat{c}_{M_1}(q_{M_2} + r_{M_1}(q_{M_2}))) \\ &= r_{M_1}(q_{M_2}), \end{aligned}$$

where the first equality follows because $\bar{c}_{M_1}$ is the postmerger cost level that would be CS-neutral if the premerger aggregate output was $\bar{q}_{M_2} + \bar{q}_{M_1} = \bar{q}_{M_2} + r_{M_1}(\bar{q}_{M_2})$, the inequality

The postmerger equilibrium lies at the intersection of this best-response curve and the group-response curve of the firms in M_2, $r_{M_2}(q_{M_1})$. With a CS-neutral merger, in which $\bar{c}_{M_1} = \hat{c}_{M_1}(Q^*)$, the postmerger best-response curve of the merged firm, $b(q_{M_2}|\hat{c}_{M_1}(Q^*))$, intersects group M_2's group-response curve at point A, so aggregate output does not change. With a CS-increasing merger, the merged firm's marginal cost $\bar{c}'_{M_1}$ is less than $\hat{c}_{M_1}(Q^*)$, so its best-response curve $b(q_{M_2}|\bar{c}'_{M_1})$ lies further to the right, shifting the equilibrium to point B. Aggregate output is larger at point B than at point A since the slope of group M_2's group-response curve, $r'_{M_2}(q_{M_1})$, is between zero and negative one. In contrast, with a CS-decreasing merger, the merged firm's marginal cost $\bar{c}''_{M_1}$ is greater than $\hat{c}_{M_1}(Q^*)$, so its best-response curve $b(q_{M_2}|\bar{c}''_{M_1})$ lies further to the left, shifting the equilibrium to point C, where aggregate output is smaller.

C. *Interactions between Mergers*

We now turn to the interactions between mergers. These interactions will play a central role when we study our dynamic model of merger review in Section III. In this subsection, we consider two potential disjoint mergers, involving firms in sets M_1 and M_2 with $M_1 \cap M_2 = \emptyset$. We will refer to these simply as merger M_1 and merger M_2. The set of firms not involved in either merger is $\mathcal{N}^c \equiv \mathcal{N} \setminus (M_1 \cup M_2)$.

Our first result establishes a certain complementarity between mergers that change consumer surplus in the same direction.[6]

Proposition 1. The sign of the CS effect of two disjoint mergers is complementary: (i) if a merger is CS-nondecreasing (and hence profitable) in isolation, it remains CS-nondecreasing (and hence profitable) if another merger that is CS-nondecreasing in isolation takes place; (ii) if a merger is CS-decreasing in isolation, it remains CS-decreasing if another merger that is CS-nonincreasing in isolation takes place.

Proof. For part i, suppose that mergers M_1 and M_2 are both CS-nondecreasing in isolation. Let Q^* denote aggregate output in the absence of either merger and let $\overline{Q}^*_i$ denote aggregate output if only merger M_i takes place. So $\overline{Q}^*_i \geq Q^*$ for $i = 1, 2$. Without loss of gen-

follows because $r'_{M_1}(q_{M_2}) \in (0, 1)$, $\hat{c}_{M_1}(q_{M_2})$ is a strictly increasing function, and $b(q_{M_2}|c)$ is strictly decreasing in c at all q_{M_2} such that $b(q_{M_2}|c) > 0$, and the last equality follows because $\hat{c}_{M_1}(q_{M_2} + r_{M_1}(q_{M_2}))$ is the cost level at which the merged firm's best response to q_{M_2} is exactly $r_{M_1}(q_{M_2})$. Likewise, at any $q_{M_2} < \bar{q}_{M_2}$ we must have $b(q_{M_2}|\bar{c}_{M_1}) < r_{M_1}(q_{M_2})$.

[6] Proposition 1 focuses on properties needed later in this section and for Sec. III. It is straightforward to show as well that (i) a CS-increasing merger M_i remains CS-increasing if a merger M_j that is CS-nondecreasing takes place provided that merger M_i remains among active firms once merger M_j takes place, and (ii) a merger among active firms that is CS-nonincreasing remains CS-nonincreasing if a merger that is CS-nonincreasing takes place; see the discussion of remark 2 in the Appendix.

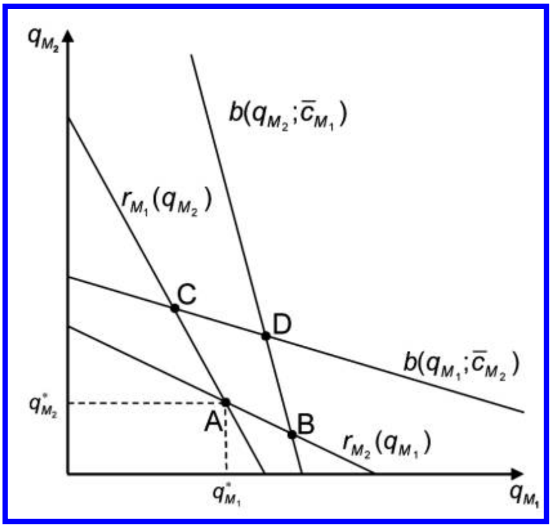

FIG. 3.—Each merger is CS-increasing in isolation and remains so if the other merger takes place.

erality, consider merger M_1. Suppose, first, that merger M_1 involves only inactive firms once merger M_2 takes place. Then, once merger M_2 takes place, merger M_1 must be CS-nondecreasing and (weakly) profitable.

Suppose, instead, that merger M_1 involves active firms once merger M_2 takes place, which also means (since $P(\overline{Q}_2^*) \leq P(Q^*)$) that it involves active firms when done in isolation. Since it is CS-nondecreasing in isolation, from corollary 1 we know that $\bar{c}_{M_1} \leq \hat{c}_{M_1}(Q^*)$. Moreover, because the threshold $\hat{c}_{M_1}(Q)$ is nondecreasing in Q, we have $\bar{c}_{M_1} \leq \hat{c}_{M_1}(\overline{Q}_2^*)$. Hence, corollary 1 implies that merger M_1 is also CS-nondecreasing once merger M_2 has taken place.

The argument for part ii follows similar lines (note that a CS-decreasing merger must involve active firms and must continue to do so after another CS-decreasing merger takes place). QED

Figure 3 illustrates the complementarity between two mergers that are CS-increasing in isolation when no other firms exist ($\mathcal{N}^c = \emptyset$). In isolation, the CS-increasing merger M_1 moves the equilibrium from point A (the intersection of the two premerger group-response curves $r_{M_1}(q_{M_2})$ and $r_{M_2}(q_{M_1})$) to point B (the intersection of best-response curve $b(q_{M_2} | \bar{c}_{M_1})$ with premerger group-response curve $r_{M_2}(q_{M_1})$), whereas the CS-increasing merger M_2 moves the equilibrium from point A to point C (the intersection of best-response curve $b(q_{M_1} | \bar{c}_{M_2})$ with premerger group-response curve $r_{M_1}(q_{M_2})$). But, conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D (the intersection of the two best-response curves $b(q_{M_2} | \bar{c}_{M_1})$ and $b(q_{M_1} | \bar{c}_{M_2})$)

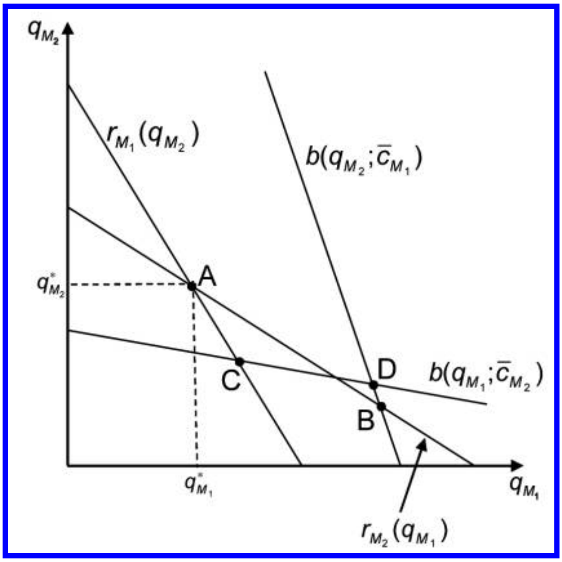

FIG. 4.—A CS-decreasing merger M_2 that becomes CS-increasing after a CS-increasing merger M_1 takes place.

along $b(q_{M_2}|\,\bar{c}_{M_1})$. Since $\partial b(q_{M_2}|\,\bar{c}_{M_1})/\partial Q_{-i} \in (-1, 0)$, aggregate output must increase with this change. Hence, conditional on merger M_1 taking place, merger M_2 remains CS-increasing. Moreover, we know from corollary 1 that it also remains profitable. Using the same type of argument, the reverse is also true: conditional on merger M_2 taking place, merger M_1 remains CS-increasing and profitable.

When we consider the dynamic model in Section III, we will also need to understand how mergers that have the opposite effects on consumer surplus (if implemented in isolation) interact. We now turn to this case. Suppose then that merger M_1 is CS-nondecreasing (and therefore profitable) in isolation whereas merger M_2 is CS-decreasing in isolation. Figure 4 illustrates that merger M_2 can become CS-increasing conditional on merger M_1 occurring. In isolation, merger M_2 moves the equilibrium from point A to point C along the premerger group-response curve $r_{M_1}(q_{M_2})$ and thus decreases industry output and consumer surplus. But after the CS-increasing merger M_1 has taken place, merger M_2 moves the equilibrium from point B to point D along best-response curve $b(q_{M_2}|\,\bar{c}_{M_1})$ and thus increases output and consumer surplus. Moreover, since it is CS-increasing once merger M_1 takes place, it is also profitable for the firms in M_2. In the dynamic model we study in Section III, this will mean that when a set of firms M_1 proposes a CS-nondecreasing merger, the approval of that merger may cause other mergers to later be proposed and approved that would not have occurred had the first merger M_1 not been proposed. It is therefore important to understand the effect of such a chain of mergers.

PROPOSITION 2. Suppose that merger M_1 is CS-nondecreasing in isolation whereas merger M_2 is CS-decreasing in isolation but CS-nondecreasing once merger M_1 has taken place. Then (i) merger M_1 is CS-increasing (and therefore strictly profitable) conditional on merger M_2 taking place and (ii) the joint profit of the firms involved in merger M_1 is strictly larger if both mergers take place than if neither merger takes place.

Proof. Consider implementing merger M_1 first followed by merger M_2. By hypothesis, consumer surplus weakly increases after each step, so the combined effect on consumer surplus of the two mergers is nonnegative. Suppose that we now reverse the order and implement merger M_2 first. Since the combined effect of the two mergers on consumer surplus is nonnegative whereas the effect of merger M_2 is strictly negative, consumer surplus must strictly increase when merger M_1 is implemented following merger M_2. Hence, part i must hold: merger M_1 is CS-increasing (and therefore strictly profitable) conditional on merger M_2 taking place.

To see that part ii holds, suppose that merger M_2 is implemented first. Since merger M_2 is CS-decreasing in isolation, it must weakly increase the profit of each firm $i \in M_1$ (the total output of all firms other than i must decrease; otherwise the fact that $\partial b(q_i|c_i)/\partial Q_{-i} \in (-1, 0)$ would imply that aggregate output increases). Since merger M_1 is strictly profitable given merger M_2, the sequence of mergers must strictly increase the joint profit of the firms in M_1. QED

Part i of proposition 2 is illustrated in figure 5 below, where merger M_1 is CS-increasing (and hence strictly profitable) in isolation and remains so conditional on merger M_2 taking place, at which point it moves the equilibrium from point C to point D along $b(q_{M_1}|\bar{c}_{M_2})$.

Part ii of proposition 2 is of some importance for our analysis in Section III. In particular, imagine that proposal and approval of merger M_1 causes merger M_2 to become CS-increasing and therefore be proposed and approved. Because the follow-on merger M_2 is CS-increasing, it is bad for the merged firm M_1. Part ii of proposition 2 tells us that despite this fact the firms in merger M_1 are better off proposing their merger even though it causes merger M_2 to occur.

REMARK 1. Observe that the logic of proposition 2 can be extended to cases with a merger M_1 that is CS-nondecreasing in isolation and a collection of mergers $M_2, \dots, M_K$ that are each CS-decreasing in isolation but form a sequence that is CS-nondecreasing at each step after merger M_1 has taken place. In such cases, merger M_1 is CS-increasing (and therefore strictly profitable) given that mergers $M_2, \dots, M_K$ have taken place, and the joint profit of the firms involved in merger M_1 is strictly larger if all these mergers take place than if none do. We will use this extension of proposition 2 in Section III.

III. Consumer Surplus–Maximizing Dynamic Merger Review

In this section, we embed the Cournot model of Section II in a dynamic model in which merger opportunities arise stochastically over time, merger proposals are endogenous, and the antitrust authority decides whether or not to approve proposed mergers. We consider the optimal merger approval policy for an antitrust authority concerned with maximizing discounted expected consumer surplus. We show that such an antitrust authority can achieve its optimal outcome using a myopic policy that in each period approves a set of mergers that maximizes consumer surplus given the current market structure, ignoring the possibility of any future mergers. Moreover, within each period, mergers can be considered one at a time and approved on the basis of whether they are CS-nondecreasing given the market structure at the time of the review.

As before, we denote the set of N firms by $\mathcal{N}$. The set of possible mergers are those in set $\{M_1, \ldots, M_K\}$, where $M_k \subseteq \mathcal{N}$ is a set of firms that may merge. We assume that these possible mergers are disjoint, that is, $M_j \cap M_k = \emptyset$ for $j \neq k$. Thus, no firm has the possibility of being part of more than one merger.[7] The assumption of disjointness is reasonable when each firm belongs to at most a single set of "natural" merger partners that can generate significant efficiencies by merging, perhaps because they use similar or complementary technologies. If all other mergers both increase market power and fail to generate efficiencies, no other mergers but these would ever optimally be approved by the antitrust authority.[8] (We discuss the disjointness assumption further in Sec. V.)

The merger process lasts for T periods. Merger M_k first becomes feasible at the start of period t with probability $p_{kt} \in [0, 1]$, where $\sum_t p_{kt} \leq 1$. Conditional on merger M_k becoming feasible in period t, the firms in M_k receive and observe a random draw of their postmerger cost $\bar{c}_{M_k}$. This cost is drawn from the set C_{kt}. One possibility allowed by this structure, of course, is that the sequence of feasible mergers and their costs is deterministic and known ex ante. However, we allow for the more realistic possibility that at any point in time neither the firms nor the antitrust authority knows with certainty what mergers will be feasible

[7] Our results can be extended to allow for a given firm to be part of several different possible mergers provided that at most one of these mergers ever becomes feasible along any path.

[8] Because it takes a strictly positive cost reduction to offset the market power increase from a merger (recall lemma 1), it is enough to justify the disjointness assumption if other mergers cannot generate large enough cost reductions.

in the future.[9] Indeed, the feasibility and cost levels of the various mergers may be correlated in any way within periods or over time. We denote the set of mergers that have become feasible up to and including period t (including their cost realizations) by $\mathfrak{F}_t$.

In each period t, all firms with feasible but not yet approved mergers decide whether to propose them or not. Previously proposed but rejected mergers can be proposed again, as can previously unproposed feasible mergers. We denote by $\mathcal{P}_t$ the set of mergers proposed in period t. The antitrust authority then responds by approving some subset of the proposed mergers. We denote by $\mathcal{A}_t$ the set of mergers approved by the end of period t; that is, $\mathcal{A}_t$ is the *market structure* at the end of the period after the merger review process for that period has concluded. Note that we must have $\mathcal{A}_{t-1} \subseteq \mathcal{A}_t \subseteq (\mathcal{A}_{t-1} \cup \mathcal{P}_t) \subseteq \mathfrak{F}_t$; the first inclusion follows because the set of approved mergers weakly grows over time, the second because only proposed mergers can be approved, and the third because only feasible mergers that have not yet been approved can be proposed.[10]

We assume that when a merger M_k becomes feasible in a period t, one of the firms in M_k is designated as the "proposer" of the merger. To keep things simple, we treat bargaining in a reduced-form manner, assuming that the proposer chooses whether to propose the merger to the antitrust authority and that if it chooses to do so, the firms in M_k split the profit gains or losses from the merger in some fixed proportions (the proportions do not matter).[11]

The antitrust authority observes the feasibility of a particular merger and its efficiency (i.e., its postmerger marginal cost) once it is proposed. For simplicity, we assume that firms observe both their own and their rivals' merger possibilities, including their efficiencies, when they become feasible. (We discuss in Sec. IV.A how our results extend if firms possess less information about rivals' mergers, e.g., observing their feasibility only once they are proposed and their efficiency only once they are approved.)

Payoffs in each period t depend only on the set of mergers $\mathcal{A}_t$ approved by the end of that period and are determined by a complete information

[9] Note that our formulation embodies another form of disjointness in merger possibilities: merger M_k receives at most one efficiency realization throughout the merger process. We relax this assumption in Sec. IV.F.

[10] In the model, we do not allow previously approved mergers to be dissolved. However, it follows from our arguments that no (approved) merged firm would want to do so.

[11] The only important feature of this assumption is that it implies that merger M_k is proposed if it raises the joint expected discounted profit of the potential merger partners and will not be proposed if it lowers it.

Cournot game, as in Section II.[12] Each agent i, whether the antitrust authority or a proposer firm, discounts future payoffs (consumer surplus or profit) according to a discount factor $\delta \leq 1$.[13]

A. *Optimality of a Myopic Merger Policy: Intuition and an Example*

Our main result shows that a "myopically CS-maximizing merger policy" (as defined formally in Sec. III.B below) is a dynamically optimal policy for the antitrust authority. In this subsection, we provide some intuition for the result and illustrate it with a simple two-period example in which there are two possible mergers. The formal statement and proof are in Section III.C.

It is useful to think of the dynamic problem facing the antitrust authority in two parts. First, imagine that the antitrust authority has total control over which mergers are implemented: rather than firms proposing mergers, suppose that the antitrust authority is aware of all feasible mergers and has the power to implement them or not as it sees fit. In that case, in which the antitrust authority's problem is a single-agent dynamic decision problem, would a myopic policy be optimal? Two features of our model imply that it would. First, the complementarity property of CS-nondecreasing mergers derived in proposition 1 means that if the antitrust authority implements in some period t a merger that is CS-nondecreasing at the time of approval, it will never regret that decision later: if other CS-nondecreasing mergers become feasible later, complementarity implies that the antitrust authority will continue to want the first merger implemented as well. Second, because feasible but not yet approved mergers remain feasible, there is never any loss from not implementing now a merger that is currently CS-decreasing, since it can always be implemented later if circumstances change.

To see these points more concretely, consider the following simple two-period example with two possible mergers.

Example 1. Suppose that in period 1 merger M_1 becomes feasible with certainty, but with an ex ante uncertain cost level $\bar{c}_{M_1}$, whereas in period 2 merger M_2 becomes feasible with certainty with an ex ante uncertain cost level $\bar{c}_{M_2}$. The industry output if no mergers occur is Q^*.

Consider which mergers the antitrust authority would like to imple-

[12] For simplicity, we assume that there is one period of competition following the merger approval decisions in the last period, period T. However, our results would also hold if instead many periods of competition followed period T.

[13] Our results continue to hold if different firms as well as the antitrust authority have differing discount factors. However, to justify our profit-splitting assumption, we need to assume that each firm in a given merger M has the same discount factor.

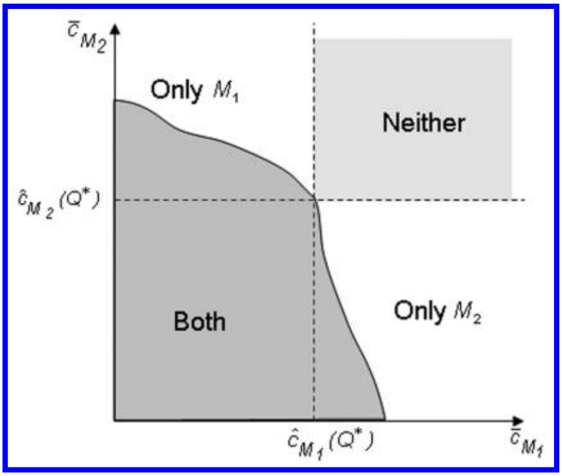

FIG. 5.—The four labeled regions show the CS-maximizing mergers in period 2 of example 1 for different postmerger cost realizations. Merger M_1 is CS-nondecreasing in isolation if postmerger costs lie on or to the left of the vertical dashed line, whereas merger M_2 is CS-nondecreasing in isolation if postmerger costs lie on or below the horizontal dashed line.

ment if it has complete control over which mergers occur. Figure 5 shows possible postmerger cost levels for the two mergers, with $\bar{c}_{M_1}$ shown on the horizontal axis and $\bar{c}_{M_2}$ on the vertical axis. Merger M_1 is CS-nondecreasing in isolation if the pair $(\bar{c}_{M_1}, \bar{c}_{M_2})$ lies on or to the left of the dashed vertical line at $\hat{c}_{M_1}(Q^*)$, whereas merger M_2 is CS-nondecreasing in isolation if the pair $(\bar{c}_{M_1}, \bar{c}_{M_2})$ lies on or below the dashed horizontal line at $\hat{c}_{M_2}(Q^*)$. The figure shows four regions, corresponding to which mergers maximize consumer surplus in period 2. If both mergers are CS-decreasing in isolation (so $\bar{c}_{M_k} > \hat{c}_{M_k}(Q^*)$ for $k = 1, 2$), then part ii of proposition 1 implies that neither should be implemented, which corresponds to the region labeled "Neither." The region labeled "Both" shows those postmerger cost pairs such that implementing both mergers maximizes consumer surplus in period 2. Note that the region in which both mergers should be implemented contains the region in which both mergers are CS-nondecreasing in isolation (corresponding to postmerger costs with $\bar{c}_{M_k} \leq \hat{c}_{M_k}(Q^*)$ for $k = 1, 2$), reflecting the complementarity property reported in part i of proposition 1.[14] Moreover, in some cases both mergers should be implemented even though one

[14] The curve forming the boundary of the area Both when $\bar{c}_{M_2} < \hat{c}_{M_2}(Q^*)$ is the set of cost pairs $\{(\bar{c}_{M_1}, \bar{c}_{M_2}) : \bar{c}_{M_1} = \hat{c}_{M_1}(Q_2^*(\bar{c}_{M_2}))\}$, where $Q_2^*(\bar{c}_{M_2})$ is aggregate output when merger M_2 with postmerger cost $\bar{c}_{M_2}$ is implemented in isolation. The curve is downward sloping because a decrease in $\bar{c}_{M_2}$ induces a larger aggregate output $Q_2^*(\bar{c}_{M_2})$ and thus a higher threshold $\hat{c}_{M_1}(Q_2^*(\bar{c}_{M_2}))$. Similarly, the downward-sloping curve forming the boundary of the area Both when $\bar{c}_{M_1} < \hat{c}_{M_1}(Q^*)$ is the set of cost pairs $\{(\bar{c}_{M_1}, \bar{c}_{M_2}) : \bar{c}_{M_2} = \hat{c}_{M_2}(Q_1^*(\bar{c}_{M_1}))\}$.

of them would be CS-decreasing in isolation because, as considered in proposition 2, a CS-increasing merger may cause another merger that is CS-decreasing in isolation to become CS-increasing. In the remaining two regions, labeled “Only M_1” and “Only M_2,” implementing just one merger maximizes period 2 consumer surplus.

Now consider whether merger M_1 should be implemented in period 1. If merger M_1 is CS-nondecreasing in isolation, implementing it in period 1 weakly increases period 1 consumer surplus. What effect does this have in period 2? Notice in figure 5 that when merger M_1 is CS-nondecreasing (so that $\bar{c}_{M_1} \leq \hat{c}_{M_1}(Q^*)$), it is optimal for merger M_1 to be implemented in period 2 regardless of the cost realization of merger M_2. Thus, in this case, merger M_1 should clearly be implemented in period 1. However, if merger M_1 is CS-decreasing in isolation, implementing it would lower period 1 consumer surplus without creating any benefit in period 2, since it could always be implemented in period 2 if that turns out to be desirable, given merger M_2’s efficiency level, $\bar{c}_{M_2}$. So if the antitrust authority could implement any merger it wanted, it would be optimal to behave myopically in period 1, implementing merger M_1 if and only if it is CS-nondecreasing in isolation.

For the second part, observe that the antitrust authority also faces an incentive problem: although it can reject mergers it does not like, it cannot compel firms to propose mergers. Rather, the antitrust authority must rely on firms to propose the mergers it wants to have implemented. However, if the antitrust authority follows a myopically CS-maximizing merger policy, it is an equilibrium for all feasible mergers to be proposed. To see the idea for why this is true, note first that if firms propose their merger and this has no effect on the other mergers that are approved, now or in the future, then the fact that, if approved, it will always be CS-nondecreasing given the other approved mergers (which follows from complementarity plus the fact that the antitrust authority is following a myopically CS-maximizing policy) means that it is a profitable merger for the firms (by corollary 1). However, if approval of their merger today causes other mergers to be approved, then by proposition 2 (and remark 1), it is better to have proposed their merger than not. Since every feasible merger is proposed in this equilibrium, the antitrust authority does in fact have total control over which mergers are implemented, so by the argument above, a myopically CS-maximizing policy is a dynamically optimal policy. (In our formal results, we also show that when the antitrust authority follows a myopically CS-maximizing merger policy, every subgame-perfect Nash equilibrium results in the same optimal sequence of consumer surpluses. Moreover, the optimality of this policy is very strong: even if the antitrust authority knew in advance the entire realized sequence of feasible mergers, it could not do better.)

B. Myopic Merger Policies

We are interested in the performance of "myopic" merger review policies, which in each period maximize consumer surplus given the set of proposed mergers and current market structure, ignoring the possibility of future mergers. Toward this end, we start by introducing the following definitions.

DEFINITION 1. A set of approved mergers $\mathcal{A}_t \subseteq (\mathcal{A}_{t-1} \cup \mathcal{P}_t) \subseteq \mathfrak{F}_t$ is *myopically CS-maximizing* for $\mathcal{P}_t$ *given market structure* $\mathcal{A}_{t-1}$ if it maximizes consumer surplus in the current period (period t) given the set of proposed mergers $\mathcal{P}_t$ and current market structure $\mathcal{A}_{t-1}$.

In our model with unchanging demand, maximizing current-period consumer surplus is equivalent to maximizing discounted consumer surplus assuming that there will be no subsequent changes in market structure.

DEFINITION 2. A *myopically CS-maximizing merger policy* is a merger approval rule that in each period t approves mergers as a function of the already approved mergers $\mathcal{A}_{t-1}$, the current set of proposed mergers $\mathcal{P}_t$, and perhaps the period t, resulting in a new market structure $\mathcal{A}_t^*(\mathcal{P}_t | \mathcal{A}_{t-1})$ that is myopically CS-maximizing for $\mathcal{P}_t$ given market structure $\mathcal{A}_{t-1}$.

While we note later in remark 2 that our main result holds for any myopically CS-maximizing merger policy, for ease of exposition we focus on the performance of the most lenient myopically CS-maximizing merger policy. In this policy, the antitrust authority resolves any indifference about mergers in favor of approval, selecting in each period the largest possible set of mergers to approve among those sets that maximize consumer surplus. We call such a set a "largest myopically CS-maximizing set."

DEFINITION 3. A set of approved mergers $\mathcal{A}_t \subseteq (\mathcal{A}_{t-1} \cup \mathcal{P}_t) \subseteq \mathfrak{F}_t$ is a *largest myopically CS-maximizing set* for $\mathcal{P}_t$ *given market structure* $\mathcal{A}_{t-1}$ if it is not contained in any other set that is myopically CS-maximizing for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$.

Given the finiteness of the set of proposed mergers $\mathcal{P}_t$, a largest myopically CS-maximizing set must always exist. In fact, our next result shows that there is a unique such "largest" set for any existing market structure $\mathcal{A}_{t-1}$ and set of proposed mergers $\mathcal{P}_t$, which we denote by $\overline{\mathcal{A}}^*(\mathcal{P}_t | \mathcal{A}_{t-1})$, and this set contains every other myopically CS-maximizing set for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$. Moreover, this set grows as the set of proposed mergers grows.

LEMMA 2. For each set of proposed mergers $\mathcal{P}_t$ and current market structure $\mathcal{A}_{t-1}$, there is a unique largest myopically CS-maximizing set $\overline{\mathcal{A}}^*(\mathcal{P}_t | \mathcal{A}_{t-1})$, and it contains every other myopically CS-maximizing set for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$. Moreover, if $\mathcal{P}_t \subset \mathcal{P}'_t$, then $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}_{t-1})$.

Proof. In the Appendix.

The monotonicity result in lemma 2 will play an important role in establishing our main result. It follows from the fact that for any two sets of proposed mergers $\mathcal{P}_t \subseteq \mathcal{P}'_t$ and corresponding myopically CS-maximizing sets $\mathcal{A}_t$ and $\mathcal{A}'_t$, the union of the two approval sets $\mathcal{A}_t \cup \mathcal{A}'_t$ results in a weakly larger CS level than either $\mathcal{A}_t$ or $\mathcal{A}'_t$. Put differently, for myopically CS-maximizing sets (which necessarily weakly increase consumer surplus), implementing one set of mergers does not change the sign of the CS effect of another set of mergers, a form of complementarity of myopically CS-maximizing sets. (This fact establishes both the uniqueness and the monotonicity results in the lemma; see the proof in the Appendix.)

The argument establishing this fact is more involved than might be expected: We already know by part i of proposition 1 that this complementarity holds for any pair of CS-nondecreasing mergers. A natural conjecture would be that a similar complementarity holds for any two sets of mergers that do not decrease consumer surplus. However, this is not true, as the following example illustrates.

Example 2. Suppose that $\mathcal{A}_t = \{M_1, M_2\}$ and $\mathcal{A}'_t = \{M_2, M_3\}$ are both sets of mergers that leave consumer surplus unchanged, while merger M_2 is CS-increasing in isolation. Thus, both M_1 and M_3 are CS-decreasing given M_2. In this case, the union $\mathcal{A}_t \cup \mathcal{A}'_t = \{M_1, M_2, M_3\}$ is CS-decreasing since M_1 is CS-decreasing once the CS-neutral set $\{M_2, M_3\}$ is approved.

However, complementarity does hold if the sets $\mathcal{A}_t$ and $\mathcal{A}'_t$ both have the property that every merger in each set is CS-nondecreasing given the other mergers in that set. (The sets in example 2 do not satisfy this property.) Since this property must hold for any myopically CS-maximizing set, this is enough to establish lemma 2. To see why complementarity holds, suppose that the sets $\mathcal{A}_t = \{M_1, M_2\}$ and $\mathcal{A}'_t = \{M_2, M_3\}$ have this property. Then since both M_1 and M_3 are CS-nondecreasing once M_2 is approved, part i of proposition 1 implies that consumer surplus is largest if the set $\{M_1, M_2, M_3\}$ is approved. For sets with more than two elements, establishing this complementarity also makes use of part ii of proposition 1.[15]

[15] For example, suppose that $\mathcal{A}_t = \{M_1, M_2\}$ and $\mathcal{A}'_t = \{M_2, M_3, M_4\}$, and every merger in each set is CS-nondecreasing given the other mergers in that set. Observe first that either M_3 or M_4 must be CS-nondecreasing given M_2; if not (i.e., if both M_3 and M_4 are CS-decreasing given M_2), then part ii of proposition 1 implies that M_3 is CS-decreasing given $\{M_2, M_4\}$ and M_4 is CS-decreasing given $\{M_2, M_3\}$, contradicting that every merger in $\mathcal{A}'_t$ is CS-nondecreasing given the other mergers in that set. Suppose, e.g., that M_3 is CS-nondecreasing given M_2. We can now use part i of proposition 1 repeatedly: starting from merger M_2, we know that both M_1 and M_3 are CS-nondecreasing, implying that M_1 remains

The most lenient myopically CS-maximizing merger policy is the policy that approves in each period the largest myopically CS-maximizing set of mergers.

DEFINITION 4. The *most lenient myopically CS-maximizing merger policy* is the myopically CS-maximizing merger policy that in each period t implements the largest myopically CS-maximizing set given the proposed mergers $\mathcal{P}_t$ and current market structure $\mathcal{A}_{t-1}$, resulting in new market structure $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$.

Note that the most lenient myopically CS-maximizing merger policy is independent of the period t since it depends only on the payoff-relevant variables $\mathcal{P}_t$ and $\mathcal{A}_{t-1}$. Importantly, the most lenient myopically CS-maximizing merger policy can also be thought of as the result of an antitrust policy that evaluates proposed mergers in an even more myopic way, making decisions on mergers within each period in a step-by-step fashion and approving a merger at each step if and only if it is CS-nondecreasing given the current market structure (including any mergers that have already been approved in that period) and continuing until no further CS-nondecreasing mergers can be identified (including mergers that may have already been examined but rejected earlier in the period). Specifically, we have the following lemma.

LEMMA 3. Suppose that the antitrust authority considers mergers within period t in a step-by-step fashion, approving mergers that are CS-nondecreasing given the current market structure until no further CS-nondecreasing mergers can be identified. Then, if $\mathcal{P}_t$ is the set of proposed mergers and $\mathcal{A}_{t-1}$ is the market structure at the start of the period, the set of approved mergers at the end of period t will be $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$.

Proof. In the Appendix.

Thus, our results will apply to any antitrust policy that considers mergers one at a time, approving each merger if it is CS-nondecreasing given the current market conditions.

The argument leading to lemma 3 follows from the same complementarity property of sets discussed above. In particular, we show that any set $\mathcal{A}_t$ that arises as a result of the step-by-step procedure also has the property that every merger in that set is CS-nondecreasing given every other merger in the set. Given this fact, if it were not contained within the largest CS-maximizing set, $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, then the union of the two sets, $\mathcal{A}_t \cup \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, would also be CS-maximizing, yielding a contradiction. However, $\mathcal{A}_t$ cannot be strictly contained within $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$: we show that if it were, there would be some additional

CS-nondecreasing given $\{M_2, M_3\}$. Since both M_1 and M_4 are then CS-nondecreasing given $\{M_2, M_3\}$, the CS level of $\mathcal{A}_t \cup \mathcal{A}'_t = \{M_1, M_2, M_3, M_4\}$ must weakly exceed that of either $\mathcal{A}_t$ or $\mathcal{A}'_t$.

merger in $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ that would be CS-nondecreasing given the set of approved mergers $\mathcal{A}_t$, and so it should be approved in the step-by-step procedure.

C. Optimality of Myopic Merger Policy: Formal Results

Corresponding to the intuition given above, our formal argument has two parts. First, we show that if all feasible but not yet approved mergers are proposed in each period—so that the antitrust authority need not worry about firms' incentives to propose mergers—then the most lenient myopically CS-maximizing merger policy maximizes discounted consumer surplus for every realized sequence of feasible mergers.

LEMMA 4. Suppose that all feasible but not yet approved mergers are proposed in each period, that is, $\mathcal{P}_1 = \mathfrak{F}_1$ and $\mathcal{P}_t = \mathfrak{F}_t \backslash \mathcal{A}_{t-1}$ for all $t > 1$. Then, the most lenient myopically CS-maximizing merger policy, which induces the approval sequence $\mathcal{A}_1 = \overline{\mathcal{A}}^*(\mathfrak{F}_1|\emptyset)$ and $\mathcal{A}_t = \overline{\mathcal{A}}^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ for all $t > 1$, maximizes discounted consumer surplus for every realization of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, \ldots, \mathfrak{F}_T)$.[16]

Proof. In the Appendix.

We establish the lemma by showing that the induced approval sequence $\{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_T\}$ coincides with the sequence $\{\overline{\mathcal{A}}^*(\mathfrak{F}_1|\emptyset), \ldots, \overline{\mathcal{A}}^*(\mathfrak{F}_T|\emptyset)\}$ that would result if the antitrust authority was not constrained by previous merger approvals and could choose in each period the market structure that maximizes consumer surplus in that period. This conclusion holds trivially when $T = 1$. Consider $T = 2$. Since $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$, the monotonicity result of lemma 2 implies that $\mathcal{A}_1 = \overline{\mathcal{A}}^*(\mathfrak{F}_1|\emptyset) \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_2|\emptyset)$; that is, in period 2 the antitrust authority does not want to undo previously approved mergers. Thus, the merger approvals in period $t = 1$ do not constrain the antitrust authority in period $t = 2$, implying that $\mathcal{A}_2 = \overline{\mathcal{A}}^*(\mathfrak{F}_2|\emptyset)$. Applying this reasoning iteratively yields the result for any number of periods T.

The second part of the argument for our main result concerns firms' incentives to propose mergers. To establish the result, we show that when the antitrust authority adopts the most lenient myopically CS-maximizing merger policy, the firms' proposal incentives are aligned with the desires of the antitrust authority. More specifically, there is a subgame-perfect Nash equilibrium for the firms in which every feasible merger is proposed in every period. Moreover, all subgame-perfect Nash equilibria result in the same (optimal) sequence of period-by-period consumer surpluses.

PROPOSITION 3. Suppose that the antitrust authority follows the most

[16] We denote by $\mathcal{A} = \emptyset$ the market structure in which no mergers have yet been approved.

lenient myopically CS-maximizing merger policy. Then: (i) All feasible mergers being proposed in each period after any history is a subgame-perfect Nash equilibrium for the firms. In this equilibrium, the outcome maximizes discounted consumer surplus for any realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, \ldots, \mathfrak{F}_T)$. (ii) For each sequence $\mathfrak{F}$, every subgame-perfect Nash equilibrium results in the same optimal sequence of period-by-period consumer surpluses.

Proof. In the Appendix.

To understand part i of the proposition, consider first the case in which $T = 1$ so that there are no future mergers. In that case, every firm is willing to propose its merger: only mergers that are CS-nondecreasing given the set of other approved mergers will be approved, and part i of proposition 1 and proposition 2 (and remark 1) imply that a CS-nondecreasing merger M is profitable (and strictly so if it results in an active firm) regardless of whether the set of other approved mergers changes because of the proposal of merger M. When $T > 1$, future mergers are possible. Nevertheless, we can apply an induction argument: in the last period, the result for $T = 1$ implies that there is a continuation equilibrium in which all mergers are proposed. In this equilibrium, the outcome in the last period is independent of previous merger proposals and approvals.[17] Given this fact, merger proposals in period $T - 1$ affect profits only in period $T - 1$. So the same logic implies that there is an equilibrium in which all feasible but not yet approved mergers are proposed in period $T - 1$. Applying this logic inductively yields the result.

To establish part ii of the proposition, we show that, in any step of the induction argument just described, any merger in the largest CS-maximizing set $\overline{\mathcal{A}}^*(\mathfrak{F}_T | \mathcal{A}_{T-1}) \backslash \mathcal{A}_{T-1}$ that will result in an active firm is sure to be approved if proposed. Since, as noted above, it is strictly profitable if approved, any such merger is sure to be proposed, which implies that the consumer surplus level is the same in all equilibria. We establish this fact by showing that there is an ordering of the mergers in the set $\overline{\mathcal{A}}^*(\mathfrak{F}_T | \mathcal{A}_{T-1}) \backslash \mathcal{A}_{T-1}$, say $(M_1, \ldots, M_K)$, that is CS-nondecreasing at each step.[18] In particular, merger M_1 is CS-nondecreasing given the set of previously approved mergers $\mathcal{A}_{T-1}$, so it is certain to be approved if proposed regardless of what other mergers are proposed. It must therefore be proposed in any equilibrium. Merger M_2 is CS-nondecreasing given that mergers $\mathcal{A}_{T-1} \cup M_1$ have been approved and is therefore certain to be approved if proposed given that merger M_1 is certain to be proposed. Merger M_2 is therefore certain to be proposed. Continuing to apply this logic inductively establishes the result.

Proposition 3 shows that a myopic merger policy that in each period

[17] This follows from the monotonicity property established in lemma 2; see the proof of lemma 4 in the Appendix.

[18] This conclusion follows from part ii of proposition 1; see lemma 5 in the Appendix.

approves the largest set of mergers that maximizes current consumer surplus (or, equivalently, maximizes discounted consumer surplus ignoring the possibility of any further changes in market structure) is dynamically optimal for the antitrust authority in that it maximizes discounted expected consumer surplus. Indeed, the proposition establishes an even stronger result: the antitrust authority could not do better even if it knew at the start of the process what the entire sequence of feasible mergers $(\mathfrak{F}_1, \dots, \mathfrak{F}_T)$ would be and could implement feasible but unproposed mergers.[19]

In addition, by lemma 3, the result implies that an even more myopic policy in which the antitrust authority considers mergers individually in a sequential fashion, myopically approving each merger if it is CS-nondecreasing given the market structure at the time of its review, is also dynamically optimal in this very strong sense.

Finally, we make the following observation.

Remark 2. While for ease of exposition we have restricted attention to the most lenient myopically CS-maximizing merger policy, dynamic optimality holds for any myopically CS-maximizing merger policy. See the Appendix for a discussion.

IV. Robustness

In this section, we discuss a number of extensions to, and limitations of, our main result.

A. *Information of Firms*

In our analysis, we have assumed that firms observe both their own and their rivals' merger possibilities, including their efficiencies, as soon as these become feasible. The conclusion of proposition 3 extends, however, to the case in which firms observe the feasibility of other mergers only when they are proposed and observe the efficiency gains of other mergers only when those mergers are approved. Formally, we have the following proposition.[20]

Proposition 4. Suppose that firms observe the feasibility of other mergers only when they are proposed and the efficiency gains of other mergers only when they are approved and that C_{kt} is a finite set for all k and t. If the antitrust authority follows the most lenient myopically

[19] Moreover, the fact that the largest myopically CS-maximizing set monotonically increases over time implies that the antitrust authority also could not do better if it could undo previously approved mergers, which we have assumed is not possible.

[20] We continue to assume that firms observe their own merger possibility when it becomes feasible and that each firm knows the initial costs of all firms at the start of period 1, so that complete-information Cournot competition in each period is justified.

CS-maximizing merger policy, then: (i) All feasible mergers being proposed in each period after any history is an extensive-form trembling-hand perfect Nash equilibrium for the firms. In this equilibrium, the outcome maximizes discounted consumer surplus for any realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, \dots, \mathfrak{F}_T)$. (ii) For each sequence $\mathfrak{F}$, every extensive-form trembling-hand perfect Nash equilibrium results in the same optimal sequence of period-by-period consumer surpluses.[21]

Proof. In the Appendix.

B. *Price Competition*

So far, we have assumed that firms compete in quantities. In this subsection, we discuss the case in which firms instead engage in Bertrand price competition. With this form of competition, our basic conclusion continues to hold, albeit in a somewhat weaker form.

One important difference between the Cournot and Bertrand models is that with Bertrand competition a merger that is CS-neutral in isolation can become CS-decreasing when another merger takes place that is CS-increasing in isolation, as the following example demonstrates.

EXAMPLE 3. Suppose that there are four firms, $N = \{1, 2, 3, 4\}$, with initial costs $c_1 = 5$, $c_2 = 10$, $c_3 = 15$, and $c_4 = 20$, and suppose that there are two possible mergers $M_1 = \{1, 3\}$ and $M_2 = \{2, 4\}$, with $\bar{c}_{M_1} = 9$ and $\bar{c}_{M_2} = 8$. If the monopoly price for a firm with marginal cost equal to 5 is greater than 10, then with no mergers firm 1 will set a price of 10 and make all the sales in the market. The cost-increasing merger M_1 is then CS-neutral in isolation since the postmerger price will still be 10. Merger M_2 is CS-increasing in isolation because it reduces firm 1's price from 10 to 8. However, once merger M_2 occurs, merger M_1 is CS-decreasing since it raises the price from 8 to 9.

This problem can be traced to the fact that a merger involving the lowest-cost firm that increases cost can be CS-neutral in the Bertrand model. The extension of our main result (proposition 3) to the Bertrand case therefore needs to restrict attention to mergers that are not cost increasing, that is, to mergers such that the postmerger marginal cost of the merged entity, $\bar{c}_M$, is no greater than the marginal cost of the most efficient merger partner, that is, $\bar{c}_M \leq \min_{i \in M} c_i$. This is a mild weakening, however, because an antitrust authority can without loss

[21] We use the extensive-form trembling-hand perfect Nash equilibrium solution concept rather than the weaker notion of sequential equilibrium to establish part ii of the result. The trembles ensure that following any history, when the true set of feasible but not yet approved mergers is $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$, every proposer firm assigns a strictly positive probability to this set being the set of feasible but not yet approved mergers.

reject any cost-increasing merger, since any such merger worsens both efficiency and the extent of market power.[22]

Another important difference from the Cournot model is that a merger that is CS-increasing may not be strictly profitable. Consider, for example, a three-firm industry in which $c_1 < c_2 < c_3$ and c_2 is below firm 1's monopoly price (so the initial equilibrium price is c_2). A cost-reducing merger of firms 2 and 3 that results in a postmerger cost above c_1 lowers the market price but leaves firms 2 and 3 with zero profit after the merger. For this reason, part ii of proposition 3 does not hold in the Bertrand model (e.g., firms 2 and 3 in this example can optimally decide not to propose their merger even if it is CS-increasing). Nevertheless, part i of proposition 3 does hold: if the antitrust authority follows a rule that approves in each period the most lenient CS-maximizing set of mergers from among those that do not increase cost, then there is an equilibrium in which all feasible mergers being proposed in each period after any history is a subgame-perfect Nash equilibrium for the firms, and the equilibrium outcome maximizes discounted consumer surplus for any realized sequence of feasible mergers $\mathfrak{F}$. (For details, see our working paper [Nocke and Whinston 2008].)

C. *Differentiated Products*

The Cournot and Bertrand analyses so far assumed a homogeneous product market. Unfortunately, extending our main results to the case of differentiated products, and hence to multiproduct firms, is not straightforward. For example, think of the extreme case in which there are two differentiated products in the market. A merger might leave overall consumer surplus unchanged while raising one price and lowering the other, and our arguments establishing the complementarity of CS-nondecreasing mergers would not go through. However, our main results do extend to the case of differentiated products if all products involved in a given merger have identical marginal costs, both premerger and postmerger. In that case, with suitable regularity conditions, price effects for all goods move in the same direction, and the complementarity results from our previous analyses carry over, as we now discuss. In our discussion, we will focus on the case of price competition with differentiated products.

Let $Q_j(\mathbf{p}_{\mathcal{N}})$ denote the demand for product j, where $\mathbf{p}_{\mathcal{N}}$ is the vector of prices, and suppose that the demand system is symmetric across prod-

[22] Formally, given any set of feasible mergers in a period, observe that it is possible to weakly improve consumer surplus starting from any set of approved mergers by instead rejecting all mergers that are cost increasing. As a result, in any period, given any set of feasible mergers, the largest CS-maximizing set from among those feasible mergers that do not increase cost maximizes consumer surplus in that period.

ucts. Moreover, assume that demand is downward sloping and strictly log concave in own price, products are demand substitutes, prices are strategic complements, and the own-price effect dominates the cross-price effects in terms of both the level of demand and its slope.[23]

For simplicity, suppose that, prior to merging, all firms produce a single product so that firm $j \in \mathcal{N}$ produces product $j \in \mathcal{N}$. After merging, the firms in the set M_k produce all the products in M_k. We assume that, prior to merging, each firm $j \in M_k$ faces the same marginal cost $c_j = c_{M_k}$ whereas after the merger all products in M_k are produced at the same marginal cost $\bar{c}_{M_k}$. This assumption ensures that any equilibrium has the property that the price of every product in the set M_k is always the same: $p_i^* = p_{M_k}^*$ for $i \in M_k$ (see Kühn and Rimler 2006). In particular, this means that we can think of each firm's strategic variable being one-dimensional, so the standard analysis of differentiated goods price competition with single-product firms (see Vives 1999) extends to our setting with multiproduct firms.

Consider a merger among active firms in set M_k, and let $\mathbf{p}_{\mathcal{N}}^*$ denote the vector of premerger equilibrium prices. Since prices are strategic complements, the merger is CS-neutral if and only if it leaves all prices unchanged, so the threshold value of postmerger marginal cost that makes this merger CS-neutral is given by

$$\begin{aligned} \hat{c}_{M_k}(\mathbf{p}_{\mathcal{N}}^*) &\equiv p_{M_k}^* - (p_{M_k}^* - c_{M_k})\left(\frac{1}{1-\Psi_i}\right), \quad i \in M_k, \\ &= c_{M_k} - (p_{M_k}^* - c_{M_k})\left(\frac{\Psi_i}{1-\Psi_i}\right), \quad i \in M_k, \end{aligned} \tag{8}$$

where

$$\Psi_i \equiv -\frac{\sum_{j\in M_k, j\neq i}[\partial Q_j(\mathbf{p}_{\mathcal{N}}^*)/\partial p_i]}{\partial Q_i(\mathbf{p}_{\mathcal{N}}^*)/\partial p_i}.$$

The term Ψ_i is familiar from merger analysis: it is the "diversion ratio" from product $i \in M_k$ to other products in M_k, defined as the share of the lost sales of product $i \in M_k$ that are captured by the other products in M_k after an increase in the price of product i. Since (by assumption) $\partial Q_i(\mathbf{p}_{\mathcal{N}}^*)/\partial p_i < 0$ and $\partial Q_j(\mathbf{p}_{\mathcal{N}})/\partial p_i > 0$ for $j \neq i$ and (from the first-order condition of profit maximization) $\sum_{j\in M_k} \partial Q_j(\mathbf{p}_{\mathcal{N}})/\partial p_i < 0$, $i \in M_k$, we have $\Psi_i \in (0, 1)$, which implies that $\hat{c}_{M_k} < c_{M_k}$. That is, for the merger to be

[23] The own effect of a price change dominates the cross effects in terms of the level of demand if $\sum_{j\in\mathcal{N}}[\partial Q_i(\mathbf{p}_{\mathcal{N}})/\partial p_j] < 0$ and in terms of the slope of demand if

$$\left|\frac{\partial^2 \ln Q_i(\mathbf{p}_{\mathcal{N}})}{\partial p_i^2}\right| > \sum_{j\in\mathcal{N}, j\neq i} \frac{\partial^2 \ln Q_i(\mathbf{p}_{\mathcal{N}})}{\partial p_j \partial p_i}$$

for all $i \in \mathcal{N}$ (see Kühn and Rimler 2006).

CS-neutral, the merger must be cost reducing and therefore profitable for the merging parties. Strategic complementarity implies that a decrease in postmerger marginal cost $\bar{c}_{M_k}$ induces all prices to fall. Consequently, a merger among active firms in M_k is CS-increasing if and only if $\bar{c}_{M_k} < \hat{c}_{M_k}$, CS-neutral if and only if $\bar{c}_{M_k} = \hat{c}_{M_k}$, and CS-decreasing if and only if $\bar{c}_{M_k} > \hat{c}_{M_k}$.

While every CS-neutral merger is profitable, it is not straightforward to show that every CS-nondecreasing merger is profitable. The complication arises because a reduction in marginal cost $\bar{c}_{M_k}$ has two opposing effects on the profits of the merged firm M_k: with the prices of all other firms held fixed, the *direct* effect of a decrease in $\bar{c}_{M_k}$ is to increase the merged firm's profit, but the *strategic* effect of a decrease in $\bar{c}_{M_k}$ is to reduce the merged firm's profit as all other firms will decrease their prices in response. One therefore needs to impose conditions on demand to ensure that the direct effect outweighs the strategic effect and a decrease in its marginal cost raises that firm's equilibrium profit. It is straightforward to check that this is indeed the case, for example, when demand is linear, $Q_j(\mathbf{p}_{\mathcal{N}}) = \alpha - \beta p_j + \gamma \sum_{i \neq j} p_i$ with $\alpha > 0$ and $\beta > (N-1)\gamma > 0$.

Let us now turn to the interaction between mergers. Our previous result on the complementarity of mergers that change consumer surplus in the same direction (proposition 2) carries over to the present setting if approving a CS-increasing merger M_l raises the threshold $\hat{c}_{M_k}$ for merger M_k, $k \neq l$ (and approving a CS-decreasing M_l reduces $\hat{c}_{M_k}$). Since a CS-increasing merger reduces all prices, this means that our complementarity result extends if demand is such that $\hat{c}_{M_k}(\mathbf{p}^*_{\mathcal{N}})$ is weakly decreasing in all prices. In the case of linear demand, for example, the diversion ratio Ψ_i is a constant, so $\hat{c}_{M_k}(P^*_N)$ depends only on, and is strictly decreasing in, $p^*_{M_k}$. It follows that complementarity holds. More generally, a sufficient condition for $\hat{c}_{M_k}(\mathbf{p}^*_{\mathcal{N}})$ to be nonincreasing in the prices of all products with positive sales is that the diversion ratio Ψ_i is nondecreasing in all prices. Provided that this complementarity holds, proposition 3 extends to this setting.

D. *Demand Shifts*

While our model had a stationary demand function, corollary 1 suggests that our main results hold provided that demand is weakly declining over time. Specifically, suppose that inverse demand in period t can be written as $P(Q; \theta_t)$, where θ_t is the publicly observable demand state realized at the beginning of period t (before mergers are proposed), which we assume is decreasing over time, that is, $\theta_t \leq \theta_{t-1}$. (These realizations may be stochastic.) For any tuple $(Q; \theta_t)$ such that $P(Q; \theta_t) > 0$, we continue to assume that $P_Q < 0$ and $P_Q + QP_{QQ} < 0$ (where subscripts denote partial derivatives); moreover, we now assume that $P_\theta > 0$ and $P_{Q\theta} \leq 0$. For example, these conditions hold if inverse demand takes the form $P(Q; \theta_t) \equiv \theta_t P(Q)$ and $P(Q)$ satisfies the conditions of assumption 1.

Let $Q^*(\mathcal{A}_t; \theta_t)$ denote the equilibrium industry output when market structure is $\mathcal{A}_t$ and the demand state is θ_t. Since inverse demand is changing over time, it is more convenient to write $\hat{c}_M$ (the postmerger marginal cost threshold that makes a merger among active firms in set M CS-neutral) as a function of equilibrium price rather than industry quantity:

$$\hat{c}_M(P^*(\mathcal{A}_t; \theta_t)) \equiv P^*(\mathcal{A}_t; \theta_t) - \sum_{i \in M} \max\{0, P^*(\mathcal{A}_t; \theta_t) - c_i\},$$

where $P^*(\mathcal{A}_t; \theta_t) \equiv P(Q^*(\mathcal{A}_t; \theta_t); \theta_t)$. Our assumptions on demand ensure that, with market structure $\mathcal{A}_t$ held fixed, a decrease in the demand state θ_t will lead to a decrease in the equilibrium price $P^*(\mathcal{A}_t; \theta_t)$.[24] This, in turn, implies that, with market structure $\mathcal{A}_t$ held fixed, the postmerger marginal cost threshold $\hat{c}_M(P^*(\mathcal{A}_t; \theta_t))$ weakly increases over time (as long as the merger involves active firms). Moreover, as before, the threshold strictly increases as a result of CS-increasing mergers in the rest of the industry. Hence, if the antitrust authority adopts a myopically CS-maximizing merger policy, a merger M that is CS-nondecreasing in period t will remain CS-nondecreasing in every future period $t' > t$. (By contrast, a merger M that is CS-decreasing in period t may now become CS-nondecreasing in some later period $t' > t$ even with market structure held fixed.)

The largest myopically CS-maximizing set of mergers now depends not only on the set of proposed mergers $\mathcal{P}_t$ and current market structure $\mathcal{A}_{t-1}$ but also on the demand state θ_t and is denoted $\overline{\mathcal{A}}^*(\mathcal{P}_t; \theta_t | \mathcal{A}_{t-1})$. As the discussion above makes clear, $\overline{\mathcal{A}}^*(\mathcal{P}_t; \theta_t | \mathcal{A}_{t-1})$ is decreasing in θ_t: if $\theta_t'' < \theta_t'$, then $\overline{\mathcal{A}}^*(\mathcal{P}_t; \theta_t' | \mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathcal{P}_t; \theta_t'' | \mathcal{A}_{t-1})$. Since $\mathfrak{F}_t \subseteq \mathfrak{F}_{t+1}$ and $\theta_t \geq \theta_{t+1}$, we therefore continue to have $\overline{\mathcal{A}}^*(\mathfrak{F}_t; \theta_t | \varnothing) \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_{t+1}; \theta_{t+1} | \varnothing)$, the critical monotonicity property identified in lemma 2. Our main result therefore extends to this environment.

[24] Summing up the first-order conditions of profit maximization and applying the implicit function theorem, we have

$$\frac{dP(Q^*; \theta_t)}{d\theta} = \frac{P_\theta (P_Q + Q^* P_{QQ}) - Q^* P_Q P_{Q\theta}}{(N+1)P_Q + Q^* P_{QQ}},$$

where Q^* is industry output and N is the number of active firms when the market structure is $\mathcal{A}_t$ and the demand state is θ_t. Under our assumptions on demand, the expression on the right-hand side is strictly positive.

E. Entry

In our analysis above, we assumed that the set of firms is fixed, except for mergers. Would our conclusions change if we allowed for firm entry? Recall that our model implies that the equilibrium price $P(Q^*)$ falls weakly over time. This suggests that if a firm does not find it profitable to enter the market at the beginning of the first period, before any mergers have become feasible, then this firm will not find it profitable to enter the market in any later period (provided that its costs have not changed). That is, allowing for free entry of firms (with unchanging costs) does not affect our results.

Moreover, suppose that new firms periodically enter the market later, for example, after discovering how to make the product. (In our model, such an entry event is equivalent to a sufficient reduction in the marginal cost of a hitherto inactive firm.) These (potentially stochastic) entry events lower the market price and leave our main result unchanged for reasons that parallel those in our discussion above of demand shifts.

F. Continuing Efficiency Improvements

In the analysis above, we assumed that when a merger, say M_k, becomes feasible, the firms in M_k receive a (random) draw of their postmerger marginal cost $\bar{c}_{M_k}$ once and for all; if merger M_k is implemented, the marginal cost of the merged entity is $\bar{c}_{M_k}$ forever after. But it seems plausible that, over time, firms involved in a (potential) merger may have more than one idea of how to create synergies, both premerger and postmerger. As we now discuss, it is possible to extend our analysis to allow for continuing efficiency improvements.

Consider the following generalization of our previous setup: As before, we assume that if merger M_k becomes feasible at the beginning of period t, then the firms in M_k receive a random draw of their postmerger marginal cost. However, we now assume that the postmerger marginal cost $\bar{c}_{M_k}$ follows a (discrete-time) stochastic process from period t onward. The stochastic process governing these additional cost draws is independent of whether the firms in M_k have already merged or not. Crucially, we assume that the postmerger marginal cost $\bar{c}_{M_k}$ weakly decreases over time.

Our previous results carry over to this generalized setting. The arguments closely parallel those in our discussion above of demand shifts: Since a reduction in merged firm M_k's marginal cost reduces the equilibrium price (and thereby reduces the postmerger marginal cost threshold $\hat{c}_{M_l}$ of every other merger M_l, $l \neq k$), the largest myopically CS-maximizing set of mergers will weakly increase over time if the antitrust authority adopts the most lenient myopically CS-maximizing merger policy, leading to our result.

Our previous results do not carry over, however, if the efficiency improvements depend on a merger's approval. For example, suppose that merger M_1 becomes feasible at time t_1 and has a postmerger cost of $\bar{c}'_{M_1}$ for the first τ periods after approval, and the lower cost level $\bar{c}''_{M_1} < \bar{c}'_{M_1}$ thereafter. Merger M_1 may then increase discounted consumer surplus but reduce consumer surplus at the time it is approved. So, if a "myopic" policy is one that approves mergers if and only if they do not lower consumer surplus at the time of approval, a myopic policy will not be optimal. An alternative definition of a myopic policy is a policy that approves mergers that do not reduce discounted consumer surplus if implemented in isolation. Even with this definition, however, a myopic policy need not be optimal. To see this, suppose that merger M_1 reduces discounted consumer surplus slightly but increases consumer surplus once the postmerger cost drops (τ periods after approval). Further, suppose that a second merger M_2, which becomes feasible at time $t_2 > t_1 + \tau$, is CS-decreasing both in isolation and when merger M_1 with the high cost $\bar{c}'_{M_1}$ is implemented but is CS-increasing if τ periods have passed since M_1 has been approved (so that M_1 has the low cost $\bar{c}''_{M_1}$). In this situation, discounted consumer surplus is maximized by approving both mergers, merger M_1 in period $t_2 - \tau$ and merger M_2 in period t_2, even though each merger lowers discounted consumer surplus when done in isolation.

G. *Fixed Costs and Exit*

So far, we have assumed that all fixed costs are sunk and that mergers have no effect on these costs. Our Cournot results extend to cases in which fixed costs are present and are possibly affected by mergers provided that (i) mergers that are CS-nondecreasing in isolation continue to be profitable in isolation and (ii) mergers do not cause active firms to shut down.

Regarding point i, recall from corollary 1 that, in the absence of fixed costs, every CS-nondecreasing merger is profitable in isolation. This result continues to be true in the presence of fixed costs when mergers generate efficiencies in fixed costs as well as marginal costs (i.e., if fixed costs never increase as a result of a merger).

If point ii is violated, proposition 1 need not hold. For example, suppose that both mergers M_1 and M_2 are CS-increasing in isolation and do not induce any firm to exit. But if both mergers are approved, then some other firm $j \in \mathcal{N} \setminus (M_1 \cup M_2)$ finds it optimal to exit. (This outcome is possible since, without exit, the market price after both mergers would be lower than after only one merger.) When the endogenous exit of firm j is taken into account, consumer surplus after both mergers might therefore be lower than after merger M_1 only, in which case merger M_2 would be CS-decreasing conditional on merger M_1. Thus, part i of proposition 1 may fail to hold, in which case a myopic policy need not be optimal.[25]

While these observations suggest that in general our main results could break down in the presence of fixed costs and endogenous exit, we can allow for exit among a competitive fringe of price-taking firms that do not take part in any mergers. To do so, we construct the competitive fringe's (long-run) supply function, $S^F(p)$, which takes potential exit (and entry) of these firms into account. The residual demand of the large, strategic firms in set $\mathcal{N}$ is then given by $R(p) \equiv D(p) - S^F(p)$, where $D(p)$ is market demand. As long as the inverse residual market demand function $P(\cdot) \equiv R^{-1}(\cdot)$ satisfies the conditions of assumption 1, our analysis and conclusions remain unchanged. Exit by these fringe producers causes no problem for our results because it does not cause discrete increases in the market price.

H. Merger Proposal and Implementation Costs

In our analysis, we have assumed that there are no costs of proposing a merger to the antitrust authority nor of implementing it once approved. Moreover, we have highlighted a subgame-perfect equilibrium in which all feasible mergers are always proposed, including some that have no chance of being approved. One might be concerned that firms would not propose such mergers if they had even the tiniest cost of making a merger proposal or implementing an approved merger. However, since every CS-nondecreasing merger is strictly profitable (provided that it results in an active firm), under the most lenient myopically CS-maximizing merger policy every subgame-perfect equilibrium outcome would still maximize discounted consumer surplus for every realized sequence of feasible mergers $\mathfrak{F}$, provided that merger proposal and implementation costs are sufficiently small. Furthermore, even if the costs of proposing and implementing a merger are not small, these costs may be swamped by merger-induced fixed cost savings (recall our discussion of fixed costs).

If, however, the costs of proposing or implementing a merger are not small and are not swamped by fixed-cost synergies, then firms may

[25] In a similar vein, Motta and Vasconcelos (2005) allow for exit in a setting with four symmetric firms and two possible disjoint cost-reducing mergers involving two firms each. Each merger is CS-decreasing in isolation because it induces the other two, nonmerging, firms to exit. But consumer surplus increases if both mergers are approved, implying that each merger becomes CS-increasing once the other merger has taken place. Thus, part ii of proposition 1 fails to hold in their model.

choose not to propose CS-increasing mergers. Indeed, this problem can arise even in the case of a single isolated merger: for example, a merger that is CS-nondecreasing in isolation may not be profitable if the cost of implementing the merger is large. Moreover, when we consider settings with merger proposal or implementation costs and multiple possible mergers, approval of one CS-nondecreasing merger may lead the firms in another CS-nondecreasing merger not to propose their merger. To see this, consider the following corollary of propositions 1 and 2.

COROLLARY 2. (i) Suppose that merger M_1, involving at least two active firms, is CS-neutral in isolation and merger M_2 is CS-increasing in isolation. Then, approving merger M_1 increases the positive effect of merger M_2 on consumer surplus but reduces the profitability of merger M_2. (ii) Suppose that merger M_1, involving at least two active firms, is CS-neutral in isolation and merger M_2 is CS-decreasing in isolation. Then approving merger M_1 increases the negative effect of merger M_2 on consumer surplus but improves the profitability of merger M_2.

Proof. Part i: From corollary 1, M_1 is CS-increasing conditional on M_2 being approved, despite being CS-neutral in isolation. Hence, approving M_1 increases the positive effect of M_2 on consumer surplus. Now, since M_1 is CS-neutral in isolation but CS-increasing once M_2 has been approved, approval of M_1 does not affect the joint premerger profit of the firms in M_2 but reduces the profit of the merged M_2. The proof of part ii proceeds along the same lines. QED

The corollary implies, in particular, that the following situation may arise: Both merger M_1 and merger M_2 are CS-increasing and profitable in isolation net of proposal and implementation costs but unprofitable once the other merger takes place (this follows since, by continuity, merger M_1 in part i of the corollary could instead be slightly CS-increasing without changing the corollary's conclusions). In that case, if merger M_1 increases consumer surplus by less than merger M_2 does, the antitrust authority will want to reject merger M_1 if it is proposed first.

I. Merger Blocking Costs

In our analysis, we have assumed that the antitrust authority's merger review process is frictionless. Suppose instead that the antitrust authority has to incur a cost of $b > 0$ whenever it blocks a merger. Further, suppose that the antitrust authority seeks to maximize discounted expected consumer surplus minus blocking costs. It turns out that the existence of these merger blocking costs implies that the optimal merger approval policy is no longer dynamically consistent. In the following discussion we therefore distinguish between the cases of no commitment and full commitment.

Consider first the case in which the antitrust authority can ex ante commit to its future policy. In particular, suppose that the antitrust authority commits to the myopically CS-maximizing merger policy analyzed in Section III, thereby ignoring any blocking costs. Then, assuming (as in Sec. III) that firms observe not only their own but also rivals' merger possibilities (including their efficiencies) whenever they become feasible, there exists an equilibrium in which the discounted expected consumer surplus minus blocking costs is maximized for any realized sequence of feasible mergers. To see this, note that in the game without blocking costs (as analyzed in Sec. III) there always exists an equilibrium in which, in each period, firms propose only those mergers that the antitrust authority approves in that period. So, along this equilibrium path, mergers are not blocked. In fact, if firms have to incur a sufficiently small but positive cost whenever they propose a merger, then this is the unique equilibrium.

Consider now the case in which the antitrust authority cannot commit to its future policy but, instead, in each period decides what set of proposed mergers to approve. It is clear that the policy outlined in Section III will not generally be dynamically consistent because the antitrust authority may want to approve a CS-decreasing merger if its consumer surplus reduction is less than the cost of blocking it. More generally, the antitrust authority's equilibrium policy (now a best response to firms' proposal strategies) may not be myopic, in the sense that a forward-looking authority may optimally consider the possibility of future mergers in making its approval decision.

As an example, suppose that there are two potential mergers, M_1 and M_2. Suppose that M_1 is CS-decreasing in isolation but the reduction in consumer surplus is slightly smaller than the blocking cost b, and suppose that M_2 is CS-increasing in isolation but CS-decreasing once M_1 has been approved. In this case, each merger in isolation should be approved. However, conditional on M_1 being proposed and M_2 not yet being proposed, the optimal forward-looking policy would consist in blocking M_1 and approving M_2 when proposed.[26] When this occurs, the antitrust authority's payoff would be the (discounted) CS effect of M_2 minus the blocking cost b. If acting myopically instead, the antitrust authority would approve M_1 and then approve M_2 if and only if the CS effect of M_2, conditional on M_1, is not worse than incurring the blocking cost; the antitrust authority's payoff would therefore be no greater than the CS effect of M_1, which is approximately $-b$.

[26] Thus, without commitment, merger-blocking costs provide one possible explanation for the often-heard argument among antitrust commentators and practitioners that "we can allow one but not both of these mergers," a sentiment that in light of proposition 1 makes little sense in the model of Secs. II and III.

J. Disjoint Mergers

Perhaps the most important limitation of our model is that mergers are disjoint. This rules out, for example, the possibility that a firm may have to choose between two merger partners or that a recently merged firm might consider merging with another still-independent firm. While disjointness of possible mergers would hold when firms have natural merger partners (as we noted earlier) and has been assumed throughout the small existing literature on antitrust review of mergers in dynamic settings (Nilssen and Sorgard 1998; Matsushima 2001; Motta and Vasconcelos 2005), it is clearly a strong assumption.

Nondisjoint mergers can cause problems for myopic policies. A first problem is that myopic approval of a CS-increasing merger today may preclude the possibility of approving an even better merger tomorrow. For example, suppose that there are two possible mergers, M_1 and M_2, with M_1 becoming feasible first. Suppose also that both mergers are CS-increasing in isolation and that firm i is involved in both mergers, $M_1 \cap M_2 = \{i\}$, implying that at most one of the two mergers can be implemented. A myopic policy would thus lead to approval of M_1 if proposed first even though M_2 may be better for consumers.

A second problem relates to firms' proposal incentives. With disjoint mergers, we saw that firms' proposal incentives were aligned with the desires of the antitrust authority because any CS-nondecreasing merger was profitable. When firms must choose among merger partners, however, they may propose the wrong merger from the antitrust authority's perspective (e.g., the most profitable merger may not be the one that maximizes consumer surplus), an issue first pointed out in a static context by Lyons (2002) and more formally studied by Armstrong and Vickers (2010) and Nocke and Whinston (2010). Firms' incentives to merge may also be distorted because of dynamic considerations when mergers are not disjoint. For example, if firms i and j have a feasible CS-increasing merger today and firms i, j, and k can merge tomorrow, firms i and j may be disadvantaged in their later bargaining with k if they have already merged and so may avoid merging until the full merger is feasible. (Whether this occurs depends on the specific bargaining process among the firms; see Segal [2003] on bargaining externalities from coalition formation.)

K. Aggregate Surplus Standard

In our analysis above, we have assumed that the antitrust authority's objective is to maximize discounted expected consumer surplus. Indeed, as pointed out in the introduction, this is close to being the legal standard in the United States and many other countries. Nevertheless, it is interesting to ask whether the antitrust authority can maximize aggregate surplus (AS) by adopting the most lenient myopically AS-maximizing merger policy.

In the homogeneous-goods Bertrand model, the answer is yes. One can prove that in the Bertrand model under the most lenient myopically AS-maximizing merger policy there is an equilibrium such that the resulting outcome maximizes discounted aggregate surplus for every realized sequence of mergers $\mathfrak{F}$.

In the homogeneous-goods Cournot model, however, the complementarity of AS-increasing mergers does not hold in general. To see this, recall that, in the Cournot model, a marginal cost reduction by a highly inefficient firm (one that produces almost no output and thus has a profit margin approximately equal to zero) necessarily reduces aggregate surplus. In contrast, a cost-reducing merger between the two most efficient firms in a market may increase aggregate surplus. Thus, complementarity can fail when a cost-reducing, AS-increasing merger by other firms in the market transforms these two firms from being the most efficient firms in the market to being the least efficient. In addition, mergers that increase aggregate surplus need not be profitable for the firms involved in them. The papers by Nilssen and Sorgard (1998) and Matsushima (2001), for example, both focus on Cournot settings with linear demand and constant marginal costs in which there are two possible mergers, each between a pair of firms, and show that a myopic policy need not be optimal for an antitrust authority interested in maximizing aggregate surplus.

V. Conclusion

In this paper, we have analyzed the antitrust authority's optimal dynamic merger approval policy in a model with Cournot competition in which merger opportunities arise stochastically over time, firms decide whether or not to propose a feasible merger, and the antitrust authority decides whether or not to approve proposed mergers. We first established that a form of complementarity exists between mergers in this Cournot setting: specifically, the sign of a merger's consumer surplus effect is unchanged if another merger whose consumer surplus effect has the same sign takes place. This result, which is of independent interest, set the stage for our main result, which showed that, in our benchmark model, an antitrust authority that wishes to maximize discounted expected consumer surplus can implement the dynamically optimal solution by adopting a completely myopic policy according to which the antitrust authority approves a merger if and only if it does not lower consumer surplus given the current market structure. In fact, the antitrust authority cannot improve on the outcome induced by the myopic policy even if it has perfect foresight about potential future mergers.

While our result on dynamic optimality of a myopic merger approval policy is robust in a number of dimensions, we have also seen that it is fragile in some others. Nonetheless, because our model gives rise to such a striking result—the optimality of myopic merger policy—we feel that it is a natural starting point for understanding the issues involved in optimal merger policy in dynamic environments. In our own continuing work (Nocke and Whinston 2010) we are exploring optimal merger policy when the nondisjointness of mergers is relaxed, so that firms may have several mergers that they can take part in.

One side implication of our model is that it provides a novel theory of merger waves (see, e.g., Fauli-Oller 2000). In contrast to much of the existing literature (e.g., Jovanovic and Rousseau 2002, 2008), our explanation of merger waves does not rely on aggregate shocks. Specifically, because of the complementarity of CS-nondecreasing mergers in our model, the arrival of a CS-increasing merger opportunity for some firms may have a domino effect by turning other feasible but currently CS-decreasing mergers into CS-nondecreasing mergers, thereby triggering a merger wave. An interesting aspect of this result is the way in which the antitrust authority's CS-maximizing merger policy affects the emergence of merger waves, since complementarity of mergers does not hold in general in the absence of this antitrust review.

In addition, our results have implications not only for horizontal mergers but also for horizontal breakups of companies into smaller firms. As the breakup of (merged) firm M can be thought of as the reverse operation of merger M, the consumer surplus effect of the breakup of firm M has the opposite sign of merger M. In contrast to merger review policy, a myopic breakup policy is, in general, not dynamically optimal. To see this, suppose that merger M_1 is CS-decreasing in isolation whereas merger M_2 is CS-increasing in isolation but CS-decreasing once merger M_1 has taken place (implying that M_1 continues to be CS-decreasing conditional on M_2 taking place). Starting from the situation in which both mergers have taken place, the breakup of each merged firm, M_1 and M_2, is CS-increasing in isolation. If M_2 is broken up first, then the breakup of M_1 is also CS-increasing as merger M_1 is, by assumption, CS-decreasing in isolation. In contrast, if M_1 is broken up first, then the breakup of M_2 is CS-decreasing as merger M_2 is CS-increasing in isolation. The complementarity result of proposition 1 therefore does not apply to breakups, and a myopic breakup policy exhibits path dependence.[27]

[27] Another issue is that the incentives of firms to propose breakups are not aligned with those of an antitrust authority seeking to maximize consumer surplus. This can be seen

Merger policy is one of the central pillars of antitrust policy. Our results show that it is possible to make progress analyzing merger review policy in richer models that recognize the dynamic nature of the merger review problem. Our hope is that our results will serve as a starting point for further studies that analyze optimal merger policy when some of the assumptions that lead to the optimality of myopic merger policy, such as the disjointness of mergers and the absence of merger blocking costs, fail to hold.

Appendix

A. *Proofs*

We begin by establishing two useful results concerning the interactions among sets of mergers. The first lemma focuses on the relationship between sequences of mergers that are CS-nondecreasing at each step and sets of mergers for which each merger is CS-nondecreasing given all the other mergers in the set. We call it the Incremental Gain Lemma.

LEMMA 5 (Incremental Gain Lemma).

i. Suppose that a set of mergers $\mathfrak{M} \equiv \{M_1, \dots, M_{J_1}\}$ has the property that every merger $M \in \mathfrak{M}$ is CS-nondecreasing if all the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M}\backslash M$) have taken place. Then for any strict subset $Y \subset \mathfrak{M}$, there exists an $M' \in \mathfrak{M}\backslash Y$ that is CS-nondecreasing if all the mergers in Y have taken place. As a result, starting from Y, there is a sequencing of the mergers in $\mathfrak{M}\backslash Y$ that is CS-nondecreasing at each step.
ii. Suppose that a sequence of mergers $M_1, \dots, M_{J_1}$ is CS-nondecreasing at each step. Then each merger $M \in \mathfrak{M} \equiv \{M_1, \dots, M_{J_1}\}$ is CS-nondecreasing if all the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M}\backslash M$) have taken place.

Proof. Part i: Suppose that the result is not true, so that every $M' \in \mathfrak{M}\backslash Y$ is CS-decreasing if all the mergers in Y have taken place. Part ii of proposition 1 implies that, with the mergers in Y taken as given, for any sequencing of the mergers in the set $\mathfrak{M}\backslash Y$, the merger implemented at each step, including the last step, is CS-decreasing. But this contradicts the hypothesis that the last merger in the sequence is CS-nondecreasing if all the other mergers in the set $\mathfrak{M}$ have taken place.

Given the existence of $M' \in \mathfrak{M}\backslash Y$ that is CS-nondecreasing if all the mergers in Y have taken place, we can update the subset Y to $Y \cup \{M'\} \subset \mathfrak{M}$ and apply the same argument again. Continuing iteratively identifies a sequencing of the mergers in $\mathfrak{M}\backslash Y$ that is CS-nondecreasing at each step starting from the subset Y.

Part ii: Consider an arbitrary merger M_j in sequence $M_1, \dots, M_{J_1}$. We will show that M_j is CS-nondecreasing given that all the mergers in $\mathfrak{M}\backslash M_j$ have taken place. For $k \geq j$, define the set $\mathfrak{M}^k = \{M_i : i \leq k\}$. Suppose that (a_k) merger M_j is CS-nondecreasing given $\mathfrak{M}^k\backslash M_j$ and that (b_k) merger M_{k+1} is CS-nonde-

from the fact that a CS-neutral breakup of an active firm is strictly unprofitable for the firm.

creasing given $\mathfrak{M}^k$. Observe that, by hypothesis, property a_k is true for $k = j$ and that property b_k holds for all k. We claim that properties a_k and b_k imply property a_{k+1}: M_j is CS-nondecreasing given $\mathfrak{M}^{k+1}\backslash M_j$. To see this, observe that if merger M_{k+1} is CS-nondecreasing given $\mathfrak{M}^k\backslash M_j$, property a_{k+1} follows from part i of proposition 1, whereas if merger M_{k+1} is CS-decreasing given $\mathfrak{M}^k\backslash M_j$, then property a_{k+1} follows from part i of proposition 2 (and the fact that M_{k+1} is CS-nondecreasing given $\mathfrak{M}^k$). Applying induction, we find that merger M_j is CS-nondecreasing given that all the mergers in $\mathfrak{M}\backslash M_j$ have taken place (property a_{J_1}). QED

Part ii of lemma 5 implies that the set of mergers resulting from a merger policy in which the antitrust authority considers mergers within period t in a step-by-step fashion, approving mergers that are CS-nondecreasing given the current market structure until no further CS-nondecreasing mergers can be identified, possesses the property that every merger in the set is CS-nondecreasing given every other merger in the set. This is also a property possessed by any myopically CS-maximizing set (if any approved merger M were CS-decreasing given the other approved mergers, then consumer surplus could be increased by not approving merger M while continuing to approve the others). The next lemma establishes two features of sets possessing this property.

LEMMA 6. Suppose that two distinct sets of mergers $\mathfrak{M}_1 \equiv \{M_1, \ldots, M_{J_1}\}$ and $\mathfrak{M}_2 \equiv \{M_1, \ldots, M_{J_2}\}$ with $\mathfrak{M}_1 \not\subseteq \mathfrak{M}_2$, not necessarily disjoint, each have the property that every merger $M \in \mathfrak{M}_i$ is CS-nondecreasing if all the other mergers in $\mathfrak{M}_i$ (those in the set $\mathfrak{M}_i\backslash M$) have taken place. Then (i) there is a merger $M'_1 \in \mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing given that all the mergers in $\mathfrak{M}_2$ have taken place, and (ii) the set of mergers $\mathfrak{M}_1 \cup \mathfrak{M}_2$ results in a level of consumer surplus that is at least as great as that of either set $\mathfrak{M}_1$ or set $\mathfrak{M}_2$.

Proof. Part i: Part i of the Incremental Gain Lemma (lemma 5) implies that there exists a merger $M'_1 \in \mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing given that all the mergers in $\mathfrak{M}_1 \cap \mathfrak{M}_2$ have taken place. It also implies that there is a sequencing of the mergers in $\mathfrak{M}_2\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$, say $M_{21}, \ldots, M_{2J_2}$, that is CS-nondecreasing at each step, given that the mergers in $\mathfrak{M}_1 \cap \mathfrak{M}_2$ have taken place. Let $\mathfrak{M}^k = \{M_{2i}: i \leq k\}$. Part i of proposition 1 implies that if merger M'_1 is CS-nondecreasing given that all the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^k$ have taken place, then (since by hypothesis merger $M_{2,k+1}$ is also CS-nondecreasing given that all the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^k$ have taken place) M'_1 is also CS-nondecreasing given that all the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^{k+1}$ have taken place. Since merger M'_1 is CS-nondecreasing if all the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^0 = (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place, applying induction yields the result (taking $k = J_1$).

Part ii: Let M'_1 be the merger identified in part i. We argue first that every merger in set $\mathfrak{M}_2 \cup \{M'_1\}$ is CS-nondecreasing if all the other mergers in that set have taken place. Part i implies that this is true for merger M'_1. Now consider any merger $M'_2 \in \mathfrak{M}_2$. By hypothesis, merger M'_2 is CS-nondecreasing given that all the mergers in set $\mathfrak{M}_2\backslash M'_2$ have taken place. If merger M'_1 is also CS-nondecreasing if all the mergers in set $\mathfrak{M}_2\backslash M'_2$ have taken place, then part i of proposition 1 implies that merger M'_2 is CS-nondecreasing if all the mergers in $(\mathfrak{M}_2\backslash M'_2) \cup \{M'_1\} = (\mathfrak{M}_2 \cup \{M'_1\})\backslash M'_2$ have taken place. If, instead, merger M'_1 is CS-decreasing if all the mergers in set $\mathfrak{M}_2\backslash M'_2$ have taken place, then part i of proposition 2 implies that this same property holds. This establishes that every merger in $\mathfrak{M}_2 \cup \{M'_1\}$ is CS-nondecreasing if all the other mergers in that set have taken place. Moreover, the level of consumer surplus with set $\mathfrak{M}_2 \cup \{M'_1\}$ is at least as large as with set $\mathfrak{M}_2$.

If $\mathfrak{M}_1 \subseteq \mathfrak{M}_2 \cup \{M'_1\}$, then the result is proven. Suppose not. Then note that sets $\mathfrak{M}_1$ and $\mathfrak{M}_2 \cup \{M'_1\}$ satisfy the hypotheses of the lemma. So we can apply the argument again for these two sets. Continuing iteratively in this fashion, we establish the result by adding to $\mathfrak{M}_2$ a sequencing of the mergers in $\mathfrak{M}_1 \setminus (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing at each step. This establishes that the level of consumer surplus is at least as high with set $\mathfrak{M}_1 \cup \mathfrak{M}_2$ as with set $\mathfrak{M}_2$. We also need to show that the level of consumer surplus in $\mathfrak{M}_1 \cup \mathfrak{M}_2$ is at least as large as in set $\mathfrak{M}_1$. If $\mathfrak{M}_1 \supseteq \mathfrak{M}_2$, so that $\mathfrak{M}_1 \cup \mathfrak{M}_2 = \mathfrak{M}_1$, this follows immediately. If instead $\mathfrak{M}_1 \not\supseteq \mathfrak{M}_2$, then we can repeat the argument above with the roles of $\mathfrak{M}_1$ and $\mathfrak{M}_2$ reversed to establish the result. QED

We now use these results to prove lemmas 2 and 3.

Proof of Lemma 2

Given proposed mergers $\mathcal{P}_t$ and market structure $\mathcal{A}_{t-1}$, let $\mathcal{A}'$ be a largest myopically CS-maximizing set and let $\mathcal{A} \neq \mathcal{A}'$ be a myopically CS-maximizing set. We will show that $\mathcal{A} \subset \mathcal{A}'$. Suppose otherwise, so that $\mathcal{A}' \subset (\mathcal{A} \cup \mathcal{A}')$. The sets $\mathcal{A}$ and $\mathcal{A}'$ satisfy the hypothesis of lemma 6. So, by part ii of lemma 6, $\mathcal{A} \cup \mathcal{A}'$ is myopically CS-maximizing as well, contradicting the assumption that $\mathcal{A}'$ is a largest myopically CS-maximizing set for $\mathcal{P}_t$ given market structure $\mathcal{A}_{t-1}$. Hence, $\mathcal{A}'$ must contain every other myopically CS-maximizing set, which also implies that $\mathcal{A}'$ is the unique largest CS-maximizing set.

For the second claim, suppose $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \not\subseteq \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}_{t-1})$. The sets $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ and $\overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}_{t-1})$ satisfy the hypothesis of lemma 6, and since $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{P}_t \cup \mathcal{A}_{t-1} \subset \mathcal{P}'_t \cup \mathcal{A}_{t-1}$, all mergers in set $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ are feasible when the set of proposed mergers is $\mathcal{P}'_t$. Thus, when the set of proposed mergers is $\mathcal{P}'_t$, approval of the mergers in set $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \cup \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}_{t-1})$ is feasible and by part ii of lemma 6 is also myopically CS-maximizing, contradicting $\overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}_{t-1})$ being the largest myopically CS-maximizing set for $\mathcal{P}'_t$ given market structure $\mathcal{A}_{t-1}$. QED

Proof of Lemma 3

Suppose that the set of proposed mergers is $\mathcal{P}_t$ and the market structure prior to period t is $\mathcal{A}_{t-1}$. Let $\mathcal{A} \subseteq \mathcal{P}_t$ denote a set of mergers resulting from a merger policy in which the antitrust authority considers mergers within period t in a step-by-step fashion, approving mergers that are CS-nondecreasing given the current market structure until no further CS-nondecreasing mergers can be identified. By part ii of lemma 5 (the Incremental Gain Lemma), every merger in $\mathcal{A}$ must be CS-nondecreasing given every other merger in the set. If $\mathcal{A} \not\subseteq \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, then part ii of lemma 6 implies that the set $\mathcal{A} \cup \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ is also myopically CS-maximizing but strictly contains set $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, a contradiction to $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ being the largest myopically CS-maximizing set for $\mathcal{P}_t$ given market structure $\mathcal{A}_{t-1}$. Hence, $\mathcal{A} \subseteq \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. If $\mathcal{A} \subset \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, part i of lemma

6 implies that once the mergers in $\mathcal{A}$ have been approved, there is a merger in $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})\backslash(\mathcal{A}\cap\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})) = \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})\backslash\mathcal{A}$ that is CS-nondecreasing given that the mergers in $\mathcal{A}$ have taken place, contradicting $\mathcal{A}$ being the result of a step-by-step merger policy that approves mergers that are CS-nondecreasing given the current market structure until no further CS-nondecreasing mergers can be identified. QED

Proof of Lemma 4

Given the realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, \ldots, \mathfrak{F}_T)$, consider the problem of maximizing discounted consumer surplus. If we ignore the monotonicity constraint that the set of approved mergers cannot shrink over time, we can choose the approved set of mergers (i.e., the market structure) in each period independently from the mergers approved in every other period. It is evident that in that case the approval sequence $\{\overline{\mathcal{A}}^*(\mathfrak{F}_1|\varnothing), \ldots, \overline{\mathcal{A}}^*(\mathfrak{F}_T|\varnothing)\}$ is optimal since it maximizes consumer surplus in every period.

Consider now the most lenient myopically CS-maximizing merger policy. We will show that this policy induces the approval sequence $\{\overline{\mathcal{A}}^*(\mathfrak{F}_1|\varnothing), \ldots, \overline{\mathcal{A}}^*(\mathfrak{F}_T|\varnothing)\}$, from which observation the result follows. To do so we will actually establish a slightly stronger fact, which will also be useful in the proof of proposition 3: If the antitrust authority follows the most lenient myopically CS-maximizing merger policy in periods $1, \ldots, t-1$ and if all feasible but not yet approved mergers are proposed in period t, the market structure at the end of period t will be $\overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$ regardless of the merger proposals that firms have made in periods $1, \ldots, t-1$.

To see this, consider an arbitrary period t and suppose that $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_{t-1}|\varnothing)$ regardless of the history of previous merger proposals (which is true if $t = 1$). If all feasible but not yet approved mergers are proposed in period t, then $\mathcal{A}_t = \overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$. We will show that $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$. Observe that the problem of myopically maximizing consumer surplus given previously approved mergers $\mathcal{A}_{t-1}$ and proposed mergers $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is a more constrained problem than the problem of myopically maximizing consumer surplus given no previously approved mergers and proposed mergers $\mathfrak{F}_t$. However, since $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_{t-1}|\varnothing) \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$ (the first inclusion follows by hypothesis and the second by lemma 2), the largest solution to this latter, less constrained, problem is feasible in the former, more constrained, problem. It must therefore also be the largest solution in the more constrained problem.[28] Hence, $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash$

[28] More generally, suppose that $\mathcal{A}_t$ is myopically CS-maximizing for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$ and that $\mathcal{A}'_t$ is myopically CS-maximizing for $\mathcal{P}'_t$ given $\mathcal{A}'_{t-1}$. If $\mathcal{A}'_{t-1} \subseteq \mathcal{A}_{t-1}$ and $(\mathcal{P}_t \cup \mathcal{A}_{t-1}) \subseteq (\mathcal{P}'_t \cup \mathcal{A}'_{t-1})$, then the level of consumer surplus under $\mathcal{A}'_t$ must be at least as great as under $\mathcal{A}_t$ (more mergers that can be approved and fewer mergers already approved make the primed problem less constrained than the unprimed one). Hence, if $\mathcal{A}'_t$ is feasible for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$—i.e., if $\mathcal{A}_{t-1} \subseteq \mathcal{A}'_t \subseteq (\mathcal{P}_t \cup \mathcal{A}_{t-1})$—then $\mathcal{A}'_t$ must also be myopically CS-maximizing for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$ and the level of consumer surplus attained in the two problems must be the same. Therefore, $\mathcal{A}_t$ is also myopically CS-maximizing for $\mathcal{P}'_t$ given $\mathcal{A}'_{t-1}$, from which it follows (by lemma 2) that $\mathcal{A}_t \subseteq \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1})$. Two conclusions follow from applying this logic, the first of which is the point made here in the text and the second of which is of relevance in the proof of proposition 3: (i) If $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1}) \subseteq (\mathcal{P}_t \cup$

$\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$, which implies that if all mergers are proposed in period t, the most lenient myopically CS-maximizing policy induces the set $\mathcal{A}_t = \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$ in period t. Note also that since $\mathcal{P}_t \subseteq \mathfrak{F}_t$, the monotonicity of the largest myopically CS-maximizing set $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ in $\mathcal{P}_t$, established in lemma 2, implies that $\mathcal{A}_t \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$, regardless of the merger proposals made up through and including period t. Thus, our induction hypothesis holds when we look at period $t+1$. Applying induction yields the result. QED

Proof of Proposition 3

Part i: The proof of the first claim is by induction. Consider a period t and suppose that starting in period $t+1$ the joint expected continuation payoff of the firms in each possible feasible merger is independent of firms' prior behavior. (Note that this is true in period T since there are no payoffs after period T.) We will establish that regardless of the previous history or rivals' proposal strategies in period t, it is optimal in period t for every proposer firm with a feasible but not yet approved merger to propose it.[29]

To see this, consider a firm that is the proposer of a feasible but not yet approved merger M_k. Note that since continuation payoffs are (by hypothesis) unaffected by period t play, it is optimal to propose the merger if proposing it maximizes the joint expected period t payoff of the firms in M_k. Let $\hat{\mathcal{P}}$ denote a realization of the set of proposed mergers in period t if merger M_k is proposed (firms in other feasible but not yet approved mergers may be using mixed strategies) and let $\hat{\mathcal{P}}_{-k} \equiv \hat{\mathcal{P}}\backslash M_k$ denote that realization without merger M_k included.

Suppose, first, that $\hat{\mathcal{P}}_{-k}$ is such that merger M_k is not approved when proposed. Then the set of approved mergers, and hence the joint period t expected payoff of the firms in M_k, is unaffected by whether merger M_k is proposed.[30]

The joint period t expected payoff of the firms in M_k is also unaffected by whether merger M_k is proposed if instead $\hat{\mathcal{P}}_{-k}$ is such that merger M_k is approved when proposed, but the merged firm M_k is inactive (produces zero output) in period t after its merger is approved. In that case, merger M_k is CS-neutral given the other mergers that are approved, which implies that $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k$ is a myopically CS-maximizing set for $\hat{\mathcal{P}}$ given $\mathcal{A}_{t-1}$. The set $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k$ is also feasible when merger M_k is not proposed, which implies that it is the largest myopically CS-maximizing set for $\hat{\mathcal{P}}_{-k}$ given $\mathcal{A}_{t-1}$.[31] So proposal of merger M_k

$\mathcal{A}_{t-1})$, then the largest myopically CS-maximizing sets in the two problems are the same, i.e., $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1})$. (ii) If the set $[\overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1}) \cap (\mathcal{P}_t \cup \mathcal{A}_{t-1})]$ is myopically CS-maximizing for $\mathcal{P}'_t$ given $\mathcal{A}'_{t-1}$, then it is the largest myopically CS-maximizing set for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$, i.e., $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) = [\overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1}) \cap (\mathcal{P}_t \cup \mathcal{A}_{t-1})]$ (this follows because any myopically CS-maximizing set $\mathcal{A}_t$ for $\mathcal{P}_t$ given $\mathcal{A}_{t-1}$ then satisfies both $\mathcal{A}_t \subseteq \overline{\mathcal{A}}^*(\mathcal{P}'_t|\mathcal{A}'_{t-1})$ and $\mathcal{A}_t \subseteq (\mathcal{P}_t \cup \mathcal{A}_{t-1})$).

29 The history prior to period t's proposal stage consists of the sequences $\mathfrak{F}^t = (\mathfrak{F}_1, \ldots, \mathfrak{F}_t)$ of feasible mergers, $\mathcal{P}^{t-1} = (\mathcal{P}_1, \ldots, \mathcal{P}_{t-1})$ of proposed mergers, and $\mathcal{A}^{t-1} = (\mathcal{A}_1, \ldots, \mathcal{A}_{t-1})$ of approved mergers. This history, which is observed by all firms, determines a subgame that starts in period t.

30 This follows formally from conclusion i in n. 27 since $\hat{\mathcal{P}}_{-k} \subseteq \hat{\mathcal{P}}$ and $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1}) \subseteq (\hat{\mathcal{P}}_{-k} \cup \mathcal{A}_{t-1})$ imply that $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})$.

31 That is, $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k = \overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$. This follows from conclusion ii in n. 27 since $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k = \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1}) \cap (\hat{\mathcal{P}}_{-k} \cup \mathcal{A}_{t-1})$.

does not affect which other mergers are approved in period t. As a result, proposal of merger M_k has no effect on the joint period t profits of the firms in M_k, which are zero in either case.

Finally, suppose that $\hat{\mathcal{P}}_{-k}$ is such that merger M_k is approved when proposed and that the merged firm M_k is active in period t after its merger is approved. We distinguish between two cases. First, suppose that $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k$. In this case, proposing merger M_k does not affect the other mergers that will be approved. Since $M_k \in \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{-1})$, the merger is CS-nondecreasing given the other mergers that will be approved and is therefore (by corollary 1) strictly profitable to propose. The other possibility (by lemma 2) is that $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1}) \subset \overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash M_k$. Part i of lemma 5 in this Appendix implies that there is a sequencing of the mergers in $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$ that is CS-nondecreasing at each step. However, since all the mergers in this set other than M_k must be CS-decreasing given that the mergers in $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$ have taken place (otherwise they would have been in $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$), merger M_k must be CS-nondecreasing given that the mergers in $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$ have occurred and must be the first merger in this sequence. By remark 1, the firms in M_k have a strictly greater profit when all the mergers in $\overline{\mathcal{A}}^*(\hat{\mathcal{P}}|\mathcal{A}_{t-1})\backslash\overline{\mathcal{A}}^*(\hat{\mathcal{P}}_{-k}|\mathcal{A}_{t-1})$ are approved than when none are. Hence, it is strictly more profitable in this case as well to propose merger M_k.

In summary, it is an optimal strategy for every feasible but not yet approved merger M_k to be proposed in period t regardless of the previous history and rivals' period t proposal strategies. The set of approved mergers at the end of period t will therefore be $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$. By the argument in the proof of lemma 4, we know that $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing)$ for any $\mathcal{A}_{t-1}$ that can arise under the most lenient myopically CS-maximizing merger policy. Thus, the market structure (and joint expected payoffs of the firms in each possible merger) at the end of period t is independent of firms' behavior prior to period t. Our induction hypothesis therefore holds when we look at period $t-1$. Applying induction starting in period T implies that in every period proposing every feasible but not yet approved merger is optimal.

Part ii: To establish the second claim, we first define two sets that form a partition of $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. Let $\overline{\mathcal{A}}_0^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ denote those mergers in $\mathcal{P}_t \cap \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ that result in merged firms that are inactive given the other mergers in $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ and let

$$\overline{\mathcal{A}}_1^*(\mathcal{P}_t|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})\backslash\overline{\mathcal{A}}_0^*(\mathcal{P}_t|\mathcal{A}_{t-1})$$

denote the complementary set. Note that approval of inactive mergers has no effect on either consumer surplus or firms' payoffs. This implies that if all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ are proposed—that is, if $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) \subseteq (\mathcal{P}_t \cup \mathcal{A}_{t-1})$—then consumer surplus and all firms' payoffs will be the same in period t as if all feasible but not yet approved mergers were proposed.

We now show that when the set of feasible but not yet approved mergers in period t is $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$, every merger in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ will be proposed. The proof is by induction. The induction hypothesis for period t is that in all future periods $\tau > t$, whenever the set of feasible but not yet approved mergers is $\mathfrak{F}_\tau\backslash\mathcal{A}_{\tau-1}$, all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_\tau\backslash\mathcal{A}_{\tau-1}|\mathcal{A}_{\tau-1})\backslash\mathcal{A}_{\tau-1}$ are proposed (which is true in period T).

Consider a merger $M_k \in \overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$. Since $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$ is a myopically CS-maximizing set for $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$ given $\mathcal{A}_{t-1}$, every merger in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$ is CS-nondecreasing given every other merger in that set. Since $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$, part i of lemma 5 (in this Appendix) implies that, starting from $\mathcal{A}_{t-1}$, there is an ordering of the mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$ that is CS-nondecreasing at each step, which we denote by $(M_1, \ldots, M_S)$. Suppose that all mergers M_s for $s < k$ are proposed when $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$ is the set of feasible and not yet approved mergers in period t. (Note that this assumption is valid when $k = 1$.) If $\hat{\mathcal{P}}_t = \{M_1, \ldots, M_k\}$, then since the sequence $(M_1, \ldots, M_k)$ is CS-nondecreasing at each step, we will have $\overline{\mathcal{A}}^*(\{M_1, \ldots, M_k\} | \mathcal{A}_{t-1}) = \{M_1, \ldots, M_k\} \cup \mathcal{A}_{t-1}$; that is, all these mergers, including merger M_k, will be approved.[32] If, instead, $\{M_1, \ldots, M_k\} \subset \hat{\mathcal{P}}_t$, then lemma 2 implies that $(\{M_1, \ldots, M_k\} \cup \mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$, so merger M_k is still approved. Since merger M_k is certain to be approved if proposed and results in an active firm, our argument in part i implies that proposal of the merger M_k is strictly profitable. Applying induction starting at $k = 1$, we see that if $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$ is the set of feasible and not yet approved mergers, all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$ will be proposed.

Applying induction starting in period T, we conclude that in every period t if the set of feasible but not yet approved mergers in period t is $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$, then every merger in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$ will be proposed in that period. The result follows. QED

Proof of Proposition 4

To consider extensive-form trembling-hand perfect Nash equilibria, we perturb the game by introducing a minimum and a maximum probability of a merger proposal at any information set of a proposer of a feasible but not yet approved merger. We examine Nash equilibrium behavior in the agent normal form as these minimum and maximum probabilities approach zero and one, respectively.

Part i: We will first establish that all feasible mergers being proposed in each period after any history is an extensive-form trembling-hand perfect Nash equilibrium for the firms. Combined with lemma 4, this yields part i of the proposition. To do so, it suffices to examine perturbed games in which at every information set of a proposer of a feasible but not yet approved merger the merger must be proposed with a probability of at least $\varepsilon > 0$ and not more than $1 - \varepsilon$, where $\varepsilon \to 0$.

The proof is by induction and follows closely that of proposition 3. Consider a period t in which the history prior to period t's proposal stage consists of the sequences $\mathfrak{F}^t = (\mathfrak{F}_1, \ldots, \mathfrak{F}_t)$ of feasible mergers, $\mathcal{P}^{t-1} = (\mathcal{P}_1, \ldots, \mathcal{P}_{t-1})$ of proposed mergers, and $\mathcal{A}^{t-1} = (\mathcal{A}_1, \ldots, \mathcal{A}_{t-1})$ of approved mergers. Note that this history also determines exactly each firm i's observed history, which we denote

[32] In particular, by part ii of lemma 5, every merger in set $\{M_1, \ldots, M_k\}$ is CS-nondecreasing given every other merger in the set. By part i of lemma 5, if a strict subset of $\{M_1, \ldots, M_k\}$ were approved, there would be a proposed but unapproved merger that could be approved without lowering consumer surplus. So all mergers in $\{M_1, \ldots, M_k\}$ will be approved in the most lenient myopically CS-maximizing merger policy.

by I_i^t. Formally, each I_i^t corresponds to an information set for firm i at the proposal stage in period t. The most important difference from the proof of proposition 3 is that the induction hypothesis is now that for any period $t < T$, starting in period $t + 1$, all feasible but not yet approved mergers will be proposed in every period with the maximum possible probability $1 - \varepsilon$.

To show that proposing its merger with the maximum possible probability is optimal in period t for every proposer firm with a feasible but not yet approved merger, consider proposer firm i at an information set I_i^t with a feasible but not yet approved merger M_k. Recall from the proof of proposition 3 that, for a given information set I_i^t and a given set of other proposed mergers $\hat{\mathcal{P}}_{-k}$, either the merger M_k is not approved when proposed (and so proposing the merger has no effect on current profits), or the merger is approved when proposed but the merged firm is inactive (and so, again, proposing the merger has no effect on current profits), or the merger is approved and results in an active firm (in which case there is a strictly positive effect on current profits). In the first case (i.e., the merger is not approved when proposed), there is also no effect on future profits of proposing the merger given the induction hypothesis. The same is true in the second case (when the merger is approved but results in an inactive firm). In the third case (where the merger is approved and results in an active firm), however, there might be an effect on future profits if $t < T$. But this effect is continuous in the size of the tremble ε and (as is clear from the proof of proposition 3) is equal to zero if $\varepsilon = 0$. Since there are at most a finite number of such information sets I_i^t and sets $\hat{\mathcal{P}}_{-k}$, there exists an $\varepsilon_t > 0$ such that, for all $\varepsilon \leq \varepsilon_t$, proposing a feasible and not yet approved merger that ends up being approved and results in an active firm in period t is strictly profitable. Hence, for $\varepsilon \leq \varepsilon_t$, proposing every feasible and not yet approved merger in period t is profitable. The same is clearly true if $t = T$, where there is no effect on future profits.

We conclude that our induction hypothesis—that all feasible and not yet approved mergers will be proposed in the future with the maximum possible probability—holds when we look at period $t - 1$ provided that $\varepsilon \leq \bar{\varepsilon}_t = \min_{\tau \geq t} \varepsilon_\tau$. Applying induction starting in period T implies that proposing every feasible but not yet approved merger in every period with the maximum possible probability is a Nash equilibrium of the agent normal form of any perturbed game with $\varepsilon \leq \bar{\varepsilon}_1$. Hence, with $\varepsilon \to 0$, proposal of every feasible but not yet approved merger in every period is an extensive-form trembling-hand perfect Nash equilibrium.

Part ii: We next show that every extensive-form trembling-hand perfect Nash equilibrium maximizes discounted consumer surplus (and results in the same sequence of period-by-period consumer surpluses) for each sequence of feasible mergers $\mathfrak{F}$. To establish this result, we restrict attention to small perturbations in which the minimum probability of a merger proposal at any information set for a proposer of a feasible but not yet approved merger is no more than $\bar{\varepsilon}_1 > 0$ (where $\bar{\varepsilon}_1$ is defined as above) and the maximum probability is no less than $1 - \bar{\varepsilon}_1$. We examine Nash equilibrium behavior in the agent normal form as the minimum and maximum probabilities approach zero and one, respectively.

We now show that if the perturbations are strictly positive but sufficiently small (in the sense defined above), then if the true set of feasible but not yet approved mergers in period t is $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$, every merger in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ will be proposed with the maximum possible probability in that period. (Recall that the set $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$, defined in the proof of part ii of proposition 3, is the set of mergers in $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ that result in active firms given the market structure $\overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$.) The result follows as we let the minimum and maximum proposal probabilities go to zero and one, respectively.

The proof is by induction. The induction hypothesis for period t is that in all future periods $\tau > t$, whenever the set of feasible but not yet approved mergers is $\mathfrak{F}_\tau\backslash\mathcal{A}_{\tau-1}$, then all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_\tau\backslash\mathcal{A}_{\tau-1}|\mathcal{A}_{\tau-1})\backslash\mathcal{A}_{\tau-1}$ are proposed with the maximum possible probability.

Suppose that $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is indeed the true set of feasible and not yet approved mergers. Let $\mathcal{I}_t(\mathfrak{F}_t\backslash\mathcal{A}_{t-1})$ denote those information sets in period t that are consistent with $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$; that is, these are the information sets that are reached for at least one sequence $(\mathfrak{F}^t, \mathcal{P}^{t-1})$ of feasible mergers and merger proposals that, given the most lenient myopically CS-maximizing merger policy, results in the set of feasible but not yet proposed mergers in period t being $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$. Consider any information set $I_i^t \in \mathcal{I}_t(\mathfrak{F}_t\backslash\mathcal{A}_{t-1})$ that belongs to the proposer of a merger $M_k \in (\mathfrak{F}_t\backslash\mathcal{A}_{t-1})$ such that $M_k \in \overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$. From our earlier argument, if all minimum proposal probabilities are no greater than $\bar{\varepsilon}_1$ and all maximum proposal probabilities are no less than $1-\bar{\varepsilon}_1$, proposing merger M_k never reduces the expected joint discounted profits of the firms in M_k. We now show that proposing merger M_k is in fact strictly profitable in expectation.

Observe, first, that in any Nash equilibrium of the agent normal form of the perturbed game, the information set I_i^t is reached with positive probability along the equilibrium path when the set of feasible but not yet proposed mergers in period t is $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$, so (in belief language) the agent choosing at this information set must assign a strictly positive probability to $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ being the set of feasible but not yet approved mergers.[33]

The rest of the argument follows closely, with some differences, the proof of part ii of proposition 3: Starting from $\mathcal{A}_{t-1}$, there is an ordering of the mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ that is CS-nondecreasing at each step, which we denote by $(M_1, \ldots, M_S)$. As in the proof of part ii of proposition 3, consider the proposal of merger M_k at I_i^t and suppose that all mergers M_s for $s<k$ are proposed with maximum probability when $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is the true set of feasible and not yet approved mergers in period t. (Note that this assumption is valid when $k = 1$.) Now, given the trembles, the proposer of M_k at I_i^t must assign a strictly positive probability to the event that the realized set of proposed mergers is $\hat{\mathcal{P}}_t = \{M_1, \ldots, M_k\}$. As in the proof of part ii of proposition 3, in this case all these mergers will be approved and will result in active firms. Hence, the proposer of M_k at I_i^t must believe that, if proposed, merger M_k will be approved and result in an active firm with strictly positive probability. But from our previous argument, if merger M_k is approved and the merged firm M_k is active, proposal of the merger is

[33] This property—that any firm with a merger in $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ must always assign a strictly positive probability to $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ being the set of feasible but not yet approved mergers—is a key step of the argument. It would not be true without the perturbations and is the reason why perturbations are needed for ensuring the proposal of all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$.

strictly profitable when the perturbations are small (in the sense described above). Applying induction starting at $k = 1$, we see that any such merger M_k will be proposed in period t with the maximum possible probability. Thus, if $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$ is the true set of feasible and not yet approved mergers, all mergers in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$ will be proposed with the maximum possible probability.

Applying induction, we conclude that in any perturbed game if the true set of feasible but not yet approved mergers in period t is $\mathfrak{F}_t \backslash \mathcal{A}_{t-1}$, then every merger in $\overline{\mathcal{A}}_1^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$ will be proposed with the maximum possible probability in that period. Taking the perturbations to zero yields the result. QED

B. *Sketch of Arguments Underlying Remark 2*

In the following, we briefly sketch the arguments leading to our claim in remark 2. To do so, we need to extend many of the results in the text. To refer to these additional results, we will append a prime to the number of the result it extends. For example, the result that extends proposition 2 will be denoted proposition 2′.

CS Effects of and Interactions between Mergers

Mirroring the statement of proposition 1, proposition 1′ states the following: (i) If a merger M_1 is CS-increasing (and hence profitable) in isolation, it remains CS-increasing (and hence profitable) if another merger M_2 that is CS-nondecreasing in isolation takes place and the merged firm M_1 remains active after merger M_2 has taken place. (ii) If a merger M_1 is CS-nonincreasing in isolation and results in an active firm, then the merger remains CS-nonincreasing if another merger M_2 that is CS-nonincreasing in isolation takes place. The proof of part i of the proposition uses the fact that for M_1 to be CS-increasing after merger M_2 takes place, the merged firm M_1 must also be active if merger M_2 does not take place, and it follows an argument similar to that in the proof of part i of proposition 1. The proof of part ii of the proposition uses the fact that if the merged firm M_1 is active in isolation, it must remain active after merger M_2 has taken place since the CS-nonincreasing merger M_2 weakly increases the equilibrium price; moreover, the CS-nonincreasing merger M_2 weakly decreases the threshold value of postmerger marginal cost, $\hat{c}_{M_1}$, that makes merger M_1 just CS-neutral, and so merger M_1 must remain CS-nonincreasing if M_2 takes place.

In proposition 2′, the hypothesis is that merger M_1 is CS-increasing (rather than CS-nondecreasing, as in proposition 2) in isolation, whereas merger M_2 is CS-nonincreasing (rather than CS-decreasing) in isolation but CS-increasing once merger M_1 has taken place. Under this modified hypothesis, the statements of parts i and ii remain unchanged. The proof of the proposition proceeds along the same lines as that of proposition 2 except for some small modifications. For instance, in the second sentence of the proof of part i, "weakly increases" is replaced by "strictly increases" and "nonnegative" by "strictly positive."

Lemma 5′ (the modified Incremental Gain Lemma) differs from lemma 5 in that "CS-nondecreasing" is replaced everywhere by "CS-increasing." For instance, the set $\mathfrak{M}$ in part i has the property that every merger $M \in \mathfrak{M}$ is CS-increasing if all the other mergers in $\mathfrak{M}$ have taken place. As a result, starting from any strict subset $Y \subset \mathfrak{M}$, there exists a sequencing of the mergers in $\mathfrak{M}\backslash Y$ that is CS-increasing at each step. The proof uses the fact that, under the hypothesis of the lemma, every merger $M \in \mathfrak{M}$ must result in an active firm for any set $\mathcal{A} \subseteq \mathfrak{M}$ of approved mergers.[34] This means that we can apply part i of proposition 1′ and use the same arguments as in the proof of lemma 5 (but with "CS-decreasing" being replaced by "CS-nonincreasing" and so on). As for part ii of lemma 5′, we need to add the assumption that every merger in the sequence remains active when all the other mergers in the sequence have taken place. That is, the statement now reads as follows: Suppose that a sequence of mergers $M_1, \ldots, M_{J_1}$ is CS-increasing at each step. Then each merger $M \in \mathfrak{M} \equiv \{M_1, \ldots, M_{J_1}\}$ is CS-increasing if all the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M}\backslash M$) have taken place, provided that each merged firm M remains active after the mergers in $\mathfrak{M}\backslash M$.

To obtain lemma 6′ from lemma 6, the hypothesis is modified so that the set $\mathfrak{M}_1$ has the property that every merger in the set is CS-increasing if all the other mergers in that set have taken place, whereas the set $\mathfrak{M}_2$ continues to have the property that every merger in the set is CS-nondecreasing if all the other mergers in that set have taken place. Lemma 6′ states that if $\mathfrak{M}_1 \not\subseteq \mathfrak{M}_2$ and if each of the mergers in set $\mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ (when done on its own) results in an active firm once all the mergers in set $\mathfrak{M}_2$ have taken place, then there exists a merger $M_1' \in \mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-increasing (rather than CS-nondecreasing, as in part i of lemma 6) given that all the mergers in $\mathfrak{M}_2$ have taken place. The proof first identifies a merger $M_1' \in \mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-increasing given that the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place. If M_1' would not be a merger among active firms once the mergers in $\mathfrak{M}_2$ have taken place, then it must be CS-increasing once the mergers in $\mathfrak{M}_2$ have taken place (since it results in an active firm). If, instead, M_1' would be a merger among active firms once the mergers in $\mathfrak{M}_2$ have taken place, then an induction argument along the same lines as that in the proof of part i of lemma 6 (now using proposition 1′) establishes the result. Note that a sufficient condition for each of the mergers in set $\mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ to result in an active firm once all the mergers in set $\mathfrak{M}_2$ have taken place is that consumer surplus under set $\mathfrak{M}_1$ is at least as large as under set $\mathfrak{M}_2$, that is, letting p_i^* denote the equilibrium price after all the mergers in set $\mathfrak{M}_i$, $i = 1, 2$, have taken place, if we have $p_2^* \geq p_1^*$. To see this, note that every merged firm $M \in \mathfrak{M}_1$ must have a cost $\bar{c}_M < p_1^*$ since, otherwise, the merger would not be CS-increasing given the other mergers in set $\mathfrak{M}_1$. But then $\bar{c}_M < p_2^*$, which implies that merger M results in an active firm once all the mergers in set $\mathfrak{M}_2$ have taken place. (A counterpart to part ii of lemma 6 is not necessary for our purposes here.)

Myopically CS-Maximizing Sets

In analogue to the largest myopically CS-maximizing set, we can define a smallest myopically CS-maximizing set for the set of proposed mergers $\mathcal{P}_t$ given current

[34] By part i of lemma 5, the price after the mergers in set $\mathcal{A}$ have taken place can be no lower than the price after all the mergers in set $\mathfrak{M}$ have taken place.

market structure $\mathcal{A}_{t-1}$ as a myopically CS-maximizing set that does not contain any other myopically CS-maximizing set.

Lemma 2′ makes the same uniqueness claim as lemma 2 but for the smallest myopically CS-maximizing set rather than for $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$: there is a unique smallest myopically CS-maximizing set, denoted $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. (In contrast to lemma 2, no monotonicity claim is made.) The proof of the uniqueness property is by contradiction. Suppose that there are two smallest myopically CS-maximizing sets, say $\underline{\mathcal{A}}$ and $\underline{\mathcal{A}}'$, with $\underline{\mathcal{A}} \neq \underline{\mathcal{A}}'$. Without loss of generality, suppose that $\underline{\mathcal{A}} \nsubseteq \underline{\mathcal{A}}'$. Since each one of the sets must have the property that every merger in the set is CS-increasing given the other mergers in that set and since the two sets must induce the same level of consumer surplus, we can apply lemma 6′ to show that there exists a merger $M' \in \underline{\mathcal{A}} \backslash (\underline{\mathcal{A}} \cap \underline{\mathcal{A}}')$ that is CS-increasing given that all the mergers in $\underline{\mathcal{A}}'$ have taken place. But then $\underline{\mathcal{A}}'$ cannot be a myopically CS-maximizing set, a contradiction.

The following result shows that the smallest myopically CS-maximizing set is contained in any other myopically CS-maximizing set and that any myopically CS-maximizing set is contained in the largest myopically CS-maximizing set.

Lemma 7. For a given proposed set of mergers, $\mathcal{P}_t$, and current market structure, $\mathcal{A}_{t-1}$, the following inclusion property holds for the myopically CS-maximizing sets:

$$\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}).$$

Proof. We have already established the second inclusion property in lemma 2. We therefore turn to the first inclusion property, $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. Let $\mathcal{A}^0 \subseteq \mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ denote the set of all those mergers in $\mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ that are CS-neutral given the other mergers in $\mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, and let $\mathcal{A}^+ \equiv \mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \backslash \mathcal{A}^0$ denote the complementary set, which has the property that every merger $M \in \mathcal{A}^+$ is CS-increasing given the other mergers in $\mathcal{A}^+$. We claim that $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{A}^+$. (In fact, $\mathcal{A}^+ = \underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, but we do not need to show this.) To see this, suppose otherwise that $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \nsubseteq \mathcal{A}^+$. Since each set, $\mathcal{A}^+$ and $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$, is myopically CS-maximizing and has the property that every merger in the set is CS-increasing given the other mergers in that set, we can apply lemma 6′ to conclude that there exists a merger $M' \in \underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \backslash (\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \cap \mathcal{A}^+)$ that is CS-increasing given that all the mergers in $\mathcal{A}^+$ have taken place. But then $\mathcal{A}^+$ cannot be myopically CS-maximizing, a contradiction. Hence, we have $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{A}^+$, which implies $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. QED

An immediate implication of lemma 7 is that we can think of set $\mathcal{A}_t^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ as consisting of the mergers in the smallest myopically CS-maximizing set $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ plus a (potentially empty) set of mergers that are CS-neutral given $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$. Thus, all myopically CS-maximizing merger policies differ from one another only in their treatment of CS-neutral mergers.[35]

[35] Note also that CS-neutral mergers are measure zero events in a model with a continuum of possible efficiency realizations.

Extension of Proposition 3

Establishing the claim of remark 2 parallels the argument leading to proposition 3 in the text. We first establish a generalization of lemma 4, lemma 4′. Specifically, lemma 4′ states that if all feasible but not yet approved mergers are proposed in each period, any myopically CS-maximizing merger policy that induces the approval sequence $\mathcal{A}_1 = \mathcal{A}_1^*(\mathfrak{F}_1|\emptyset)$ and $\mathcal{A}_t = \mathcal{A}_t^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ for all $t>1$ maximizes discounted consumer surplus for every realization of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, \ldots, \mathfrak{F}_T)$. To establish this result, we use the following lemma, which is also used to prove proposition 3′.

Lemma 8. Suppose $\mathfrak{F}_{t-1} \subseteq \mathfrak{F}_t$. If the current market structure $\mathcal{A}_{t-1}$ is such that $\mathcal{A}_{t-1} \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_{t-1}|\emptyset)$, then

$$\underline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset) \subseteq \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) \subseteq \mathcal{A}_t^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$$
$$\subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) = \overline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset).$$

Proof. The second and third inclusion properties in the display follow from lemma 7, and the equality follows from the same induction argument as in the proof of lemma 4. To see the first inclusion property, $\underline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset) \subseteq \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$, observe that $\overline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset)$ is a solution to the problem of myopically maximizing consumer surplus given that the set of proposed mergers is $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ and mergers $\mathcal{A}_{t-1}$ have previously been approved. Hence, the solution set to that problem must be a subset of the solution set of the less constrained problem of myopically maximizing consumer surplus given that the set of proposed mergers is $\mathfrak{F}_t$ and no mergers have previously been approved. As a result, the smallest solution in the unconstrained problem must be contained in the smallest solution to the constrained problem. QED

Applying lemma 8 iteratively, we see that if all feasible mergers are proposed in period t, then regardless of firms' previous behavior, the market structure in period t will contain the set $\underline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset)$ and be contained within the set $\overline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset)$, which implies that it is a solution to the problem of maximizing consumer surplus in period t given that no mergers have previously been approved. This implies that lemma 4′ holds.

Note also that if in period t all mergers in $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ are proposed when $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is the set of feasible but not yet approved mergers, then regardless of previous behavior by the firms, the market structure in period t will again contain the set $\underline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset)$ and be contained within the set $\overline{\mathcal{A}}^*(\mathfrak{F}_t|\emptyset)$. To see this, observe that if $\mathcal{P}_t$ is such that $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1} \subseteq \mathcal{P}_t \subseteq \mathfrak{F}_t\backslash\mathcal{A}_{t-1}$, then lemma 2 implies that $\overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$, whereas the fact that $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$—a set that myopically maximizes consumer surplus when all mergers in $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ are proposed—is feasible when $\mathcal{P}_t$ is proposed implies that $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) = \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$.[36] Applying lemma 8 then implies that

[36] The argument in n. 27 implies that the consumer surplus levels with approved mergers $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ and $\underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1})$ must be the same. Since each of these two sets has the property that every merger in the set is CS-increasing given the other mergers in the set, we can apply lemma 6′ to show that the two sets must be the same.

$$\underline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing) \subseteq \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1}) = \underline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathcal{P}_t|\mathcal{A}_{t-1}) \subseteq \overline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$$
$$= \overline{\mathcal{A}}^*(\mathfrak{F}_t|\varnothing).$$

Thus, if in all periods all mergers in $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ are proposed when $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is the set of feasible but not yet approved mergers, then the outcome will yield optimal period-by-period levels of consumer surplus.

These facts imply that dynamic optimality holds for any myopically CS-maximizing policy.

PROPOSITION 3′. If the antitrust authority follows a myopically CS-maximizing merger policy, then for each sequence $\mathfrak{F}$, every subgame-perfect Nash equilibrium results in the same optimal sequence of period-by-period consumer surpluses.

Proof (Sketch). The proof follows very closely that of part ii of proposition 3. It proceeds by establishing, using an induction argument, that whenever $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is the set of feasible but not yet approved mergers in period t, all mergers in set $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ will necessarily be proposed, which establishes the claim (using the argument above). One important change relative to the case of the most lenient myopically CS-maximizing merger policy is that, in any period t, future market structures may be affected by whether a given merger $M_k \in \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ is proposed today. However, these future market structure effects can involve merger M_k only if the merged firm would be inactive and can involve mergers other than M_k only in situations in which those mergers are CS-neutral given the other mergers being approved. They therefore have no effect on the joint profits of the firms in merger M_k, so we can again focus solely on current-period profit effects.

Consider the proposal of a merger $M_k \in \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ in period t under the assumption that future payoffs for the firms involved in that merger are independent of period t behavior. Since $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})$ is the smallest myopically CS-maximizing set for $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ given $\mathcal{A}_{t-1}$, every merger in $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ is CS-increasing given every other merger in that set and results in an active firm. Part i of lemma 5′ implies that there is an ordering of the mergers in $\underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ that is CS-increasing at each step, which we denote by $(M_1, \ldots, M_S)$. Suppose that all mergers M_s for $s < k$ are proposed when $\mathfrak{F}_t\backslash\mathcal{A}_{t-1}$ is the set of feasible and not yet approved mergers in period t (which is true when $k = 1$). Consider the case in which $\hat{\mathcal{P}}_t = \{M_1, \ldots, M_k\}$. We claim that all the mergers in $\{M_1, \ldots, M_k\}$ will be approved. To see this, note first that all the merged firms in $\{M_1, \ldots, M_k\}$ will be active if all are approved (since the price will be no less than if all the mergers $\{M_1, \ldots, M_S\} = \underline{\mathcal{A}}^*(\mathfrak{F}_t\backslash\mathcal{A}_{t-1}|\mathcal{A}_{t-1})\backslash\mathcal{A}_{t-1}$ are approved). Hence, by part ii of lemma 5′, every merger in $\hat{\mathcal{P}}_t$ is CS-increasing given the other mergers in that set (and given the previously approved mergers $\mathcal{A}_{t-1}$). If the antitrust authority were to approve only a (possibly empty) subset of $\hat{\mathcal{P}}_t$, part i of lemma 5′ implies that the antitrust authority could strictly increase consumer surplus by approving the other mergers in $\hat{\mathcal{P}}_t$ as well. This proves the claim that all the mergers in $\{M_1, \ldots, M_k\}$ will be approved when $\hat{\mathcal{P}}_t = \{M_1, \ldots, M_k\}$.

Consider now the case in which $\{M_1, \ldots, M_k\} \subset \hat{\mathcal{P}}_t$. From the same argument as above, the set $\{M_1, \ldots, M_k\}$ has the property that every merger in the set is

CS-increasing given the other mergers in that set, and so (by part i of lemma 5′) for any strict subset $Y \subset \{M_1, \ldots, M_k\}$, there exists a merger $M' \in \{M_1, \ldots, M_k\} \backslash Y$ that is CS-increasing given Y. We claim that $\{M_1, \ldots, M_k\} \subseteq \mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$ in any myopically CS-maximizing policy, so that all the mergers in $\{M_1, \ldots, M_k\}$ will be approved. To see this, suppose otherwise that $\{M_1, \ldots, M_k\} \not\subseteq \mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$. Note first that every merger in $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$ is CS-nondecreasing given the other mergers in that set. Since $\hat{\mathcal{P}}_t \subseteq \mathfrak{F}_t \backslash \mathcal{A}_{t-1}$, the equilibrium price under market structure $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$ must be weakly higher than under market structure $\mathcal{A}_t^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$, and the equilibrium price under market structure $\mathcal{A}_t^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$ must be the same as under market structure $\underline{\mathcal{A}}^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$ (by virtue of both sets being CS-maximizing given the same set of proposed mergers and given the same market structure). Since all the merged firms in $\{M_1, \ldots, M_k\}$ are active when the market structure is $\underline{\mathcal{A}}^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$, this implies that these firms will also be active when the market structure is $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$. Let $Y = \mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1}) \cap \{M_1, \ldots, M_k\}$. Since all the merged firms in $\{M_1, \ldots, M_k\}$ are active when market structure is $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$, lemma 6′ implies that there exists a merger $M' \in \{M_1, \ldots, M_k\} \backslash Y$ that is CS-increasing given all the mergers in $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$. But this means that the antitrust authority can strictly increase consumer surplus by approving merger M' in addition to all the mergers in $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1}) \backslash \mathcal{A}_{t-1}$, a contradiction to $\mathcal{A}_t^*(\hat{\mathcal{P}}_t | \mathcal{A}_{t-1})$ being myopically CS-maximizing. This proves the claim that all the mergers in $\{M_1, \ldots, M_k\}$ will be approved when $\{M_1, \ldots, M_k\} \subset \hat{\mathcal{P}}_t$. Hence, from the same arguments as in the proof of proposition 3, proposal of merger M_k is strictly profitable. When induction is applied starting at $k = 1$, it follows that all mergers in $\underline{\mathcal{A}}^*(\mathfrak{F}_t \backslash \mathcal{A}_{t-1} | \mathcal{A}_{t-1})$ will be proposed. QED

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