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Merger Policy with Merger Choice†

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The evaluation of proposed horizontal mergers involves a basic trade-off: mergers may increase market power but may also create efficiencies. Whether a given merger should be approved depends, as first emphasized by Williamson (1968), on a balancing of these two effects.

In most of the literature discussing horizontal merger evaluation, the assumption is that a merger should be approved if and only if it improves welfare, whether that be aggregate surplus or just consumer surplus, as is in practice the standard adopted by most antitrust authorities (see, e.g., Farrell and Shapiro 1990; McAfee and Williams 1992). Implicitly, the antitrust authority is viewed as facing a one-time merger and has no ability to commit ex ante to its approval policy. In practice, neither of these assumptions may be descriptive of reality since one merger proposal may be followed by others, and commitment to an approval rule may be possible either through legislation or through the (long-lived) antitrust authority's reputation.

This article contributes to a small literature that formally derives optimal merger approval rules. This literature started with Besanko and Spulber (1993), who discussed the optimal rule for an antitrust authority that cannot directly observe efficiencies but that recognizes that firms know this information and decide whether to propose a merger based on this knowledge. Other recent papers in this literature include Armstrong and Vickers (2010), Nocke and Whinston (2010), Ottaviani and Wickelgren (2009), and Neven and Röller (2005).

In this article, we focus on a static setting (thus ignoring dynamic issues) in which one "pivotal" firm may merge with one of a number of other firms who have differing initial marginal costs. These mergers are mutually exclusive, and each may result in a different, randomly drawn postmerger marginal cost due to merger-related synergies. The merger that is proposed is the result of a bargaining process among the firms. The antitrust authority observes the characteristics of the merger that is proposed, but neither the feasibility nor the characteristics of any mergers that are not proposed. However, the antitrust authority can commit ex ante to its merger approval rule.

We focus on an antitrust authority that wishes to maximize expected consumer surplus. Our main result characterizes the form of the antitrust authority's optimal

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policy, which we show should impose a tougher standard on mergers involving larger merger partners (in terms of their premerger market share). Specifically, the minimal acceptable improvement in consumer surplus is strictly positive for all but the smallest merger partner and is larger the greater is the merger partner's premerger share. Since in our model a greater premerger share for the merger partner is equivalent to a larger naively computed postmerger Herfindahl index (computed assuming that the merged firm's postmerger share is the sum of the merger partners' premerger shares and that the shares of outsiders do not change), another way to say this is that mergers that result in a larger naively computed postmerger Herfindahl index must generate larger improvements in consumer surplus to be approved. The form of this optimal policy is a response to a fundamental bias that we show exists in firms' proposal incentives: whenever a larger merger would create at least as large a gain for consumers as a smaller one, the larger one is proposed if both would be approved. However, if both would be approved, the larger merger will sometimes be proposed even when it is worse for consumers. The optimal policy therefore rejects some consumer surplus-enhancing larger mergers to induce firms to propose instead better smaller ones.

The closest papers to ours are Lyons (2003) and Armstrong and Vickers (2010). Lyons is the first to identify the issue that arises when firms may choose which merger to propose and to note that committing to a policy may therefore be valuable. Motivated by the horizontal merger problem, Armstrong and Vickers (2010) provide an elegant characterization of the optimal policy for a principal facing an agent who may propose a project, when the principal cannot observe the characteristics of unproposed projects and the projects are ex ante identical in terms of their distributions of possible outcomes. Our article differs from Armstrong and Vickers (2010) primarily in its focus on the optimal treatment of mergers that differ in this ex ante sense. Moreover, a key issue in our article—the bargaining process among firms—is absent in Armstrong and Vickers, as they assume there is a single agent.[1]

The article is also related to Nocke and Whinston (2010). That paper establishes conditions under which the optimal dynamic policy for an antitrust authority that wants to maximize discounted expected consumer surplus is a completely myopic policy, in which a merger is approved if and only if it does not lower consumer surplus at the time it is proposed. A key assumption for that result is that potential mergers are "disjoint," in the sense that the set of firms involved in different possible mergers do not overlap. The present article explores, in a static setting, the implications of relaxing that disjointness assumption, focusing on the polar opposite case in which all potential mergers are mutually exclusive.

The article proceeds as follows. We describe our model in Section I. In Section II we derive some basic properties characterizing the structure of the antitrust authority's merger policy design problem, most importantly demonstrating the bias in firms' proposal incentives. We also note how this same structure can be applied to

[1] Our article also contributes to the theoretical literature on delegated agency without transfers, which was initiated by Holmström (1984). Recent contributions (in addition to Armstrong and Vickers 2010) include Martimort and Semenov (2006), Alonso and Matouschek (2008), and Che, Dessein, and Kartik (2013). A key difference between Che, Dessein, and Kartik (2013) and our article is that they assume that the principal (antitrust authority) can condition its policy only on the identity of the proposed project (merger) but not on its characteristics (postmerger costs).

settings other than our baseline model, such as with efficient bargaining, alternative welfare standards, and differentiated price competition. In Section III, we derive our main result: the antitrust authority optimally imposes a tougher standard, in terms of the minimum increase in consumer surplus required for approval, the "larger" is the proposed merger. In Section IV, we show that the optimal policy may not have a cutoff structure and provide a condition for verifying whether it does. Assuming it does, we examine some comparative statics. We conclude in Section V.

I. The Model

We consider a homogeneous goods industry in which firms compete in quantities (Cournot competition). Let $\mathcal{N} = \{0, 1, 2, \dots, N\}$ denote the (initial) set of firms. All firms have constant returns to scale; firm i's marginal cost is denoted c_i. Inverse demand is given by $P(Q)$. We impose standard assumptions on demand:

ASSUMPTION 1: *For all Q such that $P(Q) > 0$, we have:*

(i) $P'(Q) + qP''(Q) < 0$ *for all* $q \in [0, Q]$;

(ii) $\lim_{Q\to\infty} P(Q) = 0$.

Under these conditions there exists a unique Nash equilibrium in quantities. Moreover, this equilibrium is "stable" [each firm i's best-response function $b_i(Q_{-i}) \equiv \arg\max_{q_i}[P(Q_{-i} + q_i) - c_i]q_i$ satisfies $b_i'(Q_{-i}) \in (-1, 0)$ whenever $b_i(Q_{-i}) > 0$, where $Q_{-i} \equiv \sum_{j\neq i} q_j$] so that comparative statics are "well behaved" (e.g., if a subset of firms jointly produce less [respectively, more] because of a change in their incentives to produce output, then equilibrium industry output will decrease [respectively, increase]). The vector of output levels in the premerger equilibrium is given by $\mathbf{q}^\circ \equiv (q_0^\circ, q_1^\circ, \dots, q_N^\circ)$, where q_i° is firm i's quantity. For simplicity, we assume that premerger marginal costs are such that all firms in $\mathcal{N}$ are active in the premerger equilibrium, i.e., $q_i^\circ > 0$ for all i. Hence, each firm i's output $(i = 0, 1, \dots, N)$ satisfies the first-order condition

$$P(Q^\circ) + P'(Q^\circ)q_i^\circ = c_i. \tag{1}$$

Aggregate output, price, consumer surplus, and firm i's profit in the premerger equilibrium are denoted $Q^\circ \equiv \sum_i q_i^\circ$, $P^\circ \equiv P(Q^\circ)$, $CS^\circ \equiv \int_0^{Q^\circ} P(s)\,ds - P^\circ Q^\circ$, and $\pi_i^\circ \equiv [P^\circ - c_i]q_i^\circ$, respectively. Firm i's market share is $s_i^\circ \equiv q_i^\circ / Q^\circ$.

We suppose that there is a set $\mathcal{K}$ of K potential mergers, each between firm 0 (the "acquirer") and a single merger partner (a "target") $k \in \mathcal{K} \subseteq \mathcal{N}$. There is a random variable $\phi_k \in \{0, 1\}$ that determines whether the merger between firm 0 and firm k is feasible $(\phi_k = 1)$ or not $(\phi_k = 0)$. We let $\theta_k \equiv \Pr(\phi_k = 1) > 0$ denote the probability that the merger is feasible. A feasible merger is described by $M_k = (k, \overline{c}_k)$, where k is the identity of the target and $\overline{c}_k$ the (realized) postmerger marginal cost, which is drawn from distribution function G_k with support $[l, h_k]$ and no mass points. The random draws of ϕ_k and $\overline{c}_k$ are independent across mergers. (We assume a common lower bound l primarily to simplify the statement of our main result, Proposition 1; we remark after that result about the effects of relaxing this restriction.) The realized set of feasible mergers is denoted $\mathcal{F} \equiv \{M_k : \phi_k = 1\}$.

If merger M_k is implemented, the vector of outputs in the resulting postmerger equilibrium is denoted $\mathbf{q}(M_k) \equiv (q_1(M_k), \ldots, q_N(M_k))$, where $q_k(M_k)$ is the output of the merged firm, aggregate output is $Q(M_k) \equiv \sum_i q_i(M_k)$, and firm i's market share is $s_i(M_k) \equiv q_i(M_k)/Q(M_k)$. We assume that all nonmerging firms remain active after any merger, so individual outputs satisfy the first-order condition

$$P(Q(M_k)) + P'(Q(M_k))q_i(M_k) = c_i \tag{2}$$

for the nonmerging firms $i \neq 0, k$ and

$$P(Q(M_k)) + P'(Q(M_k))q_k(M_k) = \overline{c}_k \tag{3}$$

for the merged firm. The postmerger profit of nonmerging firm i is given by $\pi_i(M_k) \equiv [P(Q(M_k)) - c_i]\, q_i(M_k)$, and the merged firm's profit by $\pi_k(M_k) \equiv [P(Q(M_k)) - \overline{c}_k]\, q_k(M_k)$. The induced change in consumer surplus is

$$\Delta CS(M_k) \equiv \left\{ \int_0^{Q(M_k)} P(s)\, ds - P(Q(M_k))Q(M_k) \right\} - CS^\circ.$$

We will say that a merger M_k is *CS-neutral* if $\Delta CS(M_k) = 0$, CS-*increasing* if $\Delta CS(M_k) > 0$, and *CS-decreasing* if $\Delta CS(M_k) < 0$. A merger is *CS-nondecreasing* (respectively, CS-*nonincreasing*) if it is not CS-decreasing (respectively, CS-increasing). If no merger is implemented, the status quo (or "null merger") obtains, which we denote by M°, resulting in outcome $q(M^\circ) \equiv q^\circ$, $s_i(M^\circ) \equiv q_i^\circ/Q^\circ$, and $\Delta CS(M^\circ) = 0$.

We assume that if merger M_k, $k \in \mathcal{F}$, is proposed, the antitrust authority can observe all aspects of that merger and knows as well the premerger cost levels of all firms (which can be inferred using (1) from knowledge of the demand function and observation of premerger sales). What it does *not* observe are the characteristics of any feasible mergers that are not proposed. We also assume that the antitrust authority can commit ex ante to its policy. As such, the antitrust authority commits to a merger-specific approval policy by specifying an approval (or "acceptance") set $\mathcal{A} \equiv \{M_k : \overline{c}_k \in \mathcal{A}_k\}$, where $\mathcal{A}_k \subseteq [l, h_k]$ for $k \in \mathcal{K}$ are the postmerger marginal cost levels that would lead to approval of a merger with target k. Because of our assumption of full support and no mass points, we can without loss of generality restrict attention to the case in which each $\mathcal{A}_k$ is a (finite or infinite) union of closed intervals possessing nonempty interiors, i.e., $\mathcal{A}_k \equiv \cup_{r=1}^{R}[l_k^r, h_k^r]$, where $l \leq l_k^r < h_k^r \leq h_k$ (R can be zero or infinite). On the other hand, if no merger is proposed the status quo M° remains intact.

Some remarks are in order concerning the policies that we consider: First, we confine attention to deterministic policies. One justification is that it may be hard for the antitrust authority to commit to a random rule. Second, we do not pursue a mechanism design approach. Motivated by the constraints that antitrust authorities face in the real world, we assume that the antitrust authority cannot obtain verifiable information about mergers that are not proposed. Moreover, we assume that only one of the mutually exclusive mergers can be proposed to, and evaluated by, the antitrust authority.

Given a realized set of feasible mergers $\mathcal{F}$ and the antitrust authority's approval set $\mathcal{A}$, the feasible mergers M_k that would be approved if proposed are given by the set $\mathcal{F} \cap \mathcal{A}$. A bargaining process among the firms determines which feasible merger, if any, is actually proposed. Note that this bargaining problem involves externalities as firms' payoffs depend on the identity of the target. We suppose that the bargaining process takes the form of an "offer game," as in Segal (1999), where the acquirer (firm 0)—Segal's principal—makes public take-it-or-leave-it offers. (However, see the end of Section II for a discussion of other bargaining processes and models of competition.)

In Segal (1999), the principal's offers consist of a profile of "trades" $x = (x_1, \ldots, x_K)$, with x_k the trade with agent k. Here, $x_k \in \{0, 1\}$, where $x_k = 1$ if the acquirer proposes a merger with firm k. Hence, in our model Segal's offer game simply amounts to firm 0 being able to make a take-it-or-leave-it offer of an acquisition price t_k to a single firm k of its choosing, where k is such that $M_k \in (\mathcal{F} \cap \mathcal{A})$. (Firm 0 can also choose to make no offer.) If the offer is accepted by firm k, then merger M_k is proposed to the antitrust authority, which will approve it since $M_k \in (\mathcal{F} \cap \mathcal{A})$, and firm 0 acquires the target in return for the payment t_k. If the offer is rejected, or if no offer is made, then no merger is proposed, and no payments are made.

For $k \in \mathcal{K}$, let

$$\Delta\Pi(M_k) \equiv \pi_k(M_k) - [\pi_0^\circ + \pi_k^\circ]$$

denote the change in the bilateral (i.e., joint) profit of the merging parties, firms 0 and k, induced by merger $M_k \in (\mathcal{F} \cap \mathcal{A})$. In what follows, it will also be convenient to define $\Delta\Pi(M^\circ) \equiv 0$, as no bilateral profit gain occurs when no merger happens. By choosing the payment t_k that makes firm k just indifferent between accepting and not, firm 0 can extract the entire bilateral profit gain $\Delta\Pi(M_k)$. Given the realized set of feasible and acceptable mergers, $\mathcal{F} \cap \mathcal{A}$, the merger outcome in the equilibrium of the offer game is therefore given by $M^*(\mathcal{F}, \mathcal{A})$, where

$$M^*(\mathcal{F}, \mathcal{A}) \equiv \begin{cases} \arg\max_{M_k \in (\mathcal{F} \cap \mathcal{A})} \Delta\Pi(M_k) & \text{if } \max_{M_k \in (\mathcal{F} \cap \mathcal{A})} \Delta\Pi(M_k) > 0 \\ M^\circ & \text{otherwise.} \end{cases}$$

That is, the outcome is the feasible and allowable merger M_k that maximizes the induced change in the bilateral profit of firms 0 and k, provided that change is positive; otherwise, the status quo M° remains intact.

In line with legal standards in the United States, the European Union, and many other jurisdictions, we assume that the antitrust authority acts in the consumers' interests. That is, the antitrust authority selects the approval set $\mathcal{A}$ that maximizes expected consumer surplus given that the bargaining outcome is $M^*(\cdot)$:

$$\max_{\mathcal{A}} E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A}))],$$

where the expectation is taken with respect to the set of feasible mergers, $\mathcal{F}$. (See the end of Section II for a discussion of alternative welfare standards.) To make the antitrust authority's problem interesting, and avoid certain degenerate cases, we will henceforth assume the following:

ASSUMPTION 2: *For all $k \in \mathcal{K}$, the support of the postmerger cost distribution includes both CS-increasing and CS-nonincreasing mergers: i.e.,* $\Delta CS(k, h_k) \leq 0 < \Delta CS(k, l)$.

We are interested in studying how the optimal approval set depends on the premerger characteristics of the alternative mergers. For this reason, we assume that the potential targets differ in their premerger marginal costs. To this end, we let $\mathcal{K} \equiv \{1, \ldots, K\}$ and label firms 1 through K in decreasing order of their premerger marginal costs: $c_1 > c_2 > \cdots > c_K$. Thus, in the premerger equilibrium, firm $k \in \mathcal{K}$ produces more than firm $j \in \mathcal{K}$, and has a larger market share, if $k > j$. We will say that merger M_k is *larger* than merger M_j if $k > j$, as the combined premerger market share of firms 0 and k is larger than that of firms 0 and j. Note also that the change in the naively computed Herfindahl index (calculated using premerger shares) from a merger between firms 0 and k is $2s_0^\circ s_k^\circ$.[2] Thus, a larger merger also causes a larger change in this naively computed index.

II. Structure of the Merger Policy Decision

As firms produce a homogeneous good, a merger M_k raises consumer surplus if and only if it increases aggregate output Q. The following lemma summarizes some useful properties of a *CS-neutral* merger M_k, i.e., a merger that leaves consumer surplus unchanged $[\Delta CS(M_k) = 0]$:

LEMMA 1: *Suppose merger M_k is CS-neutral. Then*

(i) *the merger causes no changes in the output of any nonmerging firm $i \notin \{0, k\}$ nor in the joint output of the merging firms* 0 *and* k;

(ii) *the merged firm's margin at the premerger and postmerger price $P(Q^\circ)$ equals the sum of the merging firms' premerger margins:*

$$P(Q^\circ) - \overline{c}_k = [P(Q^\circ) - c_0] + [P(Q^\circ) - c_k]; \tag{4}$$

(iii) *the merger is profitable for the merging firms:* $\Delta\Pi(M_k) > 0$;

(iv) *the merger increases aggregate profit:* $\sum_{i \in \mathcal{N} \setminus \{0\}} \pi_i(M_k) > \sum_{i \in \mathcal{N}} \pi_i^\circ$.

PROOF:

Part (i) follows from stability of equilibrium; part (ii) from the merged firm's first-order condition for profit maximization (3) and from summing the merger partners' premerger first-order conditions (1). Part (iii) is an implication of parts (i)

[2] Specifically, the change in the naively computed Herfindahl index induced by merger M_k is $\Delta H^{naive}(M_k) \equiv (\sum_{i \neq 0,k} (s_i^\circ)^2 + (s_0^\circ + s_k^\circ)^2) - \sum_{i=0}^N (s_i^\circ)^2 = 2s_0^\circ s_k^\circ$.

and (ii): part (ii) implies that the margin earned on each sale is larger for the merged firm than it was premerger for either merger partner. Since, by (i), the total output of the merging firms does not change, their joint profit increases with the merger. As for part (iv), note that the merger raises the bilateral (i.e., joint) profit of the merging firms 0 and k by part (iii), and it leaves the profit of any nonmerging firm unchanged (as neither price nor their output changes).

Rewriting equation (4), merger M_k is CS-neutral if the postmerger marginal cost $\overline{c}_k$ satisfies

$$\overline{c}_k = \hat{c}_k(Q^\circ) \equiv c_k - [P(Q^\circ) - c_0]. \tag{5}$$

As the following standard lemma (proof omitted) shows, reducing the merged firm's marginal cost $\overline{c}_k$ increases not only consumer surplus but also the profit of the merged firm:

LEMMA 2: *Conditional on merger M_k being implemented, a reduction in the postmerger marginal cost $\overline{c}_k$ causes aggregate output, consumer surplus, and the merged firm's profit to increase.*

Thus, conditional on merger M_k being implemented, both $\Delta CS(M_k)$ and $\Delta\Pi(M_k)$—the changes in consumer surplus and bilateral profit of the merging firms—increase when the postmerger marginal cost declines. Combined with (5), this also implies that merger M_k is CS-increasing if $\overline{c}_k < \hat{c}_k(Q^\circ)$ and CS-decreasing if $\overline{c}_k > \hat{c}_k(Q^\circ)$.

The following lemma gives a key result that indicates that there is a systematic bias in firms' proposal incentives in favor of larger mergers, relative to the interests of consumers. This bias arises in our model because an initially lower-cost firm, which produces more output, benefits more from any given cost reduction generated by a merger, and any two mergers that generate the same change in consumer surplus involve the same size cost reduction.

LEMMA 3: *Suppose two mergers, M_j and M_k with $k > j$, induce the same nonnegative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then the larger merger M_k induces a greater increase in the bilateral profit of the merger partners: i.e., $\Delta\Pi(M_k) > \Delta\Pi(M_j) > 0$.*

PROOF:

We observe first that the result is true if mergers M_j and M_k are CS neutral; i.e., if $\Delta CS(M_j) = \Delta CS(M_k) = 0$. In this case, parts (i) and (ii) of Lemma 1 imply that

$$\begin{aligned}\Delta\Pi(M_k) - \Delta\Pi(M_j) &= [(P^\circ - c_k)q_0^\circ + (P^\circ - c_0)q_k^\circ] \\ &\quad - [(P^\circ - c_j)q_0^\circ + (P^\circ - c_0)q_j^\circ] \\ &> 0,\end{aligned} \tag{6}$$

where the inequality follows because $(P^\circ - c_k) > (P^\circ - c_j)$ and $q_k^\circ > q_j^\circ$.

We next show that as the postmerger aggregate output increases above Q°, $\Delta\Pi(M_k) - \Delta\Pi(M_j)$ increases. To see this, define $\overline{c}_i(Q)$ to be the cost level of merged firm $i = j, k$ that results in output Q. From the first-order conditions (2) and (3), this cost level satisfies

$$NP(Q) + P'(Q)Q = \overline{c}_i(Q) + \sum_{r \in \mathcal{N} \setminus \{0,i\}} c_r. \tag{7}$$

Thus, $c_k < c_j$ implies that $\overline{c}_k(Q) < \overline{c}_j(Q)$ for all Q. Also define $q_i(Q)$ and $Q_{-i}(Q)$ to be, respectively, the output of the merged firm $i = j, k$ and the aggregate output of all of its rivals in the associated equilibrium. The first-order conditions (2) imply that

$$(N-1)P(Q) + P'(Q)Q_{-i}(Q) = \sum_{r \in \mathcal{N} \setminus \{0,i\}} c_r. \tag{8}$$

Using the envelope theorem, the derivative of $\Delta\Pi(M_i)$ for $i = j, k$ with respect to a differential change in the postmerger aggregate output Q is

$$\begin{aligned} \frac{d\Delta\Pi(M_i)}{dQ} &= q_i(Q)[P'(Q)Q'_{-i}(Q) - \overline{c}'_i(Q)] \\ &= -q_i(Q)[2NP'(Q) + P''(Q)(2Q - q_i(Q)], \end{aligned} \tag{9}$$

where the second equality follows by substituting

$$\begin{aligned} Q'_{-i}(Q) &= -\left(\frac{1}{P'(Q)}\right)[(N-1)P'(Q) + P''(Q)\,Q_{-i}(Q)] \\ \overline{c}'_i(Q) &= [(N+1)P'(Q) + P''(Q)\,Q], \end{aligned}$$

derived from (7) and (8). Holding Q fixed, expression (9) is larger the greater is the merged firm's output $q_i(Q)$:

$$\frac{\partial}{\partial q}\{-q[2NP'(Q) + P''(Q)(2Q - q)]\} = -2[NP'(Q) + P''(Q)(Q - q)] > 0,$$

where the inequality follows from Assumption 1. Since $\overline{c}_k(Q) < \overline{c}_j(Q)$ for all Q, the first-order condition (3) implies that $q_k(Q) > q_j(Q)$ for all Q. Hence, (9) implies that $d[\Delta\Pi(M_k) - \Delta\Pi(M_j)]/dQ > 0$, which yields the result.

Lemmas 1 to 3 imply that the possible mergers can be represented as shown in Figure 1, panel A (where there are four possible mergers; i.e., $K = 4$). In the figure, the change in the merging firms' bilateral profit, $\Delta\Pi$, is measured on the horizontal axis, and the change in consumer surplus, ΔCS, is measured on the vertical axis. The CS-increasing mergers therefore are those lying above the horizontal axis. The bilateral profit and consumer surplus changes induced by a merger between firms 0 and k, $(\Delta\Pi(M_k), \Delta CS(M_k))$, fall somewhere on the curve labeled "M_k." (The figure shows only the parts of these curves for which the bilateral profit change $\Delta\Pi$ is

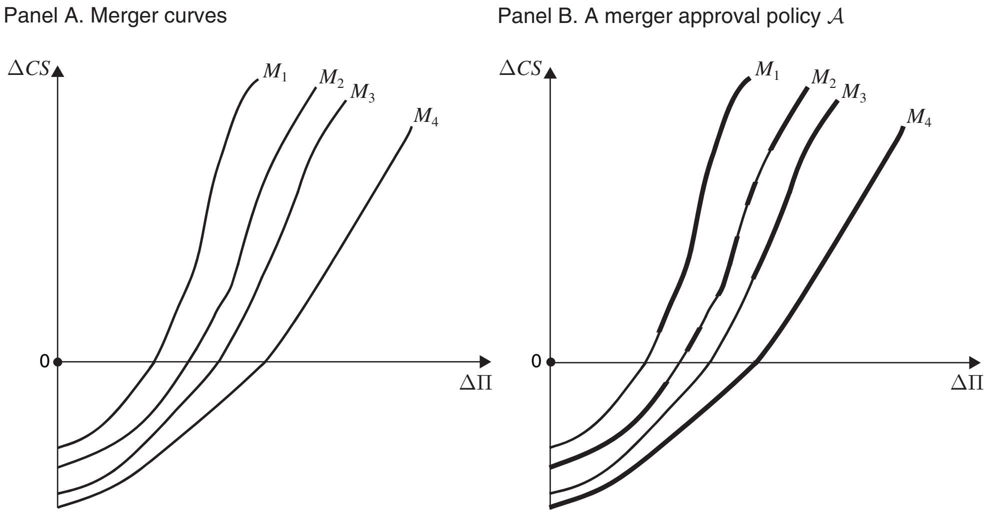

Figure 1

Notes: In panel A, each curve labeled M_j depicts the relationship between the change in consumer surplus and the change in bilateral profit for a merger between firms 0 and j, with each point on the curve corresponding to a different realization of merger M_j's postmerger marginal cost. Panel B depicts in heavy trace a possible merger approval policy $\mathcal{A}$.

nonnegative.) Since by Lemma 1 a CS-neutral merger is profitable for the merger partners, each curve crosses the horizontal axis to the right of the vertical axis. By Lemma 2, the curve for each merger M_k is upward sloping. By Lemma 3, on and above the horizontal axis the curves for larger mergers lie everywhere to the right of those for smaller mergers. (In Figure 1 the curves remain ordered below the horizontal axis, but this need not be the case.)

Note that whenever a CS-nondecreasing merger has a greater bilateral profit change than a larger merger, it also must be better for consumers: i.e., if any two CS-nondecreasing mergers M_j and M_k with $k > j$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta CS(M_k) < \Delta CS(M_j)$ (this is proven formally as Corollary 1 in the Appendix). However, if instead the larger merger has a greater bilateral surplus [i.e., if $\Delta\Pi(M_k) > \Delta\Pi(M_j)$], consumers may be better off with the smaller merger.

Figure 1, panel B shows a possible merger approval policy $\mathcal{A}$: the mergers M_k that would be approved are contained in the sections of the curves with heavy trace (the figure also shows the status quo, located at the origin). Given this policy, for any given realization of feasible mergers $\mathfrak{F}$ such that some CS-nondecreasing merger is feasible, the merger outcome $M^*(\mathfrak{F}, \mathcal{A})$ is the merger (or M°) that lies furthest to the right (having the largest increase in bilateral profit).

The characterization of optimal merger policy we present in Sections III and IV depends only on the structure of the antitrust authority's policy design problem shown in Figure 1. As such, our characterization applies to many settings in addition to the one captured by our baseline model. (Readers interested only in our baseline model can skip ahead to Section III.) In particular, suppose that each merger $M_k = (k, \overline{c}_k)$ is summarized by the identity of the acquirer and a "characteristic" $\overline{c}_k \in \mathbb{R}$ and results in a change in welfare $\Delta W(M_k)$ according to the antitrust authority's objective.[3] The status quo M° has $\Delta W(M^{\circ}) = 0$. As for bargaining, the offer game considered above is one example of what might be called a *scoring-rule bargaining process.* In a scoring-rule bargaining process, each merger M_k has a score $S(M_k)$ that is continuous in $\overline{c}_k$, and the merger that is proposed is the one with the highest score provided that is positive; otherwise the status quo M° (for which $S(M^{\circ}) \equiv 0$) obtains; that is,

$$M^*(\mathcal{F}, \mathcal{A}) \equiv \begin{cases} \arg\max_{M_k \in (\mathcal{F} \cap \mathcal{A})} S(M_k) & \text{if } \max_{M_k \in (\mathcal{F} \cap \mathcal{A})} S(M_k) > 0 \\ M^{\circ} & \text{otherwise.} \end{cases}$$

(In the offer game, the score S is the bilateral surplus $\Delta\Pi$.) A model with a scoring-rule bargaining process leads to the same structure for the antitrust authority's problem as in our model (and a figure like Figure 1, but with ΔW on the vertical axis and S on the horizontal axis) provided the following three properties hold:

- **Monotonicity:** $S(M_k)$ and $W(M_k)$ are decreasing in $\overline{c}_k$ at all $\overline{c}_k$ such that $\Delta W(M_k) \geq 0$.
- **Willingness to Propose:** $S(M_k) > 0$ if $\Delta W(M_k) = 0$.
- **Ordered Bias:** For any $k > j$, if $\Delta W(M_j) = \Delta W(M_k) \geq 0$ then $S(M_k) > S(M_j)$.

Monotonicity implies that a welfare-enhancing merger becomes more likely to be proposed if it becomes more attractive to the antitrust authority (due to a decrease in $\overline{c}_k$). Willingness to Propose says that a welfare-neutral merger will be proposed if it is the only feasible merger. Combined with Monotonicity, it implies that the antitrust authority could achieve the first-best if there were at most one feasible merger ($K = 1$) since any merger it would want to approve would be proposed in that case [i.e., $\Delta W(M_k) \geq 0$ implies $S(M_k) > 0$]. Ordered Bias says that there is an ordering of mergers such that the larger merger will be proposed when two mergers are equally attractive to the antitrust authority. It implies that the first-best cannot be achieved if $K > 1$.

Our characterization results in Sections III and IV can be applied whenever these three conditions are satisfied. For example, the following settings all satisfy these conditions:[4]

Efficient Bargaining: Suppose that bargaining is instead efficient, selecting the merger that maximizes industry profit.[5] Then the scoring rule $S(\cdot)$ is simply the change in aggregate industry profit, which we denote by $\Delta\Pi_I(M_k)$. (The welfare criterion

[3] We suppose as well that $\overline{c}_k$ is drawn from a distribution with full support and no mass points, that $\Delta W(M_k)$ is continuous in $\overline{c}_k$, and that $\Delta W(M_k)$ can be either positive or negative with positive probability.

[4] In the online Appendix we also identify situations in which these conditions hold for a collection of mutually exclusive mergers in which no single firm is part of all possible mergers. In addition, we discuss the case in which mergers may also result in reductions in fixed costs.

[5] The case of efficient bargaining is closest to the model of Armstrong and Vickers (2010) since the industry then acts like a single agent in its proposal behavior. However, even in this single agent case our model differs from Armstrong and Vickers in the fact that a proposed merger may be drawn from one of K different identifiable distributions, whose distinct treatment is our central focus. A model similar to Armstrong and Vickers would emerge if instead a fixed number of merger "ideas" were drawn i.i.d. from a distribution over all K curves (with a given firm able to receive more than one idea). However, such a model would feature an unattractive negative correlation in

ΔW is still ΔCS.) There are several bargaining processes that would lead to aggregate profit maximization:

(i) "Coasian bargaining" among all firms under complete information.

(ii) A "menu auction" in which each firm $i \neq 0$ submits a nonnegative bid $b_i(M_k) \geq 0$ to firm 0 for each merger $M_k \in (\mathcal{F} \cap \mathcal{A})$ and the status quo M°, and firm 0 then selects the merger outcome (possibly M°) that maximizes its profit, inclusive of these bids. Bernheim and Whinston (1986) show that there is an efficient equilibrium which, in this setting, implements the merger that maximizes aggregate profit.

(iii) Firm 0 committing to a sales mechanism. Jehiel, Moldovanu, and Stacchetti (1996) show that an optimal mechanism has the following structure in our setting: Firm 0 proposes to implement the aggregate profit-maximizing merger outcome $M^*(\mathcal{F}, \mathcal{A})$ and requires the payment $\pi_i\big(M^*(\mathcal{F}, \mathcal{A})\big) - \pi_i(\underline{M}_i)$ from each firm $i \neq 0$, where $\underline{M}_i$ is the merger in set $(\mathcal{F} \cap \mathcal{A})\backslash M_i$ or status quo M° that minimizes firm i's profit. If a firm i does not accept participation in the mechanism when all other firms do, then the principal commits to proposing merger $\underline{M}_i$ to the antitrust authority.[6] Given this mechanism, there is an equilibrium in which all firms participate in the mechanism, and the merger outcome is $M^*(\mathcal{F}, \mathcal{A})$.[7]

(iv) When $K = N = 2$, a first-price sealed-bid auction for firm 0 in which each firm $k \geq 1$ submits a nonnegative bid. Whenever $\mathcal{A} \subseteq \{M_k : \Delta W(M_k) \geq 0\}$ (as will always be the case), if two firms have feasible and allowable mergers, the unique undominated equilibrium of this auction has both firms bid $\min_k[\pi_k(M_k) - \pi_k(M_{-k})]$, and the firm whose merger maximizes aggregate profit wins. Moreover, this same firm wins in all Nash equilibria.

With efficient bargaining, Monotonicity holds whenever premerger cost differences are small enough that for any $k \in \mathcal{K}$ the sum of the premerger market share of firms 0 and k weakly exceeds the premerger share of any other firm, i.e., $s_0^\circ + s_k^\circ \geq \max_{j \neq 0,k} s_j^\circ$. To see why, note that multiplying the postmerger first-order condition for each firm j under merger M_k (condition (2) or (3)) by $q_j(M_k)$ and summing over j yields

$$\sum_{i \in \mathcal{N} \backslash \{0\}} \pi_i(M_k) = \big| Q(M_k)^2 P'\big(Q(M_k)\big) \big| H(M_k), \tag{10}$$

efficiency realizations across firms: learning that one merger M_k can achieve large synergies would imply that other mergers are unlikely to achieve significant synergies.

[6] Similar to Bernheim and Whinston's (1986) menu auction, firms $i \neq 0$ make payments even when they are not party to a merger.

[7] To see that firm 0 wants to propose merger $M^*(\mathcal{F}, \mathcal{A})$, note that using this type of mechanism its optimal merger proposal solves $\max_{M_k \in (\mathcal{F} \cap \mathcal{A})} \sum_{i \in \mathcal{N} \backslash \{0\}} \pi_i(M_k) - \sum_{i \neq 0} \pi_i(\underline{M}_i)$ (which is equivalent to $\max_{M_k \in (\mathcal{F} \cap \mathcal{A})} \sum_{i \in \mathcal{N} \backslash \{0\}} \pi_i(M_k)$), provided this exceeds $\sum_{i \in \mathcal{N}} \pi_i^\circ - \sum_{i \neq 0} \pi_i(\underline{M}_i)$, and is M° otherwise.

where $H(M_k) \equiv \sum_{i \in \mathcal{N} \setminus \{0\}} (s_i(M_k))^2$ is the postmerger industry Herfindahl index. Assumption 1 ensures that the first term, $|Q^2 P'(Q)|$, is increasing in Q. By Lemma 2, a reduction in postmerger marginal cost $\overline{c}_k$ leads to a larger $Q(M_k)$, so $\Delta\Pi_I(M_k) = \sum_{i \in \mathcal{N} \setminus \{0\}} \pi_i(M_k) - \sum_{i \in \mathcal{N}} \pi_i^\circ$ is decreasing in $\overline{c}_k$ if reducing the merged firm's marginal cost $\overline{c}_k$ induces an increase in $H(M_k)$. Under Assumption 1, a decrease in $\overline{c}_k$ increases the share of the merged firm and decreases the share of every other firm. Since $s_0^\circ + s_k^\circ \geq \max_{j \neq 0,k} s_j^\circ$ implies $s_k(M_k) \geq \max_{j \neq 0,k} s_j(M_k)$ for any CS-nondecreasing merger M_k, this induced change in market shares increases the postmerger Herfindahl index $H(M_k)$ (see Lemma 5 in the online Appendix). Thus, $\Delta\Pi_I(M_k)$ is decreasing in $\overline{c}_k$ if premerger cost differences are small enough, while $\Delta CS(M_k)$ is always decreasing in $\overline{c}_k$ by Lemma 2.

Willingness to Propose holds because, by Lemma 1, a CS-neutral merger M_k raises not only the bilateral profit of the merger partners but also aggregate profit.

Finally, Lemma 4 in the Appendix shows that the Ordered Bias condition is also satisfied in this case: if two mergers, M_j and M_k with $k > j$, induce the same non-negative change in consumer surplus $[\Delta CS(M_j) = \Delta CS(M_k) \geq 0]$, then the larger merger M_k induces a greater increase in aggregate profit: $\Delta\Pi_I(M_k) > \Delta\Pi_I(M_j)$.[8]

Aggregate Surplus Standard: Suppose that bargaining is efficient and the antitrust authority's welfare criterion is a weighted average of consumer surplus and aggregate surplus, so that the welfare change from merger M_k is

$$\Delta W(M_k) \equiv (1 - \lambda)\Delta CS(M_k) + \lambda \Delta AS(M_k) = \Delta CS(M_k) + \lambda \Delta\Pi_I(M_k), \tag{11}$$

where $\lambda \in [0,1]$ and $\Delta AS(M_k) \equiv \Delta CS(M_k) + \Delta\Pi_I(M_k)$ is the change in aggregate surplus.

Consider, first, Monotonicity. In general, neither ΔW nor $\Delta\Pi_I$ need increase when a firm's cost decreases. However, both must be decreasing in $\overline{c}_k$ for mergers that are W-nondecreasing if premerger marginal cost differences are sufficiently small. To see this, consider the extreme case where all firms have the same premerger marginal cost c. Then, for merger M_k to be W-nondecreasing, it must involve synergies: i.e., we must have $\overline{c}_k < c$.[9] Hence, if M_k is W-nondecreasing, then after merger M_k the merged firm is the firm with the lowest marginal cost. Reducing the merged firm's marginal cost $\overline{c}_k$ therefore increases both aggregate output Q (thereby raising $|Q^2 P'(Q)|$) and the Herfindahl index H, which from equation (10) increases $\Delta\Pi_I(M_k)$. Moreover, since $\Delta CS(M_k)$ increases, so does $\Delta W(M_k)$. By continuity of consumer surplus and aggregate industry profit in marginal costs,

[8] In the online Appendix we also show that when the bargaining outcome maximizes the joint profit of a subset of firms containing at least all the firms in $\mathcal{K}$, then these conditions hold as well. For example, this would be the case if $N > K = 2$ and the two firms in $\mathcal{K}$ engage in a first-price sealed-bid auction for firm 0.

[9] To see this, note first that if $\Delta W(M_k) \geq 0$ then $\Delta AS(M_k) > 0$: if $\Delta CS(M_k) < 0$ this follows immediately from (11), while if $\Delta CS(M_k) \geq 0$ then we know from the discussion of efficient bargaining that $\Delta\Pi_I(M_k) > 0$, implying again that $\Delta AS(M_k) > 0$. Now suppose, contrary to the claim, that $\overline{c}_k \geq c$. We can decompose the induced change in market structure into two steps: (i) a move from $N + 1$ to N firms, each with marginal cost c, and (ii) an increase in the marginal cost of one firm from c to $\overline{c}_k \geq c$. Step (i) induces a reduction in aggregate output but does not affect average production costs, so it reduces aggregate surplus. Step (ii) weakly reduces aggregate output and weakly increases average costs in the industry, so it weakly reduces aggregate surplus. Since aggregate surplus declines, so must $W(M_k)$—a contradiction to the assumption that merger M_k is W-nondecreasing.

it follows that if premerger marginal cost differences are sufficiently small, then $\Delta W(M_k) \geq 0$ implies that both $\Delta\Pi_I(M_k)$ and $\Delta W(M_k)$ are decreasing in $\overline{c}_k$.

Willingness to Propose holds for small enough cost differences because, in that case, if $\Delta W(M_k) = 0$ then $\Delta CS(M_k) < 0$ and $\Delta AS(M_k) > 0$ (see footnote 9), which implies that $\Delta\Pi_I(M_k) > 0$.

Finally, Ordered Bias holds as well: suppose that two W-nondecreasing mergers, M_j and M_k with $k > j$, induce the same change in W but the smaller merger j has the weakly larger aggregate profit level, $\Delta\Pi_I(M_j) \geq \Delta\Pi_I(M_k)$. Lemma 4 (referred to above and proven in the Appendix) would then imply that $\Delta CS(M_j) > \Delta CS(M_k)$. But then $\Delta W(M_j) > \Delta W(M_k)$, yielding a contradiction. Thus, we must have $\Delta\Pi_I(M_k) > \Delta\Pi_I(M_j)$.

Differentiated Price Competition: In the online Appendix we show that these conditions hold as well when firms instead compete in prices and each initially produces a single symmetrically differentiated good with consumers having CES or multinomial logit demand, provided that the antitrust authority has a consumer surplus standard and bargaining is efficient. The argument relies on the fact that, like the Cournot model, both of these differentiated price competition models are "aggregative games" (Corchon 1994). Moreover, if instead bargaining takes the form of the offer game, then the conditions hold provided that the potential consumer surplus gains are not too large. This follows because for CS-neutral mergers the change in aggregate profit equals the change in the bilateral profit of the merging firms (nonmerging firms are unaffected), so close enough to the horizontal axis in Figure 1 the Willingness to Propose and Ordered Bias conditions must hold (Monotonicity is always satisfied).

III. Optimal Merger Policy

We can now turn to the optimal policy of the antitrust authority. Recall that the antitrust authority can without loss restrict itself to approval sets in which the set of acceptable cost levels for a merger between firm 0 and each firm k, $\mathcal{A}_k \subseteq [l, h_k]$, is a union of closed intervals with nonempty interiors. Throughout we restrict attention to such policies.[10] Let $\overline{a}_k \equiv \max\{\overline{c}_k : \overline{c}_k \in \mathcal{A}_k\}$ denote the largest allowable postmerger cost level for a merger (i.e., the "marginal merger") between firms 0 and k. Also let $\underline{\Delta CS}_k \equiv \Delta CS(k, \overline{a}_k)$ and $\underline{\Delta\Pi}_k \equiv \Delta\Pi(k, \overline{a}_k)$ denote the changes in consumer surplus and bilateral profit, respectively, induced by that marginal merger. These are the lowest levels of consumer surplus and bilateral profit in any allowable merger between firms 0 and k. Note that $\overline{a}_k$ (and thus $\underline{\Delta CS}_k$ and $\underline{\Delta\Pi}_k$) is defined only if merger M_k is approved with positive probability, i.e., only if $k \in \mathcal{K}^+ \equiv \{k : \mathcal{A}_k \neq \varnothing\}$.

At first glance, one may be tempted to conjecture that the antitrust authority can achieve its goal by simply approving any proposed merger that is CS-nondecreasing, i.e., for every $k \geq 1$, setting $\mathcal{A}_k = [l, \overline{a}_k]$, where $\overline{a}_k$ is such that $\Delta CS(k, \overline{a}_k) = 0$. Figure 2,

[10] Thus, when we state that any optimal policy must have a particular form, we mean any optimal interval policy of this sort. There are other optimal policies that add or subtract in addition some measure zero sets of mergers, since these have no effect on expected consumer surplus.

Panel A. Naïve approval set $\mathcal{A}$ Panel B. Improvement $\mathcal{A}'$

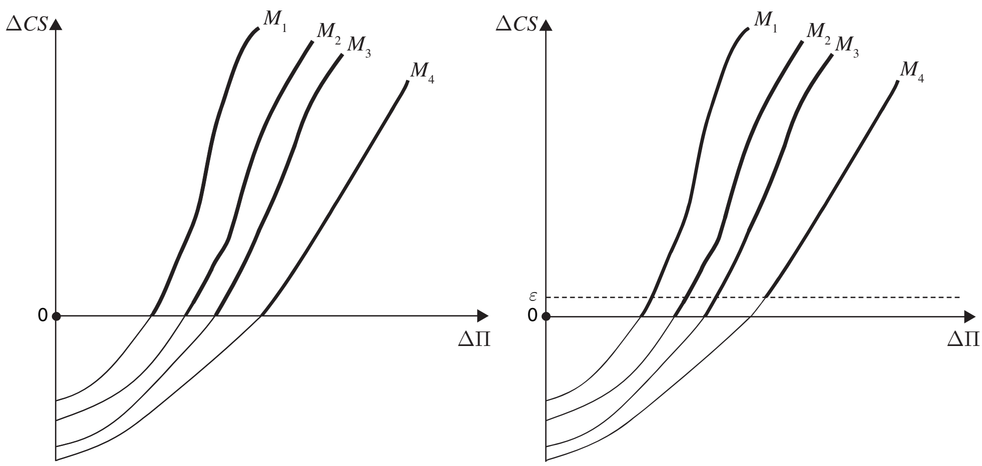

Figure 2

Notes: The "naïve" policy $\mathcal{A}$ that accepts all mergers that do not decrease consumer surplus is not optimal. Here, requiring a strictly positive increase in consumer surplus to approve merger M_4, as in policy $\mathcal{A}'$, raises expected consumer surplus.

panel A illustrates such a policy (with heavy trace) for a case in which $K = 4$. In fact, this is not an optimal policy. To see this, suppose the antitrust authority instead adopts an approval policy $\mathcal{A}'$ that imposes a slightly tougher standard on the largest merger: setting $\mathcal{A}'_k = \mathcal{A}_k$ for each merger $k < 4$, and setting $\mathcal{A}'_4 = \{M_4 : \Delta CS(M_4) \geq \varepsilon\}$ for $\varepsilon > 0$ sufficiently small. This acceptance set is shown in Figure 2, panel B. The two policies differ only in the event that the merger outcome under approval policy $\mathcal{A}$, $M^*(\mathcal{F}, \mathcal{A})$, lies in set $\mathcal{A} \backslash \mathcal{A}'$, i.e., only when $M^*(\mathcal{F}, \mathcal{A}) = M_4$ and $\Delta CS(M_4) \in [0, \varepsilon)$. Conditional on this event, the expected change in consumer surplus under approval policy $\mathcal{A}$ is bounded from above by ε, which approaches zero as ε becomes small. Under the alternative approval policy $\mathcal{A}'$, and conditioning on the same event, the firms will propose the next-most profitable acceptable merger (which must involve a target $k < 4$ or no merger). Since the two policies do not differ in their acceptance sets for such smaller mergers, the expected change in consumer surplus under $\mathcal{A}'$ thus converges to $E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F} \backslash M_4, \mathcal{A})) \mid \Delta\Pi(M^*(\mathcal{F} \backslash M_4, \mathcal{A})) \leq \underline{\Delta\Pi}_4] > 0$ as ε becomes small.[11] Hence, the expected change in consumer surplus is larger under $\mathcal{A}'$ than under the naive approval policy $\mathcal{A}$.

Since the naive policy of approving any CS-nondecreasing merger is not optimal, how should the antitrust authority construct its approval policy to maximize expected consumer surplus? Our main result is the following:[12]

PROPOSITION 1: *Any optimal approval policy $\mathcal{A}$ approves the smallest merger if and only if it is CS nondecreasing, approves only mergers $k \in \mathcal{K}^+ \equiv \{1, \ldots, \hat{K}\}$ with positive probability ($\hat{K}$ may equal K), and satisfies $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < \cdots < \underline{\Delta CS}_{\hat{K}}$ for all $k \leq \hat{K}$. That is, the lowest level of consumer surplus change that is*

[11] Recall that $\Delta\Pi(M^\circ) \equiv 0$. Hence, $\Delta\Pi(M^*(\mathcal{F} \backslash M_4, \mathcal{A})) = 0$ if $M^*(\mathcal{F} \backslash M_4, \mathcal{A}) = M^\circ$.

[12] Existence of an optimal approval policy $\mathcal{A}$ follows from Theorem 1 in Holmstrom (1984).

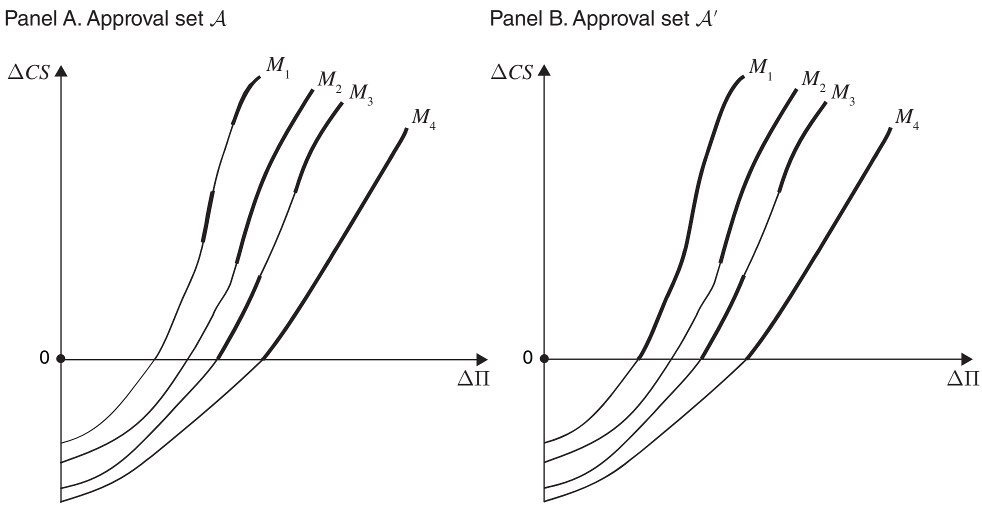

Figure 3

Note: Changing the approval set $\mathcal{A}$ by approving the smallest merger M_1 whenever it does not reduce consumer surplus, resulting in approval set $\mathcal{A}'$, raises expected consumer surplus.

acceptable to the antitrust authority equals zero for the smallest merger M_1, is strictly positive for every other merger M_k, and is monotonically increasing in the size of the merger, while the largest merger(s) may never be approved.

According to Lemma 3, there is a systematic misalignment between firms' proposal incentives and the interests of the antitrust authority: firms have an incentive to propose a merger that is larger (in terms of the target's premerger size) than the one that maximizes consumer surplus. Proposition 1 shows that to compensate for this intrinsic bias in firms' proposal incentives, the antitrust authority optimally adopts a higher minimum CS standard the larger is the proposed merger. Here we give a heuristic derivation of the result; see the formal proof in the Appendix for details. We organize our discussion in "steps" corresponding to those in the formal proof in the Appendix.

Step 1: Observe, first, that the optimal policy $\mathcal{A}$ does not approve CS-decreasing mergers: removing all CS-decreasing mergers from the approval set assures that consumer surplus does not decline, without changing the outcome when none of those CS-decreasing mergers would have been proposed.

Step 2: Next, note that every CS-nondecreasing smallest merger (M_1) must be included in the optimal approval set. If not, as in the set $\mathcal{A}$ depicted in heavy trace in Figure 3, panel A, we could change the approval set $\mathcal{A}$ by adding all CS-nondecreasing mergers M_1, resulting in the alternative approval set $\mathcal{A}'$ depicted in heavy trace in Figure 3, panel B. This change of approval sets matters only in the event in which, under $\mathcal{A}'$, a CS-nondecreasing merger M_1 would be proposed and approved while, under $\mathcal{A}$, this merger would not be approved, resulting instead in the next-most profitable (in terms of bilateral profit) allowable merger (which may

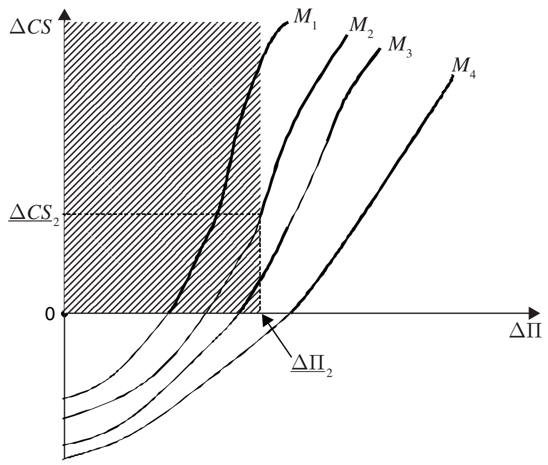

FIGURE 4

Notes: The optimal approval policy is such that the increase in consumer surplus induced by the marginal merger M_k (shown here as $\underline{\Delta CS}_2$ for $k = 2$) equals the expected consumer surplus change from the next-most profitable acceptable merger, conditional on the marginal merger being the most profitable merger in the set of feasible and acceptable mergers. The next-most profitable acceptable merger must therefore lie in the shaded region.

be the merger M°).[13] As we have already noted, this next-most profitable allowable merger must increase consumer surplus by less than merger M_1 (see Corollary 1 in the Appendix). Hence, expected consumer surplus is higher under the alternative approval set $\mathcal{A}'$ than under $\mathcal{A}$.

Step 3: In any optimal approval set $\mathcal{A}$, the consumer surplus level of the marginal merger $M_k = (k, \overline{a}_k)$, $k \in \mathcal{K}^+$, equals the expected CS-level of the next-most profitable acceptable merger, which we write as

$$E_k^{\mathcal{A}}(\overline{c}_k) \equiv E_{\mathcal{F}}\big[\Delta CS\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \,|\, M_k = (k, \overline{c}_k) \text{ and } M_k = M^*(\mathcal{F}, \mathcal{A})\big]$$
$$= E_{\mathcal{F}}\big[\Delta CS\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \,|\, M_k = (k, \overline{c}_k)$$
$$\text{and } \Delta\Pi\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \leq \Delta\Pi(M_k)\big].$$

That is, $\underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k)$ for all $k \in \mathcal{K}^+$. For example, in Figure 4, $\underline{\Delta CS}_2$ must equal $E_2^{\mathcal{A}}(\overline{a}_2)$, which is the expected level of ΔCS conditional on the next-most profitable merger being in the shaded region. To see why this indifference condition must hold, suppose first that the consumer surplus level of the marginal merger M_k is less than the expected consumer surplus level of the next-most profitable acceptable merger, i.e., $\underline{\Delta CS}_k < E_k^{\mathcal{A}}(\overline{a}_k)$. Consider changing the approval set $\mathcal{A}$ by removing all mergers M_k with $\overline{c}_k \in (\overline{a}_k - \varepsilon, \overline{a}_k]$, thereby increasing $\underline{\Delta CS}_k$. For $\varepsilon > 0$ sufficiently small,

[13] Henceforth, whenever we refer to the "next-most profitable allowable merger" or "next-most profitable acceptable merger" we mean to include the possibility that this would be the "null merger" (i.e., the status quo) M°, which has $\Delta\Pi(M^\circ) \equiv 0$.

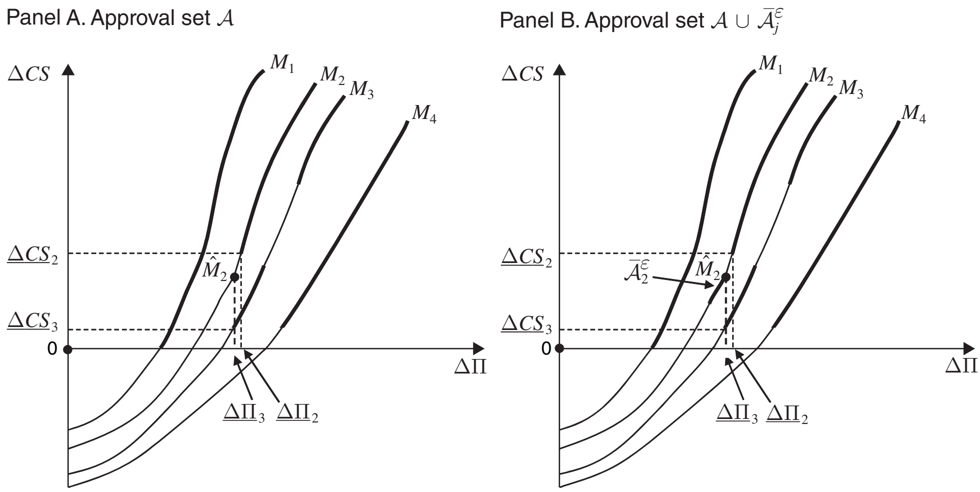

Figure 5

Note: Panel A shows a situation where $\underline{\Delta\Pi}_k$ is not increasing in k; panel B shows an improvement in the approval set.

this change in the approval set increases expected consumer surplus.[14] Similarly, if $\underline{\Delta CS}_k > E_k^{\mathcal{A}}(\overline{a}_k)$, the antitrust authority can increase expected consumer surplus by adding to the approval set all mergers M_k with $\overline{c}_k \in (\overline{a}_k, \overline{a}_k + \varepsilon)$ for $\varepsilon > 0$ sufficiently small.[15]

Step 4: Next, we can see that any optimal approval policy $\mathcal{A}$ has the property that the increase in bilateral profit induced by a marginal merger is at least as large for larger mergers: that is, $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_k$ for $j < k$, $j, k \in \mathcal{K}^+$. Panel A of Figure 5, where $\underline{\Delta\Pi}_2 > \underline{\Delta\Pi}_3$, depicts a situation where this property is not satisfied. Intuitively, the merger $\hat{M}_2$ directly above the marginal merger $(3, \overline{a}_3)$ has a higher level of ΔCS than does $(3, \overline{a}_3)$, while resulting in the same expected ΔCS if it is rejected. Hence, if $(3, \overline{a}_3)$ is approved, so should be $\hat{M}_2$ or, more precisely, so should those in the set $\overline{\mathcal{A}}_2^{\varepsilon}$ (for small ε) shown in Figure 5B.

Step 5: Next, we can show that in any optimal approval policy $\mathcal{A}$, the consumer surplus increase induced by the marginal merger is strictly greater for larger mergers, i.e., $\underline{\Delta CS}_j < \underline{\Delta CS}_k$ for $j < k$, $j, k \in \mathcal{K}^+$. A situation in which this is not true is illustrated in Figure 6, where $\underline{\Delta CS}_2 \geq \underline{\Delta CS}_3$. By the indifference condition of Step 3, $\underline{\Delta CS}_3$ must equal the expected ΔCS of the next-most profitable allowable merger, i.e., $\underline{\Delta CS}_3 = E_3^{\mathcal{A}}(\overline{a}_3)$. Now, this expectation is the weighted average of the expected ΔCS in two events. First, the next-most profitable allowable merger, say M', may be more profitable than the marginal merger $(2, \overline{a}_2)$, i.e., $\Delta\Pi(M') \in [\underline{\Delta\Pi}_2, \underline{\Delta\Pi}_3)$. In this event, M' must (by Step 4) involve a smaller target (either firm 1 or 2). Hence, the expected ΔCS in this event strictly exceeds $\underline{\Delta CS}_2$.

[14] Note that $k \in \mathcal{K}^+$ implies that $\overline{a}_k > l$, so that $\overline{a}_k - \varepsilon > l$ for $\varepsilon > 0$ sufficiently small.

[15] By Step 1 and Assumption 2, we have $\overline{a}_k < h_k$, implying that $\overline{a}_k + \varepsilon < h_k$ for $\varepsilon > 0$ sufficiently small.

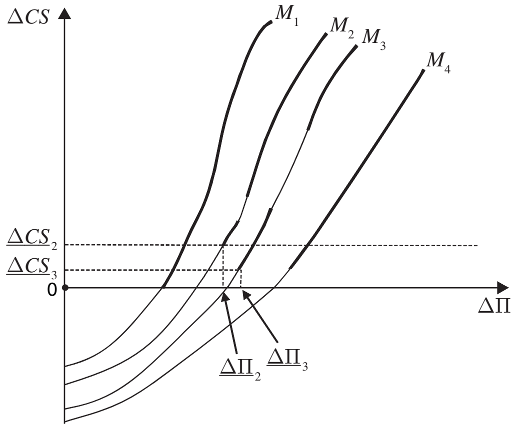

FIGURE 6

Notes: The optimal approval set is such that the consumer surplus increase induced by the marginal merger M_j is less than that by the marginal larger merger M_k, $k > j$, i.e., $\underline{\Delta CS}_j < \underline{\Delta CS}_k$. In the figure, $\underline{\Delta CS}_2 \geq \underline{\Delta CS}_3$, which is a violation of that property.

Second, the next-most profitable acceptable merger M' may be less profitable than the marginal merger $(2, \overline{a}_2)$, i.e., $\Delta\Pi(M') < \underline{\Delta\Pi}_2$. By the indifference condition of Step 4, the expected ΔCS in this event equals $\underline{\Delta CS}_2$. Taking the weighted average of these two events, we conclude that $\underline{\Delta CS}_3 = E_3^{\mathcal{A}}(\overline{a}_3) > \underline{\Delta CS}_2$, a contradiction.

Step 6: Finally, we argue that if there is a merger M_j that will never be approved under the optimal policy $\mathcal{A}$, then no larger merger M_k, $k > j$, will ever be approved either: that is, $k \notin \mathcal{K}^+$ implies $k+1 \notin \mathcal{K}^+$. The result follows by observing that the sum of marginal costs after merger (k, l) is lower than that after merger $(k+1, l)$, which implies (by (7)) that the largest possible improvement in consumer surplus under merger M_k, $\Delta CS(k, l)$, is decreasing in k (as shown in all of the figures). By arguments similar to those showing the monotonicity of $\underline{\Delta CS}_k$ in k for $k \in \mathcal{K}^+$, this implies that if merger M_k is never approved, then neither is any merger that is larger than M_k.

Remark 1: In our analysis, we have assumed that the support of the postmerger marginal cost $\overline{c}_k$ is given by $[l, h_k]$. That is, the lower bound on $\overline{c}_k$, denoted l, is the same for all mergers M_k. In the proof of Proposition 1, this assumption was used only in the last step to prove that if merger M_j is never approved, then no larger merger M_k, $k > j$, will ever be approved either. If we allow for a merger-specific lower bound l_k, Proposition 1 continues to hold as long as $\Delta CS(k, l_k) > \Delta CS(k+1, l_{k+1})$ for all $1 \leq k < K$ (which is implied by, but does not imply, $l_k \leq l_{k+1}$). In the general case where no restrictions on the merger-specific lower bounds are imposed, the main conclusion of Proposition 1 carries over: in the optimal approval policy, the smallest merger M_1 is approved if and only if it is CS-nondecreasing, while the minimum ΔCS necessary for any larger merger to be accepted is strictly positive and greater for larger mergers, i.e., $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_j < \underline{\Delta CS}_k$ for any mergers $j, k \in \mathcal{K}^+ \backslash \{1\}$ for which $j < k$. Moreover, if merger M_j is never approved, while the larger merger M_k having $k > j$ is approved with positive probability, then the maximum possible consumer surplus increase induced by merger M_j is less than the minimum consumer surplus increase necessary for approval of the larger merger M_k. That is, if $j \notin \mathcal{K}^+$ and $k \in \mathcal{K}^+$ for $j < k$, then $\Delta CS(j, l_j) < \underline{\Delta CS}_k$.

IV. Cutoff Policies

Proposition 1 shows that in any optimal policy the least efficient acceptable merger involving a target k [the marginal merger $M_k = (k, \overline{a}_k)$] involves a larger increase in consumer surplus (and larger increase in bilateral profit) the larger is the target. Moreover, the result holds for any distributions of postmerger marginal costs. However, it does not fully characterize those marginal mergers. Indeed, while we know that the marginal merger $M_k = (k, \overline{a}_k)$ satisfies the indifference condition $\Delta CS(M_k) = E_k^{\mathcal{A}}(\overline{a}_k)$, the expectation $E_k^{\mathcal{A}}(\overline{a}_k)$ depends on the acceptance sets for mergers other than k (i.e, on $\mathcal{A}_j$, $j \neq k$), whose optimal forms depend in turn on merger k's acceptance set $\mathcal{A}_k$.

Identifying the marginal merger for each target would be much simpler if we knew that the optimal policy had a "cutoff" structure, in which, for each target k, any mergers with greater efficiencies than the marginal merger are accepted. Specifically, a cutoff policy $\mathcal{A}^C$ is defined by a set of marginal cost cutoffs, $(\overline{a}_1^C, \ldots, \overline{a}_K^C)$, such that $M_k = (k, \overline{c}_k) \in \mathcal{A}^C$ if and only if $\overline{c}_k \leq \overline{a}_k^C$. In that case, Proposition 1 would imply that the marginal mergers could be found by a simple recursive procedure: accept all CS-nondecreasing mergers M_1 $\big($i.e., set $\overline{a}_1^C = \hat{c}_1(Q^\circ)\big)$, then for $k = 2, \ldots, K$ recursively identify the largest postmerger cost level $\overline{a}_k^C$ for which $\Delta CS_k(k, \overline{a}_k^C) = E_k^{\mathcal{A}^C}(\overline{a}_k^C)$, where now the expectation $E_k^{\mathcal{A}^C}(\overline{a}_k^C)$ depends only on the already determined cutoffs for mergers $M_1, \ldots, M_{k-1}$. If $\Delta CS(k, \overline{c}_k) < E_k^{\mathcal{A}^C}(\overline{c}_k)$ for all $\overline{c}_k \in [l, h_k]$, then no such cutoff exists for merger M_k, so that $\mathcal{A}_k^C = \varnothing$. Moreover, this will also imply that $\mathcal{A}_{k'}^C = \varnothing$ for all $k' > k$.

Unfortunately, however, as the following example illustrates, the optimal policy need not have a cutoff structure.

EXAMPLE 1: *Consider a four-firm industry* (*so* $N = 3$) *in which industry inverse demand is* $P(Q) = 1 - Q$. *Premerger marginal costs are* $c_0 = c_2 = 0.5$, $c_1 = 0.55$, *and* $c_3 = 0.45$, *so the premerger market shares are* $s_0 = s_2 = 1/4$, $s_1 = 1/8$, *and* $s_4 = 3/8$. *Premerger consumer surplus is* 0.8. *Firm* 0 *can merge with either firm* 1 *or firm* 2 (*so* $K = 2$). *Each of these mergers is always feasible; i.e.,* $\theta_1 = \theta_2 = 1$. *Merger* M_2 *results in a postmerger marginal cost that has a continuous density on* $[0.2, 0.5]$, *while merger* M_1*'s postmerger marginal cost is* 0.4 *with probability* 0.1 *and* 0.3 *with probability* 0.9.[16] *For these two mergers* M_1, $\big(\Delta\Pi(1, 0.4), \Delta CS(1, 0.4)\big) = (0.0227, 0.0051)$ *and* $\big(\Delta\Pi(1, 0.3), \Delta CS(1, 0.3)\big) = (0.0564, 0.0157)$, *and their unconditional expected* ΔCS *is* 0.0146. *It is straightforward to verify that, in this case, the optimal approval policy* $\mathcal{A}^*$ *is such that* $\mathcal{A}_1^* = \{0.3, 0.4\}$ *and* $\mathcal{A}_2^* = [0.2, 0.260] \cup [0.298, 0.391]$ *where* $\Delta CS(2, 0.391) = 0.0051$, $\Delta CS(2, 0.260) = 0.0146$, *and* $\Delta\Pi(2, 0.298) = 0.0564$. (*Note: the treatment of mergers* M_2 *whose change in*

[16] For simplicity, the example considers a case where, contrary to the assumptions of our model, one of the mergers has a finite support of postmerger marginal costs, and our assumptions about the upper and lower bounds of the firms' postmerger cost distributions do not hold. But the same insight would obtain if we perturbed the example so that these assumptions were satisfied. The lower bound for $\overline{c}_2$ ensures that all firms remain active after merger M_2.

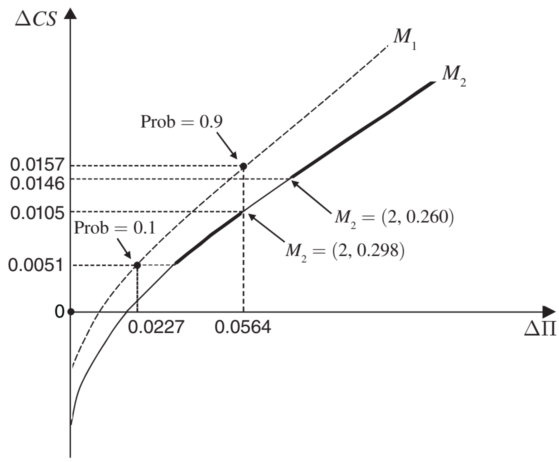

Figure 7. The Optimal Merger Approval Policy in Example 1, which Is Not a Cutoff Policy

bilateral profit is less than 0.0227 *is irrelevant because such mergers will never be proposed). This situation is illustrated in Figure 7. To see why the optimal approval policy for M_2 does not have a cutoff structure, note that for any postmerger marginal cost* $\overline{c}_2 \in (0.260, 0.298)$, *$M_2$ would always be the proposed merger if it were approved when proposed. But the induced change in consumer surplus from M_2 would be less than* 0.0146, *the expected ΔCS from M_1. However, once $\overline{c}_2$ rises just above* 0.298, *merger M_2 would be proposed only if* $M_1 = (1, 0.4)$, *so the expected ΔCS from M_1 conditional on the fact that merger* $(2, \overline{c}_2)$ *was proposed falls to* 0.0051, *and it is optimal to accept M_2. This remains true until $\overline{c}_2$ falls to* 0.391, *where* $\Delta CS(2, 0.391) = 0.0051$.

Nonetheless, our next result provides a sufficient condition that ensures that the recursively defined cutoff policy is in fact optimal. To proceed, let $\mathcal{A}^C(J) \subseteq \Pi_{k \in J}[l, h_k]$ denote the recursively defined cutoff policy when only mergers with targets in set $J \subseteq \mathcal{K}$ are possible; that is, when we suppose that there is no possibility for a merger with any target $k \notin J$. [The policy $\mathcal{A}^C(J)$ specifies $\#J$ cutoffs.] For convenience, when $J = \mathcal{K}$ we write $\mathcal{A}^C \equiv \mathcal{A}^C(\mathcal{K})$. We also let $\overline{a}_k^C(J)$ denote the cutoff level of marginal cost for a merger with target k in cutoff policy $\mathcal{A}^C(J)$ (defined for mergers accepted with positive probability).

In addition, for a set of targets $J \subseteq \mathcal{K}$, define the realized set of feasible mergers to be $\mathcal{F}_J$, and (recalling that $\Delta\Pi(M^\circ) \equiv 0$) define the function

$$ECS(\overline{\Delta\Pi}; \mathcal{A}, J) \equiv E_{\mathcal{F}_J}[\Delta CS(M^*(\mathcal{F}_J, \mathcal{A})) \mid \Delta\Pi(M^*(\mathcal{F}_J, \mathcal{A})) \leq \overline{\Delta\Pi}]$$

as the expected value of ΔCS under policy $\mathcal{A} \subseteq \Pi_{k \in J}[l, h_k]$ from the most profitable acceptable merger involving targets in set J, conditional on that merger's increase in bilateral profit being no greater than $\overline{\Delta\Pi}$.[17] Note that the structure of $\mathcal{A}$ at profit

[17] Thus, $E_k^{\mathcal{A}}(\overline{c}_k) = ECS(\Delta\Pi(k, \overline{c}_k); \mathcal{A}_{\mathcal{K}\setminus k}, \mathcal{K}\setminus k)$ where $\mathcal{A}_{\mathcal{K}\setminus k} \equiv \Pi_{j \in \mathcal{K}\setminus k} \mathcal{A}_j$.

levels above $\overline{\Delta\Pi}$ affects the value of this conditional expectation by changing the conditional distributions of postmerger marginal costs. Specifically, the probability of some merger M_j in set $\mathcal{M}_j \subseteq \{M_j : \Delta\Pi(M_j) \leq \overline{\Delta\Pi}\}$ being feasible conditional on the most profitable acceptable merger having a profit level below $\overline{\Delta\Pi}$ is $\Pr(\phi_j = 1$ and $M_j \in \mathcal{M}_j)/\big(1 - \Pr(\phi_j = 1, \Delta\Pi(M_j) > \overline{\Delta\Pi}$, and $M_j \in \mathcal{A}_j)\big)$.

We then have the following result (whose proof is in the online Appendix):

PROPOSITION 2: *Suppose that for every $J \subseteq \mathcal{K}$ with $1 \in J$ the following property holds:*[18]

(12) *Every merger* $M_k = (k, \overline{c}_k) \in \mathcal{A}^C(J)$ *has* $\Delta CS(M_k) > ECS(\Delta\Pi(M_k); \mathcal{A}^C(J\backslash k), J\backslash k)$.

Then, the recursively defined cutoff policy $\mathcal{A}^C(\cdot)$ is an optimal policy.

While Proposition 2 does not offer a condition on primitives, it allows us to verify that the recursively derived cutoff policy is optimal. The result says that this cutoff policy is an optimal policy provided that the consumer surplus change $\Delta CS(M_k)$ of each merger M_k it approves exceeds the expected ΔCS of the next-most profitable acceptable merger in policy $\mathcal{A}^C(J\backslash k)$, *the recursively defined cutoff policy for each possible set of alternative mergers $J\backslash k$* that includes merger M_1. The following example illustrates its use.

EXAMPLE 2: *Consider the same four-firm industry as in Example* 1, *but now firm* 0 *can merge with each of the other firms* (*so* $K = N = 3$). *For this industry, the naive policy marginal cost cutoffs* (*where any CS-nondecreasing merger is accepted*) *are* $\overline{a}_1^N = 0.45$, $\overline{a}_2^N = 0.40$, $\overline{a}_3^N = 0.35$. *Suppose that each merger has a* 3/4 *probability of being feasible* (*so* $\theta_k = 0.75$ *for* $k = 1,2,3$) *and that, conditional on being feasible, the postmerger marginal cost is drawn from a beta distribution with parameters* $\beta = 1$ *and* $\alpha = 5$ *and support between the merger's naive cutoff and* 0.2.[19] *With these distributions, the merger process would increase expected consumer surplus by* 6.44 *percent if there were no informational asymmetry between the firms and the antitrust authority* (*so that whichever CS-nondecreasing merger most increased consumer surplus would always be implemented*). *The cutoffs in the recursively defined cutoff policy are* $\overline{a}_1 = 0.45$, $\overline{a}_2 = 0.383$, *and* $\overline{a}_3 = 0.316$, *with associated changes in consumer surplus of* $\underline{\Delta CS}_1 = 0$, $\underline{\Delta CS}_2 = 0.00170$, *and* $\underline{\Delta CS}_3 = 0.00346$. *This policy achieves* 90.30 *percent of the first-best increase in expected consumer surplus, while the naive policy which accepts all CS-nondecreasing mergers achieves* 79.83 *percent of this amount. One can verify* (*through computation*) *that the sufficient condition of Proposition* 2 *is satisfied in this case. For example, to check condition* (12) *for merger* M_2, *first for* $J\backslash k = \{1\}$ *and then for* $J\backslash k = \{1,3\}$, *we need to*

[18] By Corollary 1, property (12) necessarily holds for $k = 1$; the assumption made here is that it holds for all $k > 1$.

[19] One can think of this situation as having a 1/4 probability of there being no CS-increasing merger, and a 3/4 probability of a CS-increasing merger. The beta distribution has a pdf $f(x\,|\,\alpha,\beta)$ that is proportional to $x^{\alpha-1}(1-x)^{\beta-1}$. Its mean is the lower bound of its support plus a fraction $\alpha/(\alpha+\beta)$ of the difference between its support's upper and lower bounds. When $\beta = 1$ and $\alpha = 5$, the pdf is an increasing function, so that small efficiency gains are more likely than large ones. The lower bound of $l = 0.2$ ensures that all firms remain active after any merger.

compare the consumer surplus level $\Delta CS(M_2)$ for each merger M_2 with $\overline{c}_2 < 0.383$ to the conditional expectation $ECS(\Delta\Pi(M_2); \mathcal{A}^C(J\backslash k), J\backslash k)$ of the consumer surplus in the next-most profitable merger that is acceptable in the recursively defined cutoff policy $\mathcal{A}^C(J\backslash k)$. For $J\backslash k = \{1,3\}$, the policy $\mathcal{A}^C(\{1,3\})$ has cutoff levels $\overline{a}_1^C(\{1,3\}) = 0.45$ and $\overline{a}_3^C(\{1,3\}) = 0.323$. Looking at two specific mergers M_2 in this case, for $M_2 = (2, 0.35)$ we have $\Delta CS(M_2) = 0.0051$ and $ECS(\Delta\Pi(M_2); \mathcal{A}^C(\{1,3\}), \{1,3\}) = 0.0025$, while for $M_2 = (2, 0.3)$ we have $\Delta CS(M_2) = 0.0103$ and $ECS(\Delta\Pi(M_2); \mathcal{A}^C(\{1,3\}), \{1,3\}) = 0.0034$.[20] *In both cases, condition (12) is satisfied.*

When cutoff rules are optimal we can also explore how changes in underlying parameters alter the nature of the optimal policy. Here we provide a result (proof in the online Appendix) on the effect of changes in the merger feasibility probabilities θ_k on the optimal policy, assuming that the optimal policy has a cutoff structure. Intuitively, lower feasibility probabilities should move the optimal policy toward the naive one. For example, as all θ_ks approach zero, the optimal policy approaches the naive policy: since there is almost no chance that two mergers are feasible, firms almost never have a choice of which merger to propose. Our result builds on this intuition:

PROPOSITION 3: *Consider an increase in the probability of merger M_k's feasibility from θ_k to $\theta_k' > \theta_k$, assuming that M_k is initially approved with positive probability (i.e., $k \leq \hat{K}$). Then, under the optimal merger approval policy, $\underline{\Delta CS}_j' = \underline{\Delta CS}_j$ for any weakly smaller merger M_j, $j \leq k$, and $\underline{\Delta CS}_j' > \underline{\Delta CS}_j$ for any larger merger M_j, $j > k$, that is approved with positive probability.*

V. Conclusion

In this paper, we have analyzed the optimal merger approval policy of an antitrust authority which seeks to maximize expected consumer surplus when there are several mutually exclusive merger possibilities and firms can choose *which* merger to propose. In our model, there is a single acquirer that can make a merger proposal to one of several, ex ante heterogeneous merger partners. While the feasibility and postmerger marginal costs of the various potential mergers is not known to the antitrust authority, the antitrust authority can observe the characteristics of the proposed merger. We have shown that in this environment the antitrust authority optimally commits to a policy that imposes a tougher standard on mergers involving firms with a larger premerger market share, or, equivalently in our model, inducing a larger increase in the naively computed Herfindahl index: the required minimum increase in consumer surplus is greater for mergers that are larger in this sense. The form of this optimal policy is a response to a fundamental bias that we have shown exists in firms' proposal incentives: larger mergers are sometimes proposed when smaller ones that would lead to greater increases in consumer surplus are available, while the reverse never happens. The optimal policy therefore rejects some consumer surplus-enhancing larger mergers to induce firms to propose better smaller ones.

[20] For postmerger costs $\overline{c}_2 > 0.339$, the next-most profitable acceptable merger in $\mathcal{A}^C(\{1,3\})$ must be M_1 or no merger, while for $\overline{c}_2 < 0.339$ it could also be M_3.

Our model and result also suggest some interesting questions for future research. For example, while in our model premerger marginal costs are taken to be exogenous, in practice they are likely to be the result of investments by the firms. To the extent that merger policy depends on premerger costs or market shares, an optimal policy needs to take account of any effects on these investments. A second issue concerns remedies. In practice, many mergers are approved subject to some remedy that is designed to improve consumer welfare, and which presumably lowers the profits of the merged firm. While imposing remedies may improve consumer welfare ex post (shifting the merger to the Northwest in $(\Delta\Pi, \Delta CS)$-space), our model suggests that insisting on remedies could also lead to changes in the mergers that firms propose. Given this fact, an interesting question is therefore the extent to which it is optimal to impose remedies as a requirement to gain approval. Finally, analyzing optimal merger policy with merger choice in a setting with a richer structure of merger possibilities than in the present model – in which all mergers involve a single pivotal firm—is an important direction for future work.

Appendix

We first state a simple but useful corollary of Lemmas 2 and 3:

COROLLARY 1: *If two CS-nondecreasing mergers M_j and M_k with $k > j$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta CS(M_k) < \Delta CS(M_j)$.*

PROOF:

Suppose instead that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\overline{c}\,'_k > \overline{c}_k$ such that $\Delta CS(k, \overline{c}\,'_k) = \Delta CS(M_j)$. But this implies (using Lemma 2 for the first inequality and Lemma 3 for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \overline{c}\,'_k) > \Delta\Pi(M_j)$, a contradiction.

LEMMA 4: *Suppose two mergers, M_j and M_k, with $k > j$, induce the same nonnegative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then, the larger merger M_k induces a greater increase in aggregate profit, i.e., $\Delta\Pi_I(M_k) > \Delta\Pi_I(M_j)$.*

PROOF:

From the discussion in the main text, the postmerger aggregate profit is given by (10). As both mergers induce the same level of consumer surplus (and thus the same Q), the first term on the right-hand side of (10) is the same for both mergers. It thus suffices to show that the larger merger M_k induces a larger value of H than the smaller merger M_j.

Now, as both mergers induce the same Q, Assumption 1 implies that the output of any firm not involved in M_j or M_k is the same under both mergers. Hence,

$$s_k(M_k) + s_j(M_k) = s_k(M_j) + s_j(M_j). \tag{A1}$$

Next, recall that a CS-nondecreasing merger increases the share of the merging firms and reduces the share of all nonmerging firms. Thus, we have $s_k(M_k) \geq s_k + s_0 > s_k(M_j)$ and $s_j(M_j) \geq s_j + s_0 > s_j(M_k)$. In addition, since total output is the same after both mergers and $c_k < c_j$, we also have $s_j(M_k) < s_k(M_j)$. By (A1), this in turn implies that $s_k(M_k) > s_j(M_j)$. Hence, the distribution of market shares after the larger merger M_k is a sum-preserving spread of those after the smaller merger M_j:

$$s_k(M_k) > \max\{s_j(M_j),\ s_k(M_j)\} \geq \min\{s_j(M_j), s_k(M_j)\} > s_j(M_k). \tag{A2}$$

Given (A1), it follows that $s_k(M_k)^2 + s_j(M_k)^2 > s_k(M_j)^2 + s_j(M_j)^2$. Since $s_i(M_k) = s_i(M_j)$ for all $i \neq 0, j, k$, this implies that H is larger after M_k than after M_j.

PROOF OF PROPOSITION 1:

The proof proceeds in a number of steps.

Step 1: We observe first that an optimal policy does not approve CS-decreasing mergers. That is, $\underline{\Delta CS}_k \geq 0$ for all $k \in \mathcal{K}^+$, where $\mathcal{K}^+$ denotes those targets for whom the probability of having a merger $M_k \in \mathcal{A}$ is strictly positive. To see this, suppose the approval set $\mathcal{A}$ includes CS-decreasing mergers, and consider the set $\mathcal{A}^+ \subseteq \mathcal{A}$ that removes any mergers in $\mathcal{A}$ that reduce consumer surplus. Since this change matters only when the bilateral profit-maximizing merger $M^*(\mathcal{F}, \mathcal{A})$ under set $\mathcal{A}$ is no longer approved under $\mathcal{A}^+$, the change in expected consumer surplus from this change in the approval policy equals $\Pr(M^*(\mathcal{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+)$, the probability of this event happening, times the conditional expectation

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A}^+)) - \Delta CS(M^*(\mathcal{F}, \mathcal{A})) \,|\, M^*(\mathcal{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+].$$

Since $\Delta CS(M^*(\mathcal{F}, \mathcal{A}^+))$ is necessarily nonnegative by construction of $\mathcal{A}^+$, and $\Delta CS(M^*(\mathcal{F}, \mathcal{A}))$ is strictly negative whenever $M^*(\mathcal{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+$, this change is strictly positive.

Step 2: Next, any smallest merger M_1 that is CS nondecreasing must be approved. To see this, suppose that the approval set is $\mathcal{A}$ but that $\mathcal{A} \subset \mathcal{A}' \equiv (\mathcal{A} \cup \{(1, \overline{c}_1) : \Delta CS(1, \overline{c}_1) \geq 0\})$. Figure 3 depicts two such sets, $\mathcal{A}$ and $\mathcal{A}'$. Because a change from $\mathcal{A}'$ to $\mathcal{A}$ matters only when the bilateral profit-maximizing merger $M^*(\mathcal{F}, \mathcal{A}')$ under $\mathcal{A}'$ is no longer approved under $\mathcal{A}$, the change in expected consumer surplus by using $\mathcal{A}'$ rather than $\mathcal{A}$ equals $\Pr(M^*(\mathcal{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A})$ times

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A}')) - \Delta CS(M^*(\mathcal{F}, \mathcal{A})) \,|\, M^*(\mathcal{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}]. \tag{A3}$$

By Corollary 1 and the fact that $\mathcal{A}' \backslash \mathcal{A}$ contains only smallest mergers (between firms 0 and 1), whenever $M^*(\mathcal{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}$ [which implies $\Delta\Pi(M^*(\mathcal{F}, \mathcal{A}')) > \Delta\Pi(M^*(\mathcal{F}, \mathcal{A}))$] we have $\Delta CS(M^*(\mathcal{F}, \mathcal{A}')) > \Delta CS(M^*(\mathcal{F}, \mathcal{A}))$, so (A3) is strictly positive. This can be seen in Figure 3 and implies in particular that $\underline{\Delta CS}_1 = 0$.

Step 3: Next, we claim that in any optimal policy, for all $k \in \mathcal{K}^+$, $\underline{\Delta CS}_k$ must equal the expected change in consumer surplus from the next-most profitable merger (i.e., from the merger with the second-highest bilateral profit change) $M^*(\mathcal{F} \backslash (k, \overline{a}_k), \mathcal{A})$, conditional on merger $M_k = (k, \overline{a}_k)$ being the most profitable merger in $\mathcal{F} \cap \mathcal{A}$. Defining the expected change in consumer surplus from the next-most profitable merger $M^*(\mathcal{F}\backslash M_k, \mathcal{A})$, conditional on merger $M_k = (k, \overline{c}_k)$ being the most profitable merger in $\mathcal{F} \cap \mathcal{A}$, to be

$$E_k^{\mathcal{A}}(\overline{c}_k) \equiv E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \,|\, M_k = (k, \overline{c}_k) \text{ and } M_k = M^*(\mathcal{F}, \mathcal{A})] \tag{A4}$$

$$= E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \,|\, M_k = (k, \overline{c}_k) \tag{A5}$$

$$\text{and } \Delta\Pi(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \leq \Delta\Pi(M_k)],$$

this means that

$$\underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k). \tag{A6}$$

In Figure 4 the possible locations of the next-most profitable merger when the most profitable merger is $M_2 = (2, \overline{a}_2)$ are shown as a shaded set. The quantity $E_2^{\mathcal{A}}(\overline{a}_2)$ is the expectation of the change in consumer surplus for the merger that has the largest change in bilateral profit among mergers other than M_2, conditional on all of these other mergers lying in the shaded region of the figure.

To see that (A6) must hold for all $k \in \mathcal{K}^+$, suppose first that $\underline{\Delta CS}_{k'} > E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$ for some $k' \in \mathcal{K}^+$ and consider the alternative approval set $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ where

$$\mathcal{A}_{k'}^{\varepsilon} \equiv \{M_k : M_k = (k', \overline{c}_{k'}) \text{ with } \overline{c}_{k'} \in (\overline{a}_{k'}, \overline{a}_{k'} + \epsilon)\}.$$

(By Step 1 and Assumption 2, we have $\overline{a}_{k'} < h_{k'}$, implying that $\overline{a}_{k'} + \varepsilon < h_{k'}$ for $\varepsilon > 0$ sufficiently small.) For any $\varepsilon > 0$, the change in expected consumer surplus from changing the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ equals $\Pr(M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon})$ times

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - \Delta CS(M^*(\mathcal{F}, \mathcal{A})) \,|\, M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}]. \tag{A7}$$

This conditional expectation can be rewritten as

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - E_{k'}^{\mathcal{A}}(\overline{c}_{k'}) \,|\, M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}], \tag{A8}$$

where $\overline{c}_{k'}$ is the realized cost level in the bilateral profit-maximizing merger $M^*(\mathcal{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. By continuity of $\Delta CS(k', \overline{c}_{k'})$ and $E_k^{\mathcal{A}}(\overline{c}_{k'})$ in $\overline{c}_{k'}$, there exists an $\overline{\varepsilon} > 0$ such that $\Delta CS(M_{k'}) > E_k^{\mathcal{A}}(\overline{c}_{k'})$ for all $M_{k'} \in \mathcal{A}_{k'}^{\varepsilon}$ provided $\varepsilon \in (0, \overline{\varepsilon}]$. For all such ε, the conditional expectation (A8) is strictly positive so this change in the approval set would strictly increase expected consumer surplus. A similar argument applies if $\underline{\Delta CS}_{k'} < E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$.

Step 4: Next, we argue that for all $j < k$ such that $j, k \in \mathcal{K}^+$ it must be that $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_k$; that is, the bilateral profit change in the marginal merger by target j must be no greater than the bilateral profit change in the marginal merger by any larger target k. Figure 5A shows a situation that violates this condition, where the marginal merger by target 3 causes a smaller bilateral profit change, $\underline{\Delta\Pi}_3$, than the marginal merger by the smaller target 2, $\underline{\Delta\Pi}_2$.

For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k \in \mathcal{K}^+, k > j} \underline{\Delta\Pi}_k$ and suppose that $\underline{\Delta\Pi}_{k'} < \underline{\Delta\Pi}_j$. We know from the previous step that $\underline{\Delta CS}_{k'} = E^{\mathcal{A}}_{k'}(\overline{a}_{k'})$. Let $\overline{c}'_j$ be the postmerger cost level satisfying $\Delta\Pi(j, \overline{c}'_j) = \underline{\Delta\Pi}_{k'}$ and consider a change in the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \overline{\mathcal{A}}^{\varepsilon}_j$ where

$$\overline{\mathcal{A}}^{\varepsilon}_j \equiv \{M_j : M_j = (j, \overline{c}_j) \text{ with } \overline{c}_j \in (\overline{c}'_j, \overline{c}'_j + \epsilon)\}.$$

The set $\overline{\mathcal{A}}^{\varepsilon}_j$ is shown in Figure 5B. The change in expected consumer surplus from this change in the approval set equals $\Pr\big(M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}^{\varepsilon}_j) \in \overline{\mathcal{A}}^{\varepsilon}_j\big)$ times

$$\text{(A9)} \qquad E_{\mathcal{F}}\big[\Delta CS\big(M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}^{\varepsilon}_j)\big) - E^{\mathcal{A}}_j(\overline{c}_j) \,|\, M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}^{\varepsilon}_j) \in \overline{\mathcal{A}}^{\varepsilon}_j\big],$$

where $\overline{c}_j$ is the realized cost level in the aggregate profit-maximizing merger $M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}^{\varepsilon}_j)$, which is a merger of firms 0 and j when the conditioning statement is satisfied. As $\varepsilon \to 0$, the expected change in (A9) converges to

$$\begin{aligned} \Delta CS(j, \overline{c}'_j) - E^{\mathcal{A}}_j(\overline{c}'_j) &= \Delta CS(j, \overline{c}'_j) - E^{\mathcal{A}}_{k'}(\overline{a}_{k'}) \\ &> \underline{\Delta CS}_{k'} - E^{\mathcal{A}}_{k'}(\overline{a}_{k'}) \\ &= 0, \end{aligned}$$

where the inequality follows from Corollary 1 since $\Delta\Pi(j, \overline{c}'_j) = \Delta\Pi_{k'}$.

Step 5: We next argue that $\underline{\Delta CS}_j < \underline{\Delta CS}_k$ for all $j, k \in \mathcal{K}^+$ with $j < k$. Suppose otherwise; i.e., for some $j, h \in \mathcal{K}^+$ with $h > j$ we have $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Define $k = \arg\min\{h \in \mathcal{K}^+ : h > j \text{ and } \underline{\Delta CS}_j \geq \underline{\Delta CS}_h\}$. Figure 6 depicts such a situation where $j = 2$ and $k = 3$.

By Step 3, we must have $E^{\mathcal{A}}_j(\overline{a}_j) = \underline{\Delta CS}_j \geq \underline{\Delta CS}_k = E^{\mathcal{A}}_k(\overline{a}_k)$. But recalling (A5), $E^{\mathcal{A}}_k(\overline{a}_k)$ can be written as a weighted average of two conditional expectations:

$$\text{(A10)} \qquad \begin{aligned} E_{\mathcal{F}}\big[\Delta CS\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \,|\, & M_k = (k, \overline{a}_k), M_k = M^*(\mathcal{F}, \mathcal{A}), \\ & \text{and } \Delta\Pi\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) < \underline{\Delta\Pi}_j\big] \end{aligned}$$

and

$$\text{(A11)} \qquad \begin{aligned} E_{\mathcal{F}}\big[\Delta CS\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \,|\, & M_k = (k, \overline{a}_k), M_k = M^*(\mathcal{F}, \mathcal{A}), \\ & \text{and } \Delta\Pi\big(M^*(\mathcal{F}\backslash M_k, \mathcal{A})\big) \in [\underline{\Delta\Pi}_j, \underline{\Delta\Pi}_k]\big]. \end{aligned}$$

Expectation (A10) conditions on the event that the next-most profitable merger other than $(k, \overline{a}_k)$ induces a bilateral profit change less than $\underline{\Delta\Pi}_j$, the bilateral profit change of merger $(j, \overline{a}_j)$. Since no merger in $\mathcal{A}$ by either target k or j can have such a profit level (since $\underline{\Delta\Pi}_k \geq \underline{\Delta\Pi}_j$ by Step 4), the expectation (A10) must exactly equal

$E_j^{\mathcal{A}}(\overline{a}_j)$. Now consider the expectation (A11). If $\Delta\Pi(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \in [\underline{\Delta\Pi}_j, \underline{\Delta\Pi}_k)$, it could be that (i) $M^*(\mathcal{F}\backslash M_k, \mathcal{A}) = (j, \overline{c}_j)$ for some $\overline{c}_j \leq \overline{a}_j$, or (ii) $M^*(\mathcal{F}\backslash M_k, \mathcal{A}) = (r, \overline{c}_r)$ for some $r < j$, or (iii) $M^*(\mathcal{F}\backslash M_k, \mathcal{A}) = (r, \overline{c}_r)$ for some $r > j$ and $r < k$. Now, in case (i) it is immediate that $\Delta CS(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \geq \underline{CS}_j$, with strict inequality whenever $\overline{c}_j < \overline{a}_j$. In case (ii), the fact that $\Delta\Pi(r, \overline{c}_r) \geq \underline{\Delta\Pi}_j$ implies by Corollary 1 that

$$\Delta CS(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) = \Delta CS(r, \overline{c}_r) > \underline{CS}_j = E_j^{\mathcal{A}}(\overline{a}_j). \tag{A12}$$

In case (iii), (A12) follows from the definition of k. Thus, expectation (A11) must strictly exceed $E_j^{\mathcal{A}}(\overline{a}_j)$, which leads to a contradiction.

Step 6: Finally, we argue that $\mathcal{K}^+ = \{1, \ldots, \hat{K}\}$ for some $\hat{K} \leq K$. To establish this fact, we show that if $k \notin \mathcal{K}^+$ and $k < K$, then $k + 1 \notin \mathcal{K}^+$. As noted in the text, we first observe that $\Delta CS(k, l) > \Delta CS(k+1, l)$. Thus, if $k + 1 \in \mathcal{K}^+$, then $\underline{\Delta CS}_{k+1} < \Delta CS(k, l)$. The result then follows by an argument similar to that in Step 5: By Step 3, $\underline{\Delta CS}_{k+1}$ must equal the expected ΔCS of the next-most profitable allowable merger, i.e., $\underline{\Delta CS}_{k+1} = E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1})$. This expectation is the weighted average of the expected ΔCS in two events: first that the next-most profitable allowable merger, say M', lies to the right of the best possible merger M_k, i.e., $\Delta\Pi(M') \in [\Delta\Pi(k, l), \underline{\Delta\Pi}_{k+1}]$ and, second, that it lies to the left of the best possible merger M_k, i.e., $\Delta\Pi(M') < \Delta\Pi(k, l)$. In the first event (which may be empty), the resulting ΔCS must exceed that of the marginal merger M_{k+1}, i.e., $\Delta CS(M') > \underline{\Delta CS}_{k+1}$. In the second possibility, the expected ΔCS in that event must weakly exceed $\Delta CS(k, l)$, as otherwise (by an argument like that in Step 3) the expected ΔCS could be increased by including all mergers $(k, \overline{c}_k)$ with $\overline{c}_k \in [l, l + \varepsilon]$, for $\varepsilon > 0$ sufficiently small, in the approval set. Taking the weighted average of these two events, it follows that $\underline{\Delta CS}_{k+1} = E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1}) \geq \Delta CS(k, l)$, a contradiction.

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